IMPROPER SIGNALING FOR OFDM UNDERLAY COGNITIVE RADIO SYSTEMS

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ABSTRACT

Improper signaling, where real and imaginary parts of the transmit signal are correlated and/or have unequal power, has received a lot of attention lately because it has been shown to increase achievable rates in many interference-limited communication systems. In this paper, we study whether improper signaling can also benefit an orthogonal frequency-division multiplexing (OFDM) underlay cognitive radio (UCR) system. We assume that the primary user (PU) transmits proper signals, while the secondary user (SU) is allowed to employ improper signaling. We consider two different rate constraints for the rate of the PU: i) the total rate of the PU, and ii) the rate of the PU in each subband. We propose an algorithm to implement improper signaling for each constraint. In both cases, we show that the benefits of improper signaling are relatively small and decrease rapidly with increasing number of subbands. This rather negative result shows that the use of improper signaling in interference scenarios needs to be justified on a case-by-case basis.

Index Terms— Improper signaling, underlay cognitive radio, OFDM systems, interference channels

1. INTRODUCTION

The performance of modern wireless communication systems is mainly limited by interference from other users. Therefore, interference management techniques have become critical for efficient spectrum usage. One way of enhancing the spectral efficiency is to employ cognitive radio (CR) systems [1]. In CR systems, licensed or primary users (PU) share the spectrum with unlicensed or secondary users (SU) provided that the PU's communications are not disturbed too much by the SU's transmission. In underlay CR (UCR) systems, the SU can transmit on the channel of the PU only if the rate of the PU is ensured to be greater than a threshold. Another approach for improving efficient spectrum usage is to employ improper signals, whose real and imaginary parts have unequal powers and/or are correlated [2]. It has been shown that improper signaling can increase the achievable rates in interference-limited systems [3–15]. In this paper, we study whether improper signaling also pays off in an UCR OFDM scenario.

Several works have investigated employing improper signaling in wireless communications. While proper signals achieve capacity in traditional communication systems [16], including point-to-point communications, the broadcast channel, and the multiple-access-channel (MAC), [3] and [4] established that, in *interference channels*, employing *improper signals* increases the degrees-of-freedom (DoF) in some cases and thus the achievable rates. In [5], the effect of improper signaling on the rate region of single-input single-output (SISO) Z-interference channels (Z-IC) was considered. The authors derived a necessary and sufficient condition for optimality of improper signaling. In [6], the authors showed that improper signaling increases the achievable rate region of the multi-antenna Z-IC. In [7], the effect of improper signaling in an UCR system has been studied. The authors proved that improper signaling can be beneficial in UCR systems, especially when the load of the primary network is relatively low. Moreover, a condition, based on channel state information (CSI), was derived for improper signaling to be superior to proper signaling. In [8], the authors applied improper Gaussian signaling to a relay scenario and showed this signaling increased the average rate. In [9,10], the authors showed that improper Gaussian signaling improved outage probabilities in underlay and overlay CR, respectively, when there are different CSI assumptions at the transmitter side.

In this paper, we consider a UCR OFDM system with improper signaling. We consider two different thresholds on the rate of the PU: i) a threshold on the total rate and ii) a threshold on the rate of each subband.[1] The reason why improper signaling can be beneficial is discussed in [7]: On the one hand, improper signaling decreases the rate of the SU. On the other hand, it allows the SU to increase its transmission power without violating the PU's rate constraint. Improper signaling is only useful if this possible power increase more than compensates for the rate loss.

Because improper signaling has been so successful at increasing the achievable rates in many interference-limited scenarios, one might expect the same in a UCR OFDM system. However, the parallel channels in OFDM allow the SU to allocate its power more flexibly compared to the single carrier case. Moreover, in relatively low load traffic, when the constraint is on the total rate of the PU, the PU can neglect some subcarriers without violating the rate constraint. This allows the SU to transmit proper signals with maximum power on those subbands. This is why our results show that the benefit of improper signaling is fairly small and decreases further with increasing number of subbands. Our study therefore reveals that the use of improper signaling in interference channels needs to be justified for each individual case.

The rest of the paper is organized as follows. In Section 2, we describe the system model and optimization problems. In Section 3, we propose algorithms for solving the optimization problems. We present numerical simulation results in Section 4.

2. SYSTEM MODEL AND PROBLEM DEFINITION

We call a zero-mean and complex random variable x proper if its *complementary-variance*, $\mathbb{E}\{x^2\}$, is zero. Otherwise, we call it improper. The *circularity coefficient* of x measures the degree of impropriety of x and is defined as $k_x = \frac{|\mathbb{E}\{x^2\}|}{\mathbb{E}\{|x|^2\}}$, $0 \leq k_x \leq 1$. If $k_x = 1$, we call x *maximally improper* [2]. The augmented covariance matrix of signal x, which is the covariance matrix of

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[1] The second constraint is more stringent and can be used when there is an interference power limit for each subband [17]. Moreover, it is more suitable for implementing orthogonal frequency-division multiple access (OFDMA).

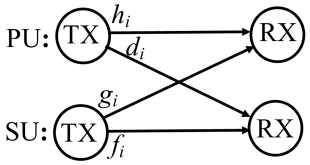

Fig. 1. The channel model for the UCR system in subband i

$[x, x^*]$, is [2]

$$\mathbf{C}_{xx} = \begin{bmatrix} p_x & k_x p_x e^{-j\phi_x} \\ k_x p_x e^{j\phi_x} & p_x \end{bmatrix}, \quad (1)$$

where p_x and ϕ_x are the variance of x and the phase of $\mathbb{E}\{x^2\}$, respectively.

