

Amplify-and-Forward Strategies in the Two-Way Relay Channel With Analog Tx–Rx Beamforming

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Abstract—In this paper, we study the multiple-input–multiple-output two-way relay channel (MIMO-TWRC) when the nodes and relay use analog beamforming. Following the amplify-and-forward (AF) strategy, the problem consists of finding the transmit and receive beamformers of the nodes and relay (as well as the power allocated to each one) that achieve the boundary of the optimal rate region. To solve it, we first express the optimal node beamformers in terms of relay beamformers and then show that the optimal rate region can efficiently be characterized using convex optimization techniques. We also extend our study to the multiple-relay scenario when the source nodes are single antenna and propose a distributed algorithm to compute the relay beamforming matrices. The proposed algorithm consists of solving two different subproblems. First, each individual TWRC is optimized independently. Next, a distributed beamforming is applied to make the signals from the relays add up coherently at the source nodes. Numerical examples are provided to illustrate the performance of the proposed techniques and to compare the performance of analog beamforming architectures against conventional MIMO schemes that operate at the baseband.

Index Terms—Analog beamforming, convex optimization, distributed beamforming, two-way relay channel (TWRC).

I. INTRODUCTION

MULTIPLE-input–multiple-output (MIMO) systems have been shown to achieve high data rates due to the exploitation of spatial diversity with the use of multiple antennas at the transmitters and/or receivers [1]. However, exploiting the full multiplexing gain of the MIMO channel requires a radio-frequency (RF) front end and two analog to digital converters for each antenna to process each data stream at baseband, which implies an increase in system size, cost, and power consumption with respect to single-input–single-output (SISO) systems. These higher costs make the deployment of commercial wireless devices difficult and motivate research on low-complexity MIMO systems, which shift part of the baseband processing to the RF domain. Applying beamforming in the RF

domain (which we call hereafter RF-MIMO) entails acquiring and processing a single data stream, and thus, the cost and power consumption are significantly reduced [2], [3]. Another approach is to consider multiple-output streams that combine analog and digital processing techniques. Applying this idea, an analog preprocessing is carried out to reduce the number of streams, which are digitally processed in a second stage. See, e.g., [4] and [5] and references therein.

For point-to-point links, the design of analog Tx–Rx beamformers for multicarrier transmissions has been thoroughly considered in [6]–[10]. The broadcast channel with orthogonal frequency-division-multiplexing transmissions is considered in [11], and we proposed in [12] an optimal transmission strategy for the RF-MIMO broadcast and multiple-access channels with single-carrier transmissions. However, there are still many open problems on analog beamforming techniques in multiuser scenarios.

On another front, cooperative and multihop communications have been a research topic of interest in recent years due to the coverage extension they provide. The two-way relay channel (TWRC) is one of the most basic multihop communication systems. The simplest TWRC consists of two source nodes that exchange information through an assisting relay node. Different protocols can be followed by the relay node, such as amplify-and-forward (AF) [13], decode-and-forward [14], [15], or compress-and-forward [15]. Hence, only two phases are needed for the exchange of a data frame: 1) a multiple access (MAC) phase and 2) a broadcast (BC) phase.

Currently, the TWRC is receiving great interest, and there are many works on optimal transmission strategies and optimal beamforming for the multiantenna case [13], [14]. In [13], the TWRC with AF strategy is considered when the source nodes are single-antenna terminals and the relay uses conventional beamforming. They compute the optimal beamforming strategy at the relay node via convex optimization techniques with fixed powers. In [17], Wang and Zhang study the conventional MIMO-TWRC and propose a suboptimal method to compute the beamforming matrices. Distributed beamforming in two-way multiple-relay networks with single-antenna relays has been considered in [18] and [19]. In [20], an iterative algorithm for computing the optimal rate region when the relays have multiple antennas is proposed. The multiuser TWRC is studied in [21] for a single multiantenna relay and in [22] for multiple single-antenna relays.

In this paper, we characterize the optimal rate region of the RF-MIMO-TWRC (i.e., the nodes and relay perform analog beamforming). This paper differs from [13] in the sense that

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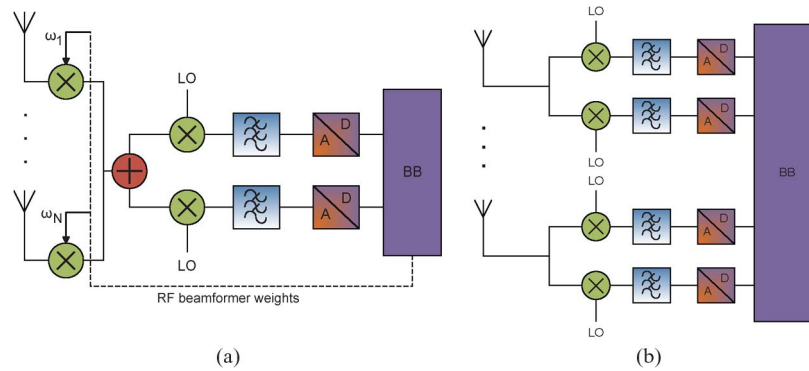


Fig. 1. RF-MIMO (left) and conventional MIMO (right) transceivers. In a RF-MIMO system, a single data stream is processed, and thus, the cost and power consumption are significantly reduced. (a) RF-MIMO architecture. (b) Conventional MIMO architecture.

the RF-MIMO architecture imposes a rank-one constraint in the relay beamforming matrix. Moreover, unlike [13], we optimize the node transmit powers and consider multiantenna nodes. We show that the optimal beamforming strategy and the power allocation problem can be solved through efficient convex optimization techniques. Next, we study beamforming strategies in the two-way multiple-relay channel (TWMRC), when the source nodes are single antenna. The main difference with other works is the rank-one constraint on the relay beamforming matrices imposed by the RF domain spatial processing. Furthermore, unlike [20], we propose a distributed algorithm that avoids the need of global channel state information (CSI). The proposed algorithm is based upon the results derived for the TWRC. We show that, although being suboptimal, it performs very close to the boundary of the optimal rate region. In addition, as we show in [23], it can easily be extended to the conventional MIMO case.

We use the following notation: Bold upper and lower case letters denote matrices and vectors, respectively; light-faced lower case letters denote scalar quantities. We use \mathbf{A}^H , \mathbf{A}^* , and \mathbf{A}^T to denote Hermitian, conjugate, and transpose of \mathbf{A} , respectively; $\text{Tr}(\mathbf{A})$ denotes the trace of \mathbf{A} , and $\text{rank}(\mathbf{A})$ denotes the rank of \mathbf{A} . For vectors, $\|\mathbf{a}\|$ denotes the Euclidean norm of \mathbf{a} ; and for complex scalars, $|a|$ denotes the absolute value of a . The optimal solution of an optimization problem is indicated by $(\cdot)^*$.

II. RADIO-FREQUENCY MULTIPLE-INPUT-MULTIPLE-OUTPUT ARCHITECTURE

In this section, we summarize the RF-MIMO concept and architecture and point out the main differences with respect to conventional MIMO systems that apply spatial processing in the digital or baseband domain. The RF-MIMO architecture is shown in Fig. 1(a). The transmitter (the receiver operates analogously) applies a set of complex weights $\boldsymbol{\omega} = [\omega_1, \dots, \omega_N]^T$, which represent the gain factors and phase shifts at each antenna and focus the energy beam in the proper direction. These complex weights can be applied by means of a variable gain amplifier followed by a phase shifter or by a vector modulator that amplifies inphase and quadrature signal components independently [2]. As the spatial processing is performed in the RF domain, a single RF chain is needed, thus reducing the costs

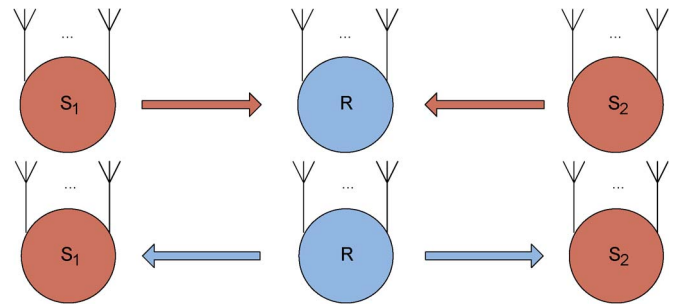


Fig. 2. TWRC with AF strategy and two-phase protocol. In the MAC phase (top), the source nodes send their messages to the relay node, whereas in the BC phase (bottom), the relay node retransmits a linear composition of the received signal.

and power consumption with respect to conventional MIMO architectures, which require a number of RF chains equal to the number of antennas, to perform the spatial processing at baseband, as shown in Fig. 1(b). Consequently, the system complexity can be reduced from $\mathcal{O}(\max(N_T, N_R))$, where N_T and N_R are the number of transmit and receive antennas, respectively, to $\mathcal{O}(1)$, which is a complexity of the same order as a SISO system [2]. On the other hand, the multiplexing gain of the RF-MIMO architecture is limited to one since only one data stream is processed. Nevertheless, we have shown in [24] that they have full diversity and array gains. Moreover, in the low and medium signal-to-noise ratio (SNR) regimes or in highly correlated MIMO channels, the performance improvement of conventional MIMO architectures over RF-MIMO schemes due to spatial multiplexing is negligible.

The channel estimation in RF-MIMO systems can be performed by using orthogonal beamformers at both sides of the link. Hence, the $N_T N_R$ SISO channels are estimated by means of a training phase based on time-division multiple access.

For further details, see [2].