We consider a UCR system, in which both primary and secondary users employ OFDM with N subbands. It is assumed that the PU transmits proper Gaussian signals, while the SU is allowed to employ improper Gaussian signaling. Since only the SU transmits improper signals, the phase can be chosen as $\phi_x = 0$ without loss of generality. Thus, the augmented covariance matrices of the signal transmitted by the PU and SU in subband i are $\mathbf{P}_i = p_i\mathbf{I}$ and

$$\mathbf{Q}_i = \begin{bmatrix} q_i & k_i q_i \\ k_i q_i & q_i \end{bmatrix},$$

respectively, where p_i, q_i, and k_i denote the transmission power of the PU, transmission power of the SU, and circularity coefficient of the SU in subband i, respectively. Hence, the rates of the PU and SU in subband i are [18]

$$R_{p,i} = \frac{1}{2}\log_2 |\mathbf{I} + (\sigma^2\mathbf{I} + \mathbf{G}_i\mathbf{Q}_i\mathbf{G}_i^H)^{-1}\mathbf{H}_i\mathbf{P}_i\mathbf{H}_i^H|,$$

$$R_{s,i} = \frac{1}{2}\log_2 |\mathbf{I} + (\sigma^2\mathbf{I} + \mathbf{D}_i\mathbf{P}_i\mathbf{D}_i^H)^{-1}\mathbf{F}_i\mathbf{Q}_i\mathbf{F}_i^H|, \quad (2)$$

respectively, where σ^2, $\mathbf{H}_i$, $\mathbf{G}_i$, $\mathbf{F}_i$, and $\mathbf{D}_i$ are the variance of the additive noise, PU-PU, SU-PU, SU-SU and PU-SU channels, respectively (see Fig. 1). Note that the channel matrices are diagonal and can be written as a product of the channel coefficient and the identity matrix (e.g., $\mathbf{H}_i = h_i\mathbf{I}$).

In this paper, our goal is to find a transmission strategy, $\{\mathbf{Q}_i\}_{i=1}^N$, for the SU that maximizes its rate, R_s, under the constraint that the rate of the PU, R_p, is ensured to be above a threshold, $\overline{R}$. It is assumed that the transmission power of the PU is given, and the power budget of the SU is $Q_{\max}$. The optimization problem can be formulated as

$$\begin{aligned}
\underset{\{\mathbf{Q}_i\}_{i=1}^N}{\text{maximize}} \quad & R_s = \sum_{i=1}^N R_{s,i} && (3a)\\
\text{s.t.} \quad & R_p \geqslant \overline{R}, && (3b)\\
& \sum_{i=1}^N \text{Tr}(\mathbf{Q}_i) \leqslant 2Q_{\max}, && (3c)\\
& \mathbf{Q}_i \succeq \mathbf{0}, \quad \text{for} \quad i = 1,2,...,N, && (3d)\\
& [\mathbf{Q}_i]_{11} = [\mathbf{Q}_i]_{22}, \quad \text{for} \quad i = 1,2,...,N, && (3e)
\end{aligned}$$

where $[\mathbf{Q}_i]_{kk}$ is the kth element in the kth row of $\mathbf{Q}_i$. We consider two different scenarios, in which the rate constraints are different. In the first scenario, the total rate is greater than a threshold, and thus, (3b) is equivalent to $\sum_{i=1}^N R_{p,i} \geqslant \overline{R}$. In the second, the rate in each subband is greater than a threshold, and thus, (3b) is replaced by the N constraints $R_{p,i} \geqslant \overline{R}_i$, for $i = 1,2,...,N$.

3. PROPOSED TRANSMISSION STRATEGIES

3.1. Constraint on the total rate

We now solve the optimization problem in (3) with the rate constraint $\sum_{i=1}^N R_{p,i} \geqslant \overline{R}$. This problem is not convex since the rate of the PU is a convex function of $\{\mathbf{Q}_i\}_{i=1}^N$ rather than concave. In order to solve the problem, we use the general inner approximation approach [19]. In this approach, the optimization problem is approximated by a convex optimization problem, and solved iteratively. At each iteration, the non-concave constraint is approximated by a concave function. It is known that this approach converges to to a point satisfying the Karush-Kuhn-Tucker (KKT) conditions [20]. Note that the initial point should be in the feasible set of the original problem. Since the channel and PU's power matrices are scaled identity matrices, we will show that (3e) is automatically satisfied in each iteration if the initial point satisfies (3e). Thus, we can safely drop (3e) and consider only (3a)-(3d) in each iteration.

In order to solve the described optimization problem, at each iteration we approximate the rate of the PU by an affine function, which is the closest concave function. We use the first-order term in the Taylor series expansion of the rate of the PU with respect to $\mathbf{Q}_i$ at point $\mathbf{Q}_i^{l-1}$, which is given by the previous iteration. That is

$$R_{p,i}(\mathbf{Q}_i) \simeq R_{p,i}(\mathbf{Q}_i^{l-1}) + \text{Tr}\Big[\nabla_{\mathbf{Q}_i} R_{p,i}(\mathbf{Q}_i^{l-1})\Big(\mathbf{Q}_i - \mathbf{Q}_i^{l-1}\Big)\Big], \quad (4)$$

where $\nabla_{\mathbf{Q}_i} R_{p,i}(\mathbf{Q}_i^{l-1})$ is the derivative of the PU rate with respect to $\mathbf{Q}_i$, which is

$$\begin{aligned}
\nabla_{\mathbf{Q}_i} R