III. SYSTEM MODEL

We consider the TWRC depicted in Fig. 2, where two source nodes equipped with N_S antennas¹ establish a bidirectional communication through a relay node with N_R antennas. The

¹The extension to source nodes with different number of antennas is straightforward.

two multiantenna nodes and the relay perform beamforming in the RF domain, which is called analog beamforming. We use the two-phase TWRC protocol that was also adopted in [13] and [17] and assume perfect CSI at every node. In the MAC phase, both source nodes simultaneously transmit to the relay node. Due to the restrictions of the RF-MIMO architecture, the nodes are able to transmit a single data stream. Then, assuming flat-fading channels, the signal received at the relay node can be written as

$$\mathbf{y}_R = \mathbf{H}_1 \mathbf{v}_1 \sqrt{p_1} s_1 + \mathbf{H}_2 \mathbf{v}_2 \sqrt{p_2} s_2 + \mathbf{r}_R \quad (1)$$

where $(\mathbf{v}_1, \mathbf{v}_2) \in \mathbb{C}^{N_S \times 1}$ are the unit-norm analog transmit beamformers of nodes S_1 and S_2 , respectively; $(\mathbf{H}_1, \mathbf{H}_2) \in \mathbb{C}^{N_R \times N_S}$ are the channel matrices; and s_1 and s_2 are the symbols transmitted by nodes S_1 and S_2 , respectively, which are assumed to be distributed as $\mathcal{CN}(0, 1)$; p_1 and p_2 are the transmit powers of each source node; and $\mathbf{r}_R \sim \mathcal{CN}(0, \sigma^2 \mathbf{I})$ represents the noise at the relay node. In the BC phase, the relay node performs the AF strategy to linearly process the received signal (1). The RF-MIMO architecture, unlike the conventional MIMO scheme studied in [13], imposes a rank-one constraint in the relay beamforming matrix (as the relay is only able to process one data stream). Then, the (analog) relaying matrix can be expressed as

$$\mathbf{B} = \mathbf{v}_R \mathbf{u}_R^H \quad (2)$$

where \mathbf{u}_R and \mathbf{v}_R are the $N_R \times 1$ receive and transmit beamformers, respectively. Notice that, unlike [13], we consider multiple antennas at the nodes and relay.

Without loss of generality, we can assume $\|\mathbf{v}_R\| = 1$ and $\|\mathbf{v}_i\| = 1, i = 1, 2$. Thus, the transmit power of the relay node is given by

$$p_R(p_1, p_2, \mathbf{u}_R, \mathbf{v}_1, \mathbf{v}_2) = p_{\text{ef}_1} + p_{\text{ef}_2} + \sigma^2 \|\mathbf{u}_R\|^2 \quad (3)$$

where we have defined the effective power of S_i as

$$p_{\text{ef}_i} = p_i \left| \mathbf{u}_R^H \mathbf{H}_i \mathbf{v}_i \right|^2, \quad i = 1, 2. \quad (4)$$

If channel reciprocity holds, the signal received by the first source node is

$$y_1 = p_1 \mathbf{u}_1^H \mathbf{H}_1^T \mathbf{v}_R \mathbf{u}_R^H \mathbf{H}_1 \mathbf{v}_1 s_1 + \mathbf{u}_1^H \mathbf{H}_1^T \mathbf{v}_R \sqrt{p_{\text{ef}_2}} s_2 + \tilde{r}_1 \quad (5)$$

where $\mathbf{u}_1 \in \mathbb{C}^{N_S \times 1}$ and $\tilde{r}_1 \sim \mathcal{CN}(0, \sigma^2 [1 + \|\mathbf{u}_R\|^2 |\mathbf{u}_1^H \mathbf{H}_1^T \mathbf{v}_R|^2])$ are the unit-norm analog receive beamformer and the equivalent noise at S_1 , respectively. Notice that the first term on the right-hand side of (5) is the self-interference of S_1 , which is caused by its own transmission. Providing that $\mathbf{u}_1^H \mathbf{H}_1^T \mathbf{v}_R \mathbf{u}_R^H \mathbf{H}_1 \mathbf{v}_1$ is perfectly known at S_1 , the self-interference can be perfectly removed from y_1 . Therefore, the received signal at the nodes, after suppressing the self-interference, is given by

$$y_1 = \mathbf{u}_1^H \mathbf{H}_1^T \mathbf{v}_R \sqrt{p_{\text{ef}_2}} s_2 + \tilde{r}_1 \quad (6)$$

$$y_2 = \mathbf{u}_2^H \mathbf{H}_2^T \mathbf{v}_R \sqrt{p_{\text{ef}_1}} s_1 + \tilde{r}_2. \quad (7)$$

Finally, the achievable bidirectional rate pairs, which are denoted by R_{12} (link from S_1 to S_2 , through the relay node) and R_{21} (link from S_2 to S_1 , through the relay node), are given by

$$R_{12} \leq \frac{1}{2} \log_2 \left[1 + \frac{p_{\text{ef}_1} |\mathbf{u}_2^H \mathbf{H}_2^T \mathbf{v}_R|^2}{\sigma^2 (1 + \|\mathbf{u}_R\|^2 |\mathbf{u}_2^H \mathbf{H}_2^T \mathbf{v}_R|^2)} \right] \quad (8)$$

$$R_{21} \leq \frac{1}{2} \log_2 \left[1 + \frac{p_{\text{ef}_2} |\mathbf{u}_1^H \mathbf{H}_1^T \mathbf{v}_R|^2}{\sigma^2 (1 + \|\mathbf{u}_R\|^2 |\mathbf{u}_1^H \mathbf{H}_1^T \mathbf{v}_R|^2)} \right]. \quad (9)$$

To sum up, the RF-MIMO-TWRC under study differs from the conventional MIMO-TWRC in [13] in the following:

- We consider multiple antennas not only at the relay but at the source nodes as well.
- We optimize the transmit power of the source nodes.
- The RF-MIMO architecture imposes a rank-one relaying matrix, while in [13], the matrix has no rank constraints.

These differences make the problem a nontrivial extension of the algorithm proposed in [13]. In the next section, we derive the optimal beamforming vectors to be applied at the source nodes and define the achievable rate region of the RF-MIMO-TWRC. Finally, we propose an iterative algorithm based upon convex optimization techniques to compute the boundary of the optimal rate region.

IV. OPTIMAL RATE REGION

A. Optimal Node Beamformers

For a fixed transmit beamformer at the relay \mathbf{v}_R , the BC phase reduces to a single-input–multiple-output channel. Thus, the optimal strategy for both nodes is matched filtering with respect to their channels or maximum ratio combining [25]. Therefore, the optimal unit-norm analog receive beamformers are

$$\mathbf{u}_i = \frac{\mathbf{H}_i^T \mathbf{v}_R}{\|\mathbf{H}_i^T \mathbf{v}_R\|}, \quad i = 1, 2. \quad (10)$$

Similarly, for a fixed receive beamforming at the relay node \mathbf{u}_R , the MAC phase reduces to a multiple-input–single-output channel. To achieve the boundary of the optimal rate region, each source node must control its effective power p_{ef_i} . According to (4), this task can be done by varying either the source node power or its transmit beamformer \mathbf{v}_i . Hence, if the source nodes perform maximum ratio transmission (MRT) [25], the effective power can be controlled solely by the source node power p_i . Thus, the MRT is always optimal. The MRT beamformers of the source nodes are given by

$$\mathbf{v}_i = \frac{\mathbf{H}_i^H \mathbf{u}_R}{\|\mathbf{H}_i^H \mathbf{u}_R\|}, \quad i = 1, 2. \quad (11)$$

In summary, the optimal Tx–Rx node beamformers can be written in terms of the Tx–Rx relay beamformers. Notice that (10) and (11) imply that a feedback channel must exist between the relay and the nodes. According to that, the source node

beamformers can be computed at the relay and then sent them back to the source nodes.

With these considerations, the rates in (8) and (9) can be rewritten as

$$R_{12} \leq \frac{1}{2} \log_2 \left[1 + \frac{p_{ef_1} \|\mathbf{H}_2^T \mathbf{v}_R\|^2}{\sigma^2 \left(1 + \|\mathbf{u}_R\|^2 \|\mathbf{H}_2^T \mathbf{v}_R\|^2 \right)} \right] \quad (12)$$

$$R_{21} \leq \frac{1}{2} \log_2 \left[1 + \frac{p_{ef_2} \|\mathbf{H}_1^T \mathbf{v}_R\|^2}{\sigma^2 \left(1 + \|\mathbf{u}_R\|^2 \|\mathbf{H}_1^T \mathbf{v}_R\|^2 \right)} \right] \quad (13)$$

where now, $p_{ef_i} = p_i \|\mathbf{H}_i^H \mathbf{u}_R\|^2$. Including the power constraints at the nodes and the relay, we can now define the achievable rate region of the RF-MIMO-TWRC as

$$\mathcal{R}(P_1, P_2, P_R) \triangleq \bigcup_{\substack{p_1 \leq P_1 \\ p_2 \leq P_2}} \left[\bigcup_{\substack{\|\mathbf{v}_R\|=1 \\ p_R(p_1, p_2, \mathbf{u}_R) \leq P_R}} \{R_{12}, R_{21}\} \right] \quad (14)$$

with R_{12} and R_{21} defined in (12) and (13), respectively. Note that, as we show in [26], the number of variables can be further reduced by expressing the beamformers in terms of an orthogonal basis for the column range of the channels. This parameterization is particularly useful when $N_R > 2N_S$. In this paper, however, we will consider the standard parameterization in terms of the channels and the relay beamformers.

B. Computing the Optimal Rate Region

The achievable rate region in (14) can be characterized by solving a weighted sum-rate maximization problem (WS-Rmax). This approach assigns different weights to source nodes to establish a priority between them. Hence, varying the weights, every point on the rate region boundary can be computed. However, the WSRmax problem is nonconvex [27], and thus, it is very difficult to solve. In [28], an alternative method is proposed to compute the boundary rate tuples of the optimal rate region called rate profile, which was also applied in [13] to the TWRC with single-antenna source nodes. Applying this idea, the rate at each node can be expressed as a fraction of the sum rate, i.e., $[R_{21}, R_{12}]^T = R_{\text{sum}}[\beta, 1 - \beta]^T$, where $[\beta, 1 - \beta]^T$ is the rate profile vector. Since R_{21} and R_{12} monotonically grow with the SNR, we can express the rate profile in terms of an SNR profile as $[\gamma_1, \gamma_2]^T = \gamma_{\text{sum}}[\alpha, 1 - \alpha]^T$, where γ_1 and γ_2 are the SNRs at the receivers of S_1 and S_2 , respectively. Thus, for a fixed value of α between 0 and 1, we can compute a boundary point of the optimal rate region by solving the following optimization problem:

$$\begin{aligned} & \underset{\substack{p_1, p_2, \mathbf{v}_R \\ \mathbf{u}_R, \gamma_{\text{sum}}}}{\text{maximize}} && \gamma_{\text{sum}} \\ \text{subject to :} && \frac{p_1 \|\mathbf{H}_1^H \mathbf{u}_R\|^2 \|\mathbf{H}_2^T \mathbf{v}_R\|^2}{\sigma^2 \left(1 + \|\mathbf{u}_R\|^2 \|\mathbf{H}_2^T \mathbf{v}_R\|^2 \right)} \geq (1 - \alpha) \gamma_{\text{sum}} \\ && \frac{p_2 \|\mathbf{H}_2^H \mathbf{u}_R\|^2 \|\mathbf{H}_1^T \mathbf{v}_R\|^2}{\sigma^2 \left(1 + \|\mathbf{u}_R\|^2 \|\mathbf{H}_1^T \mathbf{v}_R\|^2 \right)} \geq \alpha \gamma_{\text{sum}} \end{aligned}$$

$$\begin{aligned} & p_1 \|\mathbf{H}_1^H \mathbf{u}_R\|^2 + p_2 \|\mathbf{H}_2^H \mathbf{u}_R\|^2 + \sigma^2 \|\mathbf{u}_R\|^2 \leq P_R \\ & \|\mathbf{v}_R\| = 1 \\ & p_1 \leq P_1 \\ & p_2 \leq P_2. \end{aligned} \quad (15)$$

For a fixed value of γ_{sum} , the foregoing problem is equivalent to a power minimization problem with SNR constraints as follows:

$$\begin{aligned} & \underset{p_1, p_2, \mathbf{v}_R, \mathbf{u}_R}{\text{minimize}} && p_1 \|\mathbf{H}_1^H \mathbf{u}_R\|^2 + p_2 \|\mathbf{H}_2^H \mathbf{u}_R\|^2 + \sigma^2 \|\mathbf{u}_R\|^2 \\ \text{subject to :} && \frac{p_1 \|\mathbf{H}_1^H \mathbf{u}_R\|^2 \|\mathbf{H}_2^T \mathbf{v}_R\|^2}{\sigma^2 \left(1 + \|\mathbf{u}_R\|^2 \|\mathbf{H}_2^T \mathbf{v}_R\|^2 \right)} \geq (1 - \alpha) \gamma_{\text{sum}} \\ && \frac{p_2 \|\mathbf{H}_2^H \mathbf{u}_R\|^2 \|\mathbf{H}_1^T \mathbf{v}_R\|^2}{\sigma^2 \left(1 + \|\mathbf{u}_R\|^2 \|\mathbf{H}_1^T \mathbf{v}_R\|^2 \right)} \geq \alpha \gamma_{\text{sum}} \\ && \|\mathbf{v}_R\| = 1 \\ && p_1 \leq P_1 \\ && p_2 \leq P_2. \end{aligned} \quad (16)$$

The solution of the foregoing problem provides a feasible point of the optimal rate region if and only if $p_R^* \leq P_R$, where p_R^* is the optimal power value. It turns out that the boundary of the optimal rate region can be obtained by a bisection method over γ_{sum} , solving problem (16) in each step, as indicated in Algorithm 1.

Algorithm 1 Algorithm for computing one point of the optimal rate region boundary of the RF-MIMO-TWRC

Select the weight $0 \leq \alpha_0 \leq 1$ and the desired tolerance, δ .

Initialize: $\gamma_{\text{sum}}^{\min} = 0$ and $\gamma_{\text{sum}}^{\max} = \gamma_{\text{sum}}^{\text{UB}}$.

repeat

$$\gamma_{\text{sum}} = (1/2)(\gamma_{\text{sum}}^{\min} + \gamma_{\text{sum}}^{\max}).$$

Solve problem (16) for $\alpha = \alpha_0$.

if $p_R^* \leq P_R$ **then**

$$\gamma_{\text{sum}}^{\min} = \gamma_{\text{sum}}.$$

else

$$\gamma_{\text{sum}}^{\max} = \gamma_{\text{sum}}.$$

end if

until $(\gamma_{\text{sum}}^{\max} - \gamma_{\text{sum}}^{\min}) \leq \delta$.

A reasonable value of the SNR upper bound $\gamma_{\text{sum}}^{\text{UB}}$ can be obtained through the upper bound derived in [13]. This upper bound is computed considering the optimal beamforming of both links independently. In the analog beamforming case, the optimal strategy at the relay node is transmitting and receiving through the principal eigenvectors of the channels. Hence, an SNR upper bound for the RF-MIMO-TWRC is

$$\gamma_{\text{sum}}^{\text{UB}} = P_R \sigma_1^2 \sigma_2^2 \left(\frac{P_2}{P_R \sigma_1^2 + P_2 \sigma_2^2 + \frac{1}{2}} + \frac{P_1}{P_R \sigma_2^2 + P_1 \sigma_1^2 + \frac{1}{2}} \right) \quad (17)$$

where σ_i is the largest singular value of \mathbf{H}_i , $i = 1, 2$.

The optimization problem (16) is nonconvex, but a solution can be found through a relaxed semidefinite program (SDP), as we show in the next section.

C. Semidefinite Relaxation

Defining the Hermitian rank-one matrices \mathbf{V}_R and \mathbf{U}_R as

$$\begin{aligned}\mathbf{U}_R &= \mathbf{u}_R \mathbf{u}_R^H \\ \mathbf{V}_R &= \mathbf{v}_R \mathbf{v}_R^H\end{aligned}\quad (18)$$

the optimization problem (16) can be written as

$$\begin{aligned}\underset{p_1, p_2, \mathbf{U}_R, \mathbf{V}_R}{\text{minimize}} \quad & p_1 \text{Tr}(\mathbf{R}_1 \mathbf{U}_R) + p_2 \text{Tr}(\mathbf{R}_2 \mathbf{U}_R) + \sigma^2 \text{Tr}(\mathbf{U}_R) \\ \text{subject to:} \quad & p_1 \text{Tr}(\mathbf{R}_1 \mathbf{U}_R) - (1 - \alpha) \gamma_{\text{sum}} \sigma^2 \text{Tr}(\mathbf{U}_R) \\ & \geq \frac{(1 - \alpha) \gamma_{\text{sum}} \sigma^2}{\text{Tr}(\mathbf{R}_2^* \mathbf{V}_R)} \\ & p_2 \text{Tr}(\mathbf{R}_2 \mathbf{U}_R) - \alpha \gamma_{\text{sum}} \sigma^2 \text{Tr}(\mathbf{U}_R) \\ & \geq \frac{\alpha \gamma_{\text{sum}} \sigma^2}{\text{Tr}(\mathbf{R}_1^* \mathbf{V}_R)} \\ & \text{Tr}(\mathbf{V}_R) = 1 \\ & \mathbf{V}_R \succeq 0 \\ & \mathbf{U}_R \succeq 0 \\ & \text{rank}(\mathbf{V}_R) = 1 \\ & \text{rank}(\mathbf{U}_R) = 1 \\ & p_1 \leq P_1 \\ & p_2 \leq P_2\end{aligned}\quad (19)$$

where $\mathbf{R}_i = \mathbf{H}_i \mathbf{H}_i^H$. The foregoing problem can be shown to be still nonconvex due to several reasons. First, the cross products between \mathbf{U}_R and the power variables (p_1 and p_2) make the first two constraints nonconvex. However, we can overcome it by optimizing the effective powers instead. On the other hand, the rank-one constraints are nonconvex, although we can find a solution relaxing these two constraints, which is called a relaxed SDP. With these considerations, the relaxed SDP of (19) is given by

$$\begin{aligned}\underset{\substack{p_{\text{ef}1}, p_{\text{ef}2} \\ \mathbf{U}_R, \mathbf{V}_R}}{\text{minimize}} \quad & p_{\text{ef}1} + p_{\text{ef}2} + \sigma^2 \text{Tr}(\mathbf{U}_R) \\ \text{subject to:} \quad & p_{\text{ef}1} - (1 - \alpha) \gamma_{\text{sum}} \sigma^2 \text{Tr}(\mathbf{U}_R) \geq \frac{(1 - \alpha) \gamma_{\text{sum}} \sigma^2}{\text{Tr}(\mathbf{R}_2^* \mathbf{V}_R)} \\ & p_{\text{ef}2} - \alpha \gamma_{\text{sum}} \sigma^2 \text{Tr}(\mathbf{U}_R) \geq \frac{\alpha \gamma_{\text{sum}} \sigma^2}{\text{Tr}(\mathbf{R}_1^* \mathbf{V}_R)} \\ & \text{Tr}(\mathbf{V}_R) = 1 \\ & \mathbf{V}_R \succeq 0 \\ & \mathbf{U}_R \succeq 0 \\ & p_{\text{ef}1} \leq P_1 \text{Tr}(\mathbf{R}_1 \mathbf{U}_R) \\ & p_{\text{ef}2} \leq P_2 \text{Tr}(\mathbf{R}_2 \mathbf{U}_R).\end{aligned}\quad (20)$$

Note that the last two inequalities are the power constraints of the source nodes, according to the definition of the effective powers in (4). Notice also that the right-hand side of the two first constraints is a particular case of the quadratic-over-linear convex function [27]. Consequently, the foregoing problem is

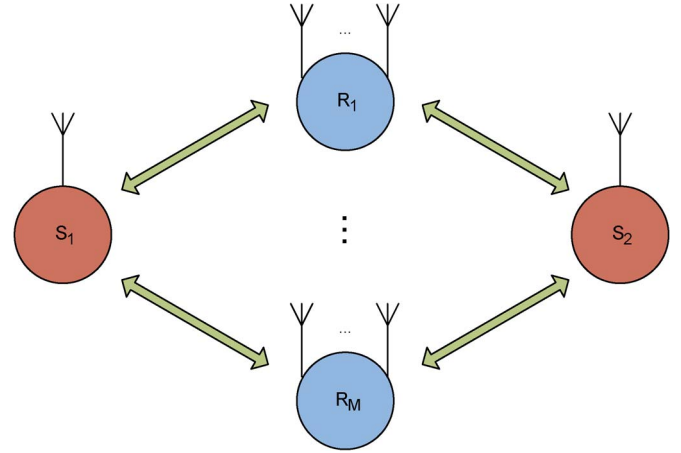


Fig. 3. TWMRC.

convex and can be efficiently solved. Following the lines in [29] and [30], in Appendix A, it is shown that the relaxed problem (20) always admits a rank-one solution, which obviously solves the original nonconvex problem (16), and which can be found, from any other solution of (20), by means of the algorithm presented in [30]. Given the rank-one solution of (20), the optimal powers assigned to each source node p_1^* and p_2^* are given by

$$\begin{aligned}p_1^* &= \frac{p_{\text{ef}1}^*}{\text{Tr}(\mathbf{R}_1 \mathbf{U}_R^*)} \\ p_2^* &= \frac{p_{\text{ef}2}^*}{\text{Tr}(\mathbf{R}_2 \mathbf{U}_R^*)}.\end{aligned}\quad (21)$$

Note that, in contrast to [13], we also optimize the power of each source node and thus completely characterize the achievable rate region of the RF-MIMO-TWRC without resorting to an exhaustive search on the transmit powers.

V. EXTENSION TO THE TWO-WAY MULTIPLE RELAY CHANNEL

In this section, we study analog beamforming strategies for the TWMRC (RF-MIMO-TWMRC), i.e., a TWRC where the source nodes exchange information through the assistance of M RF-MIMO relay nodes, as shown in Fig. 3. Due to the complexity of this problem, we consider a scenario where the source nodes are single-antenna terminals with fixed transmission power, i.e., $N_S = 1$, $p_1 = P_1$ and $p_2 = P_2$, whereas the relays are RF-MIMO nodes equipped with $N_R = N$ antennas each.² Unlike the existing algorithms for computing the optimal rate region of the conventional MIMO-TWMRC [20], we derive in this section an algorithm that significantly reduces the feedback among relays; hence, it is very interesting from a practical point of view. Moreover, we show that the proposed algorithm performs very close to the optimal rate region.

Considering the sum-power constraint, i.e., $\sum_{i=1}^M p(\mathbf{B}_i) \leq P_R$, where $p(\mathbf{B}_i)$ is the power transmitted by the i th relay,

²The extension to relays with different number of antennas is straightforward.

the achievable rate region of the RF-MIMO-TWRC can be expressed as

$$\mathcal{R}(P_1, P_2, P_R) \triangleq \bigcup_{\mathbf{B}_1, \dots, \mathbf{B}_M: \sum_{i=1}^M p(\mathbf{B}_i) \leq P_R} \{R_{12}, R_{21}\} \quad (22)$$

where R_{12} and R_{21} are now given by

$$R_{12} \leq \frac{1}{2} \log_2 \left[1 + \frac{P_1 \left| \sum_{i=1}^M \mathbf{h}_{1i}^T \mathbf{B}_i \mathbf{h}_{2i} \right|^2}{\sigma^2 \sum_{i=1}^M \left(1 + \|\mathbf{B}_i^H \mathbf{h}_{2i}^*\|^2 \right)} \right] \quad (23)$$

$$R_{21} \leq \frac{1}{2} \log_2 \left[1 + \frac{P_2 \left| \sum_{i=1}^M \mathbf{h}_{2i}^T \mathbf{B}_i \mathbf{h}_{1i} \right|^2}{\sigma^2 \sum_{i=1}^M \left(1 + \|\mathbf{B}_i^H \mathbf{h}_{1i}^*\|^2 \right)} \right] \quad (24)$$

$$p(\mathbf{B}_i) = P_1 \|\mathbf{B}_i \mathbf{h}_{1i}\|^2 + P_2 \|\mathbf{B}_i \mathbf{h}_{2i}\|^2 + \sigma^2 \text{Tr}(\mathbf{B}_i^H \mathbf{B}_i) \quad (25)$$

where $(\mathbf{h}_{1i}, \mathbf{h}_{2i}) \in \mathbb{C}^{N \times 1}$ are the channel vectors from source nodes S_1 and S_2 , respectively, to the i th relay.

A. Relay Beamforming Matrices

One of the key ideas of the proposed algorithm follows from the ensuing proposition.

Proposition 1: The bidirectional transmission rates in (23) and (24) can be expressed as

$$R_{12} \leq \frac{1}{2} \log_2 \left(1 + \frac{P_1 |\mathbf{c}_2^H \mathbf{h}_2^{\text{eq}}|^2}{\sigma^2 \|\mathbf{c}_2\|^2} \right) \quad (26)$$

$$R_{21} \leq \frac{1}{2} \log_2 \left(1 + \frac{P_2 |\mathbf{c}_1^H \mathbf{h}_1^{\text{eq}}|^2}{\sigma^2 \|\mathbf{c}_1\|^2} \right) \quad (27)$$

where $(\mathbf{c}_1, \mathbf{c}_2) \in \mathbb{C}^{M \times 1}$. Vectors \mathbf{h}_1^{eq} and \mathbf{h}_2^{eq} are given by

$$\mathbf{h}_1^{\text{eq}} = \left[\frac{\mathbf{h}_{11}^T \tilde{\mathbf{B}}_1 \mathbf{h}_{21}}{\sqrt{1 + \|\tilde{\mathbf{B}}_1^H \mathbf{h}_{11}^*\|^2}}, \dots, \frac{\mathbf{h}_{1M}^T \tilde{\mathbf{B}}_M \mathbf{h}_{2M}}{\sqrt{1 + \|\tilde{\mathbf{B}}_M^H \mathbf{h}_{1M}^*\|^2}} \right] \quad (28)$$

$$\mathbf{h}_2^{\text{eq}} = \left[\frac{\mathbf{h}_{21}^T \tilde{\mathbf{B}}_1 \mathbf{h}_{11}}{\sqrt{1 + \|\tilde{\mathbf{B}}_1^H \mathbf{h}_{21}^*\|^2}}, \dots, \frac{\mathbf{h}_{2M}^T \tilde{\mathbf{B}}_M \mathbf{h}_{1M}}{\sqrt{1 + \|\tilde{\mathbf{B}}_M^H \mathbf{h}_{2M}^*\|^2}} \right] \quad (29)$$

where $\tilde{\mathbf{B}}_i$ is the normalized beamforming matrix of the i th relay, which satisfies $p(\tilde{\mathbf{B}}_i) = P_R$. The equivalent beamformers \mathbf{c}_1 and \mathbf{c}_2 satisfy

$$\mathbf{D}_1^{-\frac{1}{2}} \mathbf{c}_1 = \mathbf{D}_2^{-\frac{1}{2}} \mathbf{c}_2 = \mathbf{g} \quad (30)$$

where $\mathbf{D}_j = \mathbf{I} + \text{diag}(\|\tilde{\mathbf{B}}_1^H \mathbf{h}_{j1}^*\|^2, \dots, \|\tilde{\mathbf{B}}_M^H \mathbf{h}_{jM}^*\|^2)$, $j = 1, 2$, and $\mathbf{g} = [g_1, \dots, g_M]^T$ is the unit-norm distributed beamforming such that

$$\mathbf{B}_i = g_i^* \tilde{\mathbf{B}}_i, \quad i = 1, \dots, M. \quad (31)$$

Proof: The proof can be found in Appendix B. ■

The proposition states that, using the parameterization in (31), the TWRC can be viewed as an interference-free

broadcast channel with equivalent channels (28) and (29) and equivalent transmit beamformers \mathbf{c}_1 and \mathbf{c}_2 , which are linked through the constraint (30). Relaxing this constraint leads to the following corollary.

Corollary 1: An upper bound of the optimal rate region of the RF-MIMO-TWRC is given by

$$\mathcal{R}^{\text{UB}}(P_1, P_2, P_R) = \bigcup_{\substack{\tilde{\mathbf{B}}_i: p(\tilde{\mathbf{B}}_i) \leq P_R \\ i=1, \dots, M}} \{R_{12}^{\text{UB}}, R_{21}^{\text{UB}}\} \quad (32)$$

where R_{12}^{UB} and R_{21}^{UB} are

$$R_{12}^{\text{UB}} = \frac{1}{2} \log_2 \left(1 + \frac{P_1}{\sigma^2} \|\mathbf{h}_2^{\text{eq}}\|^2 \right) \quad (33)$$

$$R_{21}^{\text{UB}} = \frac{1}{2} \log_2 \left(1 + \frac{P_2}{\sigma^2} \|\mathbf{h}_1^{\text{eq}}\|^2 \right). \quad (34)$$

Note that the union in (32) is taken under individual power constraint at each relay, i.e., an upper bound of the optimal rate region with sum-power constraint can be obtained by the union of rate regions satisfying the individual power constraint.

To derive a suboptimal method that approaches the optimal rate region defined in (22), we first obtain the beamformers that produce the upper bound (32), whose norm will be modified in a second step to satisfy the sum-power constraint. As for the achievable rate region of the TWRC with single relay, this upper bound can be characterized using a rate profile method, which is able to find all the boundary points. Therefore, each point of the upper bound, for some weight $0 \leq \alpha \leq 1$, can be found by solving the following optimization problem:

$$\begin{aligned} & \underset{\tilde{\mathbf{B}}_1, \dots, \tilde{\mathbf{B}}_M}{\text{maximize}} && \gamma_{\text{sum}} \\ & \text{subject to :} && \frac{P_2}{\sigma^2} \|\mathbf{h}_1^{\text{eq}}\|^2 \geq \alpha \gamma_{\text{sum}} \\ & && \frac{P_1}{\sigma^2} \|\mathbf{h}_2^{\text{eq}}\|^2 \geq (1 - \alpha) \gamma_{\text{sum}} \\ & && p(\tilde{\mathbf{B}}_i) \leq P_R, \quad i = 1, \dots, M \\ & && \text{rank}(\tilde{\mathbf{B}}_i) = 1, \quad i = 1, \dots, M. \end{aligned} \quad (35)$$

The foregoing problem is nonconvex and cannot efficiently be solved in its current form. Moreover, (35) cannot be rewritten in a convex form by invoking the same arguments as in (15) due to the contribution of all relays in the first two constraints. Using Lagrange multipliers, the first two constraints can be placed in the objective function as

$$\begin{aligned} & \underset{\tilde{\mathbf{B}}_1, \dots, \tilde{\mathbf{B}}_M}{\text{maximize}} && \gamma_{\text{sum}} + \lambda_1^* \left(\frac{P_2}{\sigma^2} \|\mathbf{h}_1^{\text{eq}}\|^2 - \alpha \gamma_{\text{sum}} \right) \\ & && + \lambda_2^* \left[\frac{P_1}{\sigma^2} \|\mathbf{h}_2^{\text{eq}}\|^2 - (1 - \alpha) \gamma_{\text{sum}} \right] \\ & \text{subject to :} && p(\tilde{\mathbf{B}}_i) \leq P_R, \quad i = 1, \dots, M \\ & && \text{rank}(\tilde{\mathbf{B}}_i) = 1, \quad i = 1, \dots, M \end{aligned} \quad (36)$$

where λ_1^* and λ_2^* are the optimal values of the dual variables associated with the first and second constraint, respectively.

The foregoing problem is a weighted sum SNR maximization (WSSNRmax) with weights λ_1^* and λ_2^* . If strong duality holds, the optimal solution of the dual problem (36) is also the optimal solution of the original problem (35). Notice that the WSSNRmax is able to find all the boundary points if the region is convex, and therefore, strong duality holds if and only if the upper bound is a convex region. In the case of a nonconvex region, the dual problem (36) yields an upper bound of the original problem (35), which is also an upper bound of the achievable rate region. In this case, the dual problem provides the convex hull of the rate region, whose boundary points can always be achieved by means of time sharing.

In (36), each relay has an individual power constraint, and according to the definition of the equivalent channels in (28) and (29), there is no coupling between the normalized beamforming matrices $\tilde{\mathbf{B}}_i$ (note that the relay beamforming matrices that are a solution of (35) are normalized so as to satisfy individual power constraint, and their norm will be modified in a second step to satisfy the sum-power constraint at the relays, as explained in Section V-B). Hence, (36) can be divided into M parallel optimization problems, where each one is given by

$$\begin{aligned} & \underset{\tilde{\mathbf{B}}_i}{\text{maximize}} && \omega \frac{P_2}{\sigma^2} |h_{1(i)}^{\text{eq}}|^2 + (1 - \omega) \frac{P_1}{\sigma^2} |h_{2(i)}^{\text{eq}}|^2 \\ & \text{subject to} && p(\tilde{\mathbf{B}}_i) \leq P_R, \\ & && \text{rank}(\tilde{\mathbf{B}}_i) = 1 \end{aligned} \quad (37)$$

for each $i = 1, \dots, M$, where $h_{j(i)}^{\text{eq}}$ is the i th entry of \mathbf{h}_j^{eq} , $j = 1, 2$, and $\omega = \lambda_1^*/(\lambda_1^* + \lambda_2^*)$. Varying ω between 0 and 1, all the points of the convex hull can be obtained. Note that, according to (28) and (29), $(P_2/\sigma^2)|h_{1(i)}^{\text{eq}}|^2$ and $(P_1/\sigma^2)|h_{2(i)}^{\text{eq}}|^2$ are the SNRs at S_1 and S_2 , respectively, when only the i th relay is transmitting. Thus, each of the M preceding problems consists of optimizing each relay in the absence of the others. Hence, the upper bound in (32) can be obtained by applying Algorithm 1 for each of the M relays, thus requiring only local CSI.

B. Relay Combining Algorithm (RCA)

To summarize, the proposed algorithm divides the original problem into two different subproblems. First, the normalized beamforming matrices $\tilde{\mathbf{B}}_i$ are computed by solving each of the M individual TWRC using Algorithm 1. To solve this problem, each relay needs only local CSI. Once the beamforming matrices are obtained, the relays cooperate, exchange their equivalent channels, and compute the distributed beamformer \mathbf{g} . Finally, the actual relay beamforming matrices are obtained according to (31). The distributed beamformer controls not only the power but also the phase of the signal received by the source nodes. As we show in the following theorem, the optimal solutions of (37) satisfy a key property.

Theorem 1: Let $\tilde{\mathbf{B}}_i^*$ be the solution of (37) for any $0 \leq \omega \leq 1$. Then

$$\arg\left(\mathbf{h}_{1i}^T \tilde{\mathbf{B}}_i^* \mathbf{h}_{2i}\right) = \arg\left(\mathbf{h}_{2i}^T \tilde{\mathbf{B}}_i^* \mathbf{h}_{1i}\right) \quad (38)$$

where $\arg(a)$ denotes the phase of the complex scalar a .

Proof: The proof can be found in Appendix C. ■

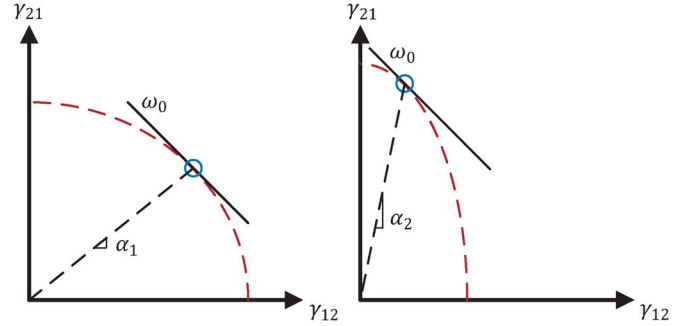


Fig. 4. Example of the SNR regions achieved in a TWRC with two relays when only the first relay (left) or the second relay (right) is transmitting. The relays must be combined for the same point, ω_0 , which is achieved by different SNR profile weights.

The theorem states that, when the beamformers are optimized independently, the phase difference between both links through a given relay (from S_1 to S_2 , and from S_2 to S_1) is equal to zero. Note that the theorem is valid only if channel reciprocity holds, i.e., if the channel remains unchanged during both transmission phases. Proposition 1 and Theorem 1 suggest that, if each relay is optimized independently and the signals from the relays are combined coherently through the distributed beamformer \mathbf{g} , the resulting rate region will be very close to the optimum. To better understand this idea, consider the following. Relaxing constraint (30) yields the upper bound of the achievable rate region in Corollary 1, which is obtained by optimizing \mathbf{c}_1 and \mathbf{c}_2 independently and therefore yielding a different \mathbf{g} for each node. Consequently, the difference between the upper bound and the achievable rate region is to be determined by the difference between the optimal \mathbf{g} for each node. According to Theorem 1, the distributed beamformer \mathbf{g} controls only the power transmitted by each relay, and therefore, it is a real vector. This fact, along with the symmetry of the TWRC (i.e., the signal from each source node travels through all channels of the network), suggest that the optimal \mathbf{g} for each user will be close to one another. To see this, suppose that the first relay has better channel quality than the rest of them. Then, it is likely that both source nodes want this relay to transmit more power than the others. Thus, we expect a small gap between the upper bound and the performance of the proposed method. In other words, the proposed technique is expected to be quasi-optimal.

There is an important consideration that must be taken into account. To obtain the matrices $\tilde{\mathbf{B}}_i$ for a certain point of the rate region, the WSSNRmax problem in (37) must be solved for $i = 1, \dots, M$ and for a given weight ω_0 . For the sake of clarity, let us denote this set of beamforming matrices as $\{\tilde{\mathbf{B}}_1(\omega_0), \dots, \tilde{\mathbf{B}}_M(\omega_0)\}$. To find this solution, Algorithm 1, based on SNR profile, can be applied. The SNR profile method, however, uses a different set of weights α . Nevertheless, the mapping between α and ω is nonlinear and different for each relay, as we show in Fig. 4. As depicted in the figure, the SNR profile can be geometrically interpreted as a straight line starting in the origin with a slope of α that crosses the SNR region at one point, whereas the WSSNRmax can be viewed as a straight line tangent to the SNR region at one point and with a slope given by ω . Suppose that we set $\alpha_i = \alpha_0$ for $i = 1, \dots, M$. Then,

we achieve the set of solutions $\{\tilde{\mathbf{B}}_1(\alpha_0), \dots, \tilde{\mathbf{B}}_M(\alpha_0)\} = \{\tilde{\mathbf{B}}_1(\omega_1), \dots, \tilde{\mathbf{B}}_M(\omega_M)\}$. Therefore, Algorithm 1 must be modified so as to calculate the value of α_i , α_i^* , such that $\{\tilde{\mathbf{B}}_1(\alpha_1^*), \dots, \tilde{\mathbf{B}}_M(\alpha_M^*)\} = \{\tilde{\mathbf{B}}_1(\omega_0), \dots, \tilde{\mathbf{B}}_M(\omega_0)\}$. As the relationship between α and ω is monotonous increasing, ω_{α_i} can be made arbitrarily close to ω_0 by iteratively updating α_i in the direction $\omega_0 - \omega_{\alpha_i}$, applying Algorithm 1 at each iteration. The associated weight ω_{α_i} can be computed through the dual variables associated to the SNR constraints of (20). Let λ_1^* and λ_2^* be the optimal values of the dual variables associated the SNR constraints of nodes S_1 and S_2 , respectively, for a given α . Then, the initial problem (15) is equivalent to

$$\begin{aligned} & \underset{\substack{P_{\text{ef}_1}, P_{\text{ef}_2} \\ \mathbf{U}_R, \mathbf{V}_R}}{\text{maximize}} \quad \gamma_{\text{sum}} + \lambda_1^* \left\{ p_{\text{ef}_1} - (1 - \alpha)\gamma_{\text{sum}} \right. \\ & \quad \left. \times \frac{\sigma^2 [1 + \text{Tr}(\mathbf{U}_R)\text{Tr}(\mathbf{R}_2^* \mathbf{V}_R)]}{\text{Tr}(\mathbf{R}_2^* \mathbf{V}_R)} \right\} \\ & \quad + \lambda_2^* \left\{ p_{\text{ef}_2} - \alpha\gamma_{\text{sum}} \right. \\ & \quad \left. \times \frac{\sigma^2 [1 + \text{Tr}(\mathbf{U}_R)\text{Tr}(\mathbf{R}_1^* \mathbf{V}_R)]}{\text{Tr}(\mathbf{R}_1^* \mathbf{V}_R)} \right\} \\ \text{subject to: } & \quad \text{Tr}(\mathbf{V}_R) = 1 \\ & \quad \mathbf{V}_R \succeq 0 \\ & \quad \mathbf{U}_R \succeq 0 \\ & \quad p_{\text{ef}_1} \leq P_1 \text{Tr}(\mathbf{R}_1 \mathbf{U}_R) \\ & \quad p_{\text{ef}_2} \leq P_2 \text{Tr}(\mathbf{R}_2 \mathbf{U}_R) \\ & \quad p_{\text{ef}_1} + p_{\text{ef}_2} + \sigma^2 \text{Tr}(\mathbf{U}_R) \leq P_R. \end{aligned} \quad (39)$$

If we define the new dual variables $\tilde{\lambda}_1^*$ and $\tilde{\lambda}_2^*$ as

$$\begin{aligned} \tilde{\lambda}_1^* &= \lambda_1^* \frac{\sigma^2 [1 + \text{Tr}(\mathbf{U}_R^*) \text{Tr}(\mathbf{R}_2^* \mathbf{V}_R^*)]}{\text{Tr}(\mathbf{R}_2^* \mathbf{V}_R^*)} \\ \tilde{\lambda}_2^* &= \lambda_2^* \frac{\sigma^2 [1 + \text{Tr}(\mathbf{U}_R^*) \text{Tr}(\mathbf{R}_1^* \mathbf{V}_R^*)]}{\text{Tr}(\mathbf{R}_1^* \mathbf{V}_R^*)} \end{aligned} \quad (40)$$

then the objective function of (39) turns into a weighted sum of the SNR of the nodes with weights $\tilde{\lambda}_1^*$ and $\tilde{\lambda}_2^*$. Note that the dual variables defined above satisfy the Karush–Kuhn–Tucker conditions of the original nonconvex problem (15), i.e., they are the optimal solution of the dual problem of (15). Finally, the associated weight is given by

$$\omega_\alpha = \frac{\tilde{\lambda}_1^*}{\tilde{\lambda}_1^* + \tilde{\lambda}_2^*}. \quad (41)$$

Once the normalized beamforming matrices are computed, the distributed beamformer \mathbf{g} must be obtained. The optimal distributed beamformer is the solution of the problem

$$\mathbf{g}^* = \arg \max_{\|\mathbf{g}\|=1} \left\{ \omega \frac{P_2 \left\| (\mathbf{h}_1^{\text{eq}})^H \mathbf{D}_1^{\frac{1}{2}} \mathbf{g} \right\|^2}{\left\| \mathbf{D}_1^{\frac{1}{2}} \mathbf{g} \right\|^2} + (1 - \omega) \frac{P_1 \left\| (\mathbf{h}_2^{\text{eq}})^H \mathbf{D}_2^{\frac{1}{2}} \mathbf{g} \right\|^2}{\left\| \mathbf{D}_2^{\frac{1}{2}} \mathbf{g} \right\|^2} \right\}. \quad (42)$$

Note that, in the extreme points, i.e., when $\omega = 0$ or $\omega = 1$, the foregoing expression turns into a generalized Rayleigh quotient, which has a closed-form solution. However, in the general case, when $0 < \omega < 1$, (42) is a sum of two generalized Rayleigh quotients, and the problem cannot be solved in closed form. The problem of distributed beamforming in two-way multiple-relay networks, which is also called collaborative beamforming, has been studied in [18] and [20], where a convex optimization technique based upon the SNR profile and the bisection method has been proposed. However, a suboptimal but closed-form distributed beamformer can be computed as a linear combination of the optimal solution in the extreme points, i.e.,

$$\begin{aligned} \mathbf{g} &= \frac{\tilde{\mathbf{g}}}{\|\tilde{\mathbf{g}}\|} \\ \tilde{\mathbf{g}} &= \omega \frac{P_2}{\sigma^2} \mathbf{D}_1^{-\frac{1}{2}} \mathbf{h}_1^{\text{eq}} + (1 - \omega) \frac{P_1}{\sigma^2} \mathbf{D}_2^{-\frac{1}{2}} \mathbf{h}_2^{\text{eq}}. \end{aligned} \quad (43)$$

Moreover, as \mathbf{g} only controls the power transmitted by each relay (see Theorem 1), the foregoing solution performs very close to the optimal. Finally, the proposed algorithm for the RF-MIMO-TWMRC, which we call RCA, is summarized in Algorithm 2.

Algorithm 2 Relay combining algorithm

Select the weight $0 \leq \omega_0 \leq 1$, the desired tolerance, δ , and the step size, ϵ .

for $i = 1, \dots, M$ **do**

repeat

Apply Algorithm 1 for $\alpha_i = \omega_0$ and compute ω_{α_i} using (40) and (41).

$$\alpha_i = \alpha_i + \epsilon(\omega_0 - \omega_{\alpha_i}).$$

until $|\omega_0 - \omega_{\alpha_i}| \leq \delta$

end for

Compute the distributed beamforming, \mathbf{g} , using (43) for $\omega = \omega_0$.

VI. NUMERICAL EXAMPLES

In this section, we present some examples to illustrate the achievable rates of the analog beamforming architecture and to compare them with those achieved by conventional MIMO schemes. The elements of \mathbf{H}_1 , \mathbf{H}_2 , \mathbf{h}_{1i} , and \mathbf{h}_{2i} ($i = 1, \dots, M$) are independent identically distributed zero-mean circular complex Gaussian random variables with unit variance. We consider equal power constraints for the nodes and relay, i.e., $P_R = P_1 = P_2 = P$, and define the SNR as $\text{SNR} = 10 \log_{10}(P/\sigma^2)$. Without loss of generality, we take $\sigma^2 = 1$.

First, we compare the achievable rate region when the nodes use analog beamforming against conventional (baseband) MIMO beamforming. The optimal rate region with conventional MIMO is obtained through the algorithm proposed in [13]. This algorithm does not allow power optimization and requires the source nodes to be single antenna terminals. Thus, we focus on the following scenario: $N_s = 1$, $N_R = 4$, and fixed powers. For convenience of analysis, we consider one channel

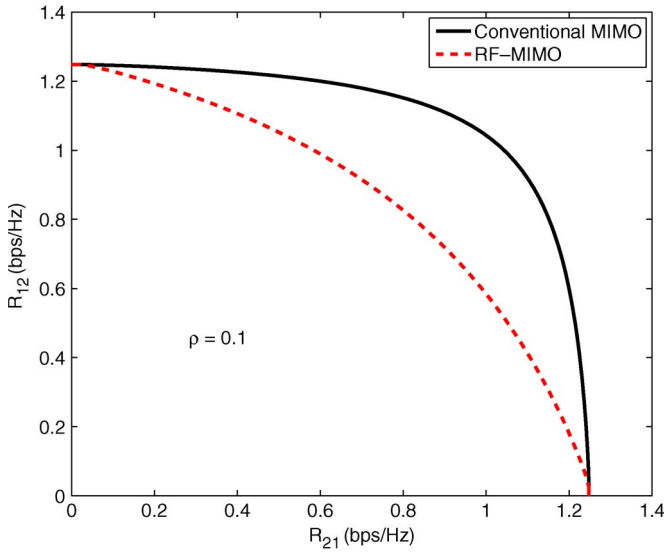


Fig. 5. Comparison between the achievable rates of conventional MIMO and RF-MIMO schemes when $N_s = 1$ and $N_R = 4$ for $\rho = 0.1$. The optimal rate region of conventional MIMO-TWRC has been computed using the algorithm proposed in [13].

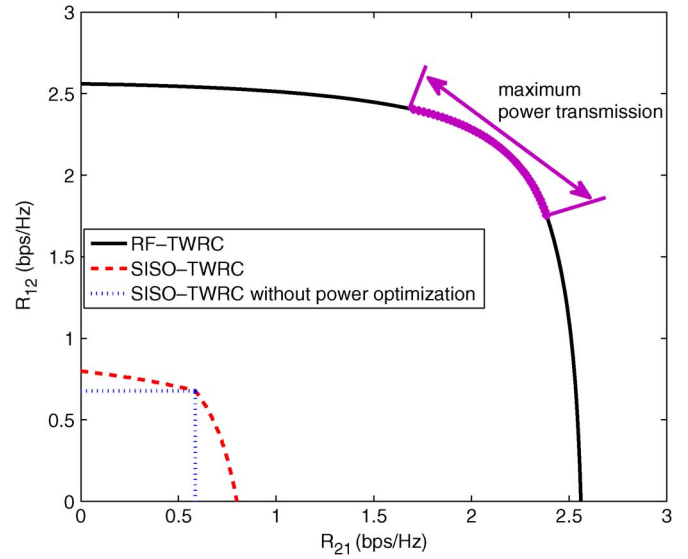


Fig. 7. Optimal rate region of the 4×2 RF-MIMO-TWRC with SNR of 10 dB. The dashed line depicts the achievable region of the SISO case with and without power optimization.

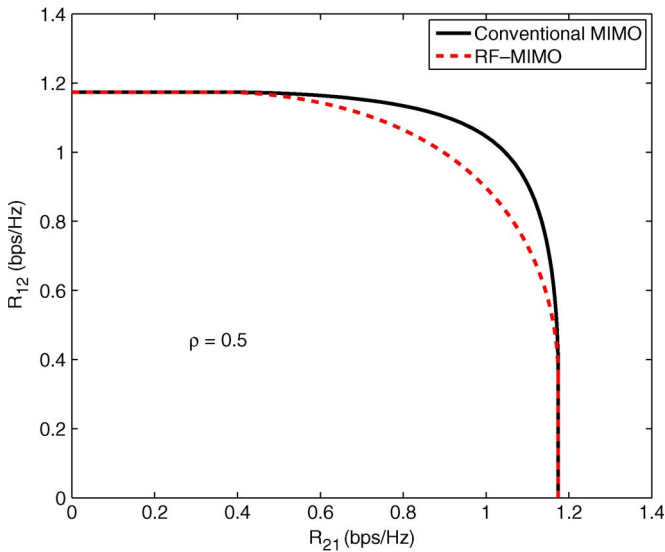


Fig. 6. Comparison between the achievable rates of conventional MIMO and RF-MIMO schemes when $N_s = 1$ and $N_R = 4$ for $\rho = 0.5$. The optimal rate region of conventional MIMO-TWRC has been computed using the algorithm proposed in [13].

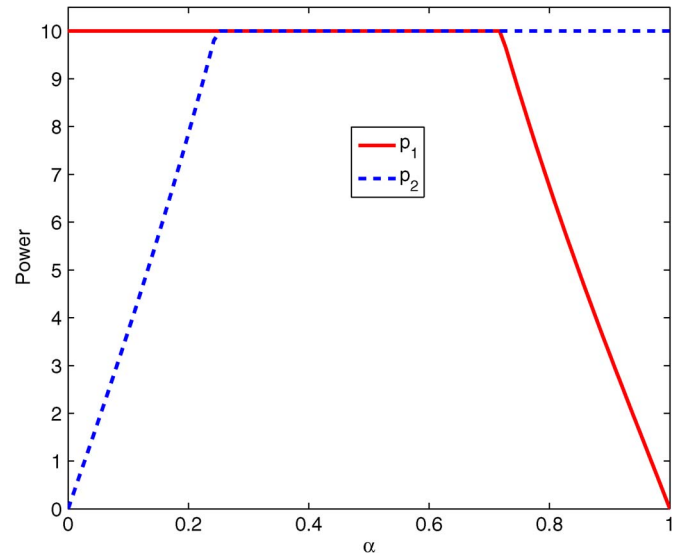


Fig. 8. Power allocation of the 4×2 RF-MIMO-TWRC with SNR of 10 dB. Note that only a subset of the boundary rate-tuples is achieved when both nodes are simultaneously transmitting at maximum power. Moreover, there is at least one node transmitting at maximum power at each boundary point.

realization of \mathbf{h}_1 normalized to have unit norm. Channel vector \mathbf{h}_2 is obtained such that $\|\mathbf{h}_2\| = 1$ and $|\mathbf{h}_1^H \mathbf{h}_2|^2 = \rho$, where ρ is the squared cosine of the angle formed between \mathbf{h}_1 and \mathbf{h}_2 . Figs. 5 and 6 show the achievable rate region of both schemes for $\rho = 0.1$ and $\rho = 0.5$, respectively. The SNR is equal to 10 dB, and the relay node is equipped with four antennas. As ρ increases, the channel vectors tend to be collinear, and the gap between analog and conventional beamforming goes to 0.

Now, we consider multiantenna nodes and optimize the power transmitted by the source nodes. In Fig. 7, we show the optimal rate region of a RF-MIMO-TWRC channel when $N_R = 4$ and $N_S = 2$, and in Fig. 8, we show the actual power transmitted by the source nodes. The SNR has a value of 10 dB, and the energies of the channels are equal to unity. We observe

that there are some points of the boundary that are achieved when the source nodes do not transmit at maximum power. This result can easily be explained in terms of the transmit power of the relay node p_R . From (3), p_R is a function of the effective powers of both source nodes. As the relay node has a power constraint P , the greater is the effective power of a node, the lower is the effective power of the other. Thus, maximum power transmission is optimal only in a subset of the optimal rate region boundary. Fig. 7 also shows the optimal rate region of the single-antenna (SISO) case using the first antenna of each terminal, with and without power optimization (in the latter case, both nodes always transmit at maximum power). We clearly observe the enlargement of the optimal rate region when the nodes are multiantenna terminals performing analog

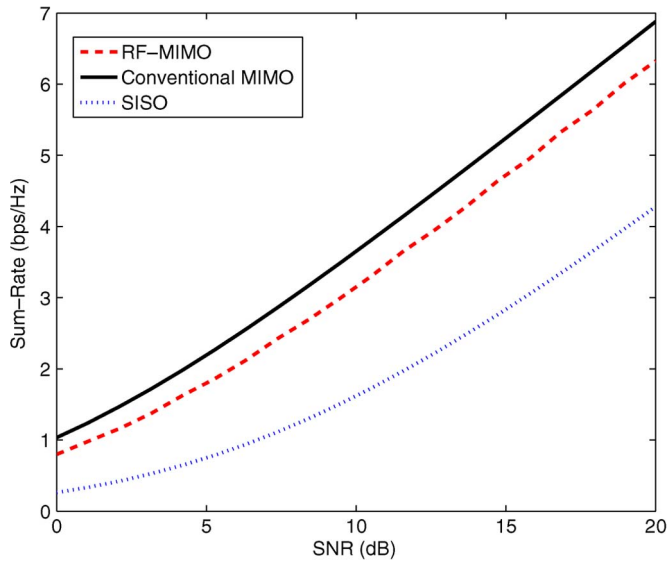


Fig. 9. Sum rate through Monte Carlo simulation (specifically, we average the results of 100 channel realizations) for the TWRC channel with $N_R = 4$ and $N_S = 1$ when no power optimization is performed.

beamforming. Moreover, optimizing the power of the source nodes noticeably improves the achievable rate region of the SISO-TWRC.

We also evaluate the sum rate versus the SNR of the RF-MIMO-TWRC through Monte Carlo simulation (specifically, we average the results of 100 channel realizations) and compare it with the conventional MIMO-TWRC and SISO cases when the first antenna of each terminal is used. The sum rate is computed using exhaustive search over α , and we follow the algorithm proposed in [13] for the conventional MIMO-TWRC. Thus, we consider single antenna nodes and no power optimization, i.e., $N_S = 1$ and $p_1 = p_2 = P$. Fig. 9 shows the sum rate when the relay is equipped with $N_R = 4$ antennas. We observe that the RF-MIMO-TWRC and the conventional MIMO-TWRC perform closely, as well as the enlargement in the sum rate with respect to the SISO case.

Regarding the TWMRC, Fig. 10 shows the performance of the RCA algorithm when $M = 4$ relays equipped with $N = 2$ antennas each. The SNR is equal to 10 dB. It can be noticed that the proposed algorithm performs very closely to the upper bound, computed according to (32). As the boundary of the optimal rate region of the RF-MIMO-TWMRC lies within this upper bound and the RCA region, the proposed algorithm performs very closely to the boundary of the optimal rate region. This result can be explained by Theorem 1 and Proposition 1. First, by Theorem 1, we know that the signals from the relays add up coherently at both nodes simultaneously, and thus, the distributed beamformer \mathbf{g} only controls the transmit power of each relay. Second, from Proposition 1 (and the ensuing corollary), an upper bound can be found by assuming a different distributed beamformer for each node. However, as \mathbf{g} only controls the transmit power of each relay and the network is symmetric (i.e., channel reciprocity holds and, consequently, a relay having bad channel conditions affects both nodes), the optimal \mathbf{g} for each node will be almost the same; hence, the gap between the achievable rate region and the upper bound is

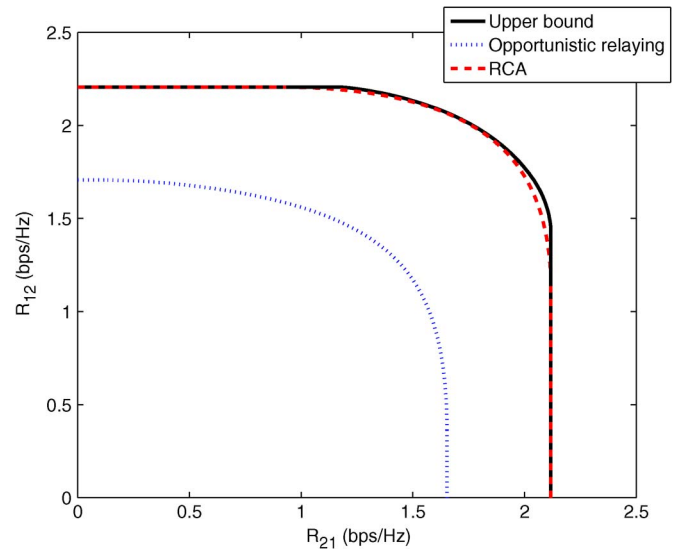


Fig. 10. Achievable rate region of RCA algorithm when $M = 4$, $N = 2$, and SNR = 10 dB. It can be noticed that the RCA performs very closely to the optimal rate region.

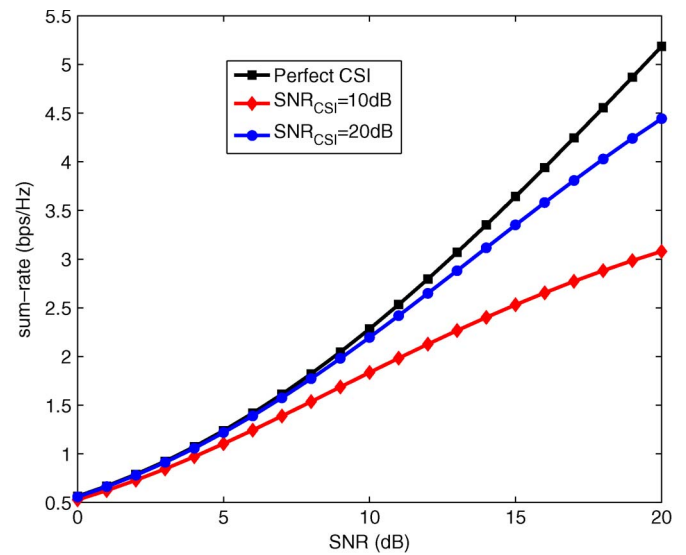


Fig. 11. Impact of CSI errors in the sum-rate performance of one channel realization and averaged over 100 different channel estimates.

very small. Fig. 10 also shows the achievable rate region of the opportunistic relaying scheme [31], which consists of selecting only the relay with the best channel conditions. We clearly observe the enlargement of the rate region when multiple relays are used.

Finally, we show in Fig. 11 the impact of CSI errors in the sum-rate performance for a TWMRC comprised of $M = 2$ relay nodes with $N_R = 2$ antennas each. The estimate of node j and relay i is given by $\hat{\mathbf{h}}_{ij} = \sqrt{1-\epsilon} \mathbf{h}_{ij} + \sqrt{\epsilon} \mathbf{h}_{ij}^n$, where \mathbf{h}_{ij} is the actual channel, $\mathbf{h}_{ij}^n \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$, and $0 \leq \epsilon \leq 1$. Thus, we define the SNR of the channel estimate as $\text{SNR}_{\text{CSI}} = 10 \log_{10}((1-\epsilon)/\epsilon)$. Although the proposed algorithm is not a robust design, it can be noticed that the impact of CSI errors is not significant. When the SNR increases, the sum-rate degradation also increases due to the fact that the self-interference is not perfectly canceled.

VII. CONCLUSION

RF-MIMO transceivers that apply beamforming in the analog domain are of practical interest due to the reduced system size, cost, and power consumption in comparison with conventional MIMO architectures. In this paper, we have studied a basic MIMO-TWRC, when the two nodes and the relay are RF-MIMO terminals. Our main contribution has been to show that the optimal beamforming vectors and the power allocation can efficiently be computed using convex optimization techniques through an iterative algorithm based on a bisection method. We have also shown that the rate gap between analog beamforming and conventional MIMO schemes, when the source nodes are single-antenna terminals and no power optimization is performed, approaches 0 as the collinearity coefficient between the channels increases. We have extended our study to the TWMRC, when the source nodes are single-antenna terminals. We have proposed a distributed algorithm that combines the results of the TWRC with distributed beamforming techniques. The proposed algorithm avoids the need for global CSI and performs very close to the optimal rate region boundary. We have shown that the use of multiple relays performing analog beamforming notably increases the achievable rate region of the RF-MIMO-TWRC.

APPENDIX A

PROOF OF THE RANK-ONE SOLUTION OF (20)

Suppose that the optimal solution of (20) \mathbf{U}_R^* has rank $r > 1$. If there exists an equivalent rank-one solution, the following must hold:

$$\begin{aligned}\text{Tr}(\mathbf{R}_1 \tilde{\mathbf{U}}_R) &= \text{Tr}(\mathbf{R}_1 \mathbf{U}_R^*) \\ \text{Tr}(\mathbf{R}_2 \tilde{\mathbf{U}}_R) &= \text{Tr}(\mathbf{R}_2 \mathbf{U}_R^*) \\ \text{Tr}(\tilde{\mathbf{U}}_R) &= \text{Tr}(\mathbf{U}_R^*)\end{aligned}\quad (44)$$

where $\tilde{\mathbf{U}}_R$ is a rank-one matrix. Through the matrix decomposition theorem for Hermitian matrices [29], [30], given the Hermitian matrices \mathbf{R}_1 and \mathbf{R}_2 , there exists a decomposition of \mathbf{U}_R^* , $\mathbf{U}_R^* = \sum_{k=1}^r \mathbf{u}_R^{(k)} (\mathbf{u}_R^{(k)})^H$, such that $\text{Tr}(\mathbf{R}_1 \tilde{\mathbf{U}}_R^{(k)}) = \text{Tr}(\mathbf{R}_1 \mathbf{U}_R^*)$ and $\text{Tr}(\mathbf{R}_2 \tilde{\mathbf{U}}_R^{(k)}) = \text{Tr}(\mathbf{R}_2 \mathbf{U}_R^*)$ for all $k = 1, \dots, r$, where $\tilde{\mathbf{U}}_R^{(k)} = r \mathbf{u}_R^{(k)} (\mathbf{u}_R^{(k)})^H$ is a rank-one matrix. Thus, there exist r rank-one matrices that satisfy the first two conditions in (44). Taking into account that the traces of \mathbf{U}_R^* and $\tilde{\mathbf{U}}_R^{(k)}$ are, respectively, given by $\text{Tr}(\mathbf{U}_R^*) = \sum_{k=1}^r \|\mathbf{u}_R^{(k)}\|^2$ and $\text{Tr}(\tilde{\mathbf{U}}_R^{(k)}) = r \|\mathbf{u}_R^{(k)}\|^2$, and assuming without loss of generality $\|\mathbf{u}_R^{(1)}\|^2 \geq \|\mathbf{u}_R^{(2)}\|^2 \geq \dots \geq \|\mathbf{u}_R^{(r)}\|^2$, it follows that $\text{Tr}(\mathbf{U}_R^*) \geq \text{Tr}(\tilde{\mathbf{U}}_R^{(r)})$. On the other hand, as \mathbf{U}_R^* is an optimal solution of (20), $\text{Tr}(\mathbf{U}_R^*) \leq \text{Tr}(\tilde{\mathbf{U}}_R^{(r)})$ must hold. Thus, to satisfy both inequalities, the traces must be equal, i.e., $\text{Tr}(\mathbf{U}_R^*) = \text{Tr}(\tilde{\mathbf{U}}_R^{(r)})$. Therefore, the rank-one matrix $\tilde{\mathbf{U}}_R^{(r)}$ is also optimal and equivalent to \mathbf{U}_R^* .

Similarly, suppose that the optimal solution of (20) \mathbf{V}_R^* has rank $r > 1$. Then, the same arguments can be invoked to prove the existence of an optimal rank-one matrix.

APPENDIX B

PROOF OF PROPOSITION 1

The achievable SNR at S_1 is given by

$$\gamma_1 = \frac{P_2 \left| \sum_{i=1}^M \mathbf{h}_{1i}^T \mathbf{B}_i \mathbf{h}_{2i} \right|^2}{\sigma^2 \left(1 + \sum_{i=1}^M \|\mathbf{B}_i^H \mathbf{h}_{1i}^*\|^2 \right)} \quad (45)$$

where the beamforming matrices satisfy total power constraint, i.e., $\sum_{i=1}^M p(\mathbf{B}_i) \leq P$. If we parameterize the beamforming matrices as (31), the SNR achieved by S_1 can be rewritten as

$$\gamma_1 = \frac{P_2 \left| \sum_{i=1}^M g_i^* \mathbf{h}_{1i}^T \tilde{\mathbf{B}}_i \mathbf{h}_{2i} \right|^2}{\sigma^2 \sum_{i=1}^M |g_i|^2 \left(1 + \|\tilde{\mathbf{B}}_i^H \mathbf{h}_{1i}^*\|^2 \right)}. \quad (46)$$

Using (30), γ_1 can be expressed as

$$\gamma_1 = \frac{P_2 |\mathbf{c}_1^H \mathbf{h}_1^{\text{eq}}|^2}{\sigma^2 \|\mathbf{c}_1\|^2}. \quad (47)$$

The same arguments can be applied to the SNR achieved by S_2 .

APPENDIX C

PROOF OF THEOREM 1

It is intuitively clear that the relay beamformers lie in the subspace spanned by the channel vectors. Thus, without loss of generality, the beamformers used by the i th relay can be written as $\mathbf{v}_{Ri} = \sqrt{\epsilon_{1i}} \mathbf{h}_{1i}^* + \sqrt{\epsilon_{2i}} e^{j\psi_{1i}} \mathbf{h}_{2i}^*$ and $\mathbf{u}_{Ri} = \sqrt{\alpha_{1i}} \mathbf{h}_{1i} + \sqrt{\alpha_{2i}} e^{j\psi_{2i}} \mathbf{h}_{2i}$, where $(\epsilon_{ji}, \alpha_{ji}, \psi_{ji}) \in \mathbb{R}$, $j = 1, 2$, and $\tilde{\mathbf{B}}_i^* = \mathbf{v}_{Ri} \mathbf{u}_{Ri}^H$. Then, assuming without loss of generality that the channels have unit norm, it follows that

$$\begin{aligned}\mathbf{h}_{1i}^T \tilde{\mathbf{B}}_i^* \mathbf{h}_{2i} &= \sqrt{\epsilon_{1i} \alpha_{1i} \rho_i} e^{j(\phi_i + \psi_{2i})} + \sqrt{\epsilon_{2i} \alpha_{1i} \rho_i} e^{j(\psi_{1i} + \psi_{2i})} \\ &\quad + \sqrt{\epsilon_{1i} \alpha_{2i}} + \sqrt{\epsilon_{2i} \alpha_{2i} \rho_i} e^{j(\psi_{1i} - \phi_i)}\end{aligned}\quad (48)$$

$$\begin{aligned}\mathbf{h}_{2i}^T \tilde{\mathbf{B}}_i^* \mathbf{h}_{1i} &= \sqrt{\epsilon_{1i} \alpha_{1i} \rho_i} e^{j(\phi_i - \psi_{1i})} + \sqrt{\epsilon_{1i} \alpha_{2i} \rho_i} e^{-j(\psi_{1i} + \psi_{2i})} \\ &\quad + \sqrt{\epsilon_{2i} \alpha_{1i}} + \sqrt{\epsilon_{2i} \alpha_{2i} \rho_i} e^{-j(\psi_{2i} + \phi_i)}\end{aligned}\quad (49)$$

where $\rho_i = |\mathbf{h}_{1i}^H \mathbf{h}_{2i}|^2$, $0 \leq \rho_i \leq 1$, and $\phi_i = \arg(\mathbf{h}_{1i}^H \mathbf{h}_{2i})$. Therefore, if the following expression holds:

$$\psi_{1i} = \phi_i \quad (50)$$

$$\psi_{2i} = -\phi_i \quad (51)$$

then $\arg(\mathbf{h}_{1i}^T \tilde{\mathbf{B}}_i^* \mathbf{h}_{2i}) = \arg(\mathbf{h}_{2i}^T \tilde{\mathbf{B}}_i^* \mathbf{h}_{1i}) = 0$. Hence, we prove in the following that (50) and (51) hold when the optimal beamforming matrices $\tilde{\mathbf{B}}_i^*$ are applied.

Analyzing the derivatives of the equivalent channels subject to norm constraint on the beamformers, i.e., $(|\mathbf{h}_{1i}^T \tilde{\mathbf{B}}_i^* \mathbf{h}_{2i}|^2 /$

$\|\mathbf{v}_{R_i}\|^2\|\mathbf{u}_{R_i}\|^2$) and $(\|\mathbf{h}_{2i}^T\tilde{\mathbf{B}}_i^*\mathbf{h}_{1i}\|^2/\|\mathbf{v}_{R_i}\|^2\|\mathbf{u}_{R_i}\|^2)$, with respect to ψ_{1i} and ψ_{2i} , we have

$$\left. \begin{aligned} \frac{\partial}{\partial \psi_{1i}} \left(\frac{\|\mathbf{h}_{1i}^T\tilde{\mathbf{B}}_i^*\mathbf{h}_{2i}\|^2}{\|\mathbf{v}_{R_i}\|^2\|\mathbf{u}_{R_i}\|^2} \right) = 0 \\ \frac{\partial}{\partial \psi_{1i}} \left(\frac{\|\mathbf{h}_{2i}^T\tilde{\mathbf{B}}_i^*\mathbf{h}_{1i}\|^2}{\|\mathbf{v}_{R_i}\|^2\|\mathbf{u}_{R_i}\|^2} \right) = 0 \end{aligned} \right\} \Rightarrow \psi_{1i} = \phi_i \quad (52)$$

$$\left. \begin{aligned} \frac{\partial}{\partial \psi_{2i}} \left(\frac{\|\mathbf{h}_{1i}^T\tilde{\mathbf{B}}_i^*\mathbf{h}_{2i}\|^2}{\|\mathbf{v}_{R_i}\|^2\|\mathbf{u}_{R_i}\|^2} \right) = 0 \\ \frac{\partial}{\partial \psi_{2i}} \left(\frac{\|\mathbf{h}_{2i}^T\tilde{\mathbf{B}}_i^*\mathbf{h}_{1i}\|^2}{\|\mathbf{v}_{R_i}\|^2\|\mathbf{u}_{R_i}\|^2} \right) = 0 \end{aligned} \right\} \Rightarrow \psi_{2i} = -\phi_i \quad (53)$$

i.e., the projection of the beamformers onto the channels is simultaneously improved by choosing $\psi_{1i} = -\phi_i$ and $\psi_{2i} = \phi_i$. This proves that (50) and (51) hold when the optimal beamformers are applied, and, consequently, the theorem.

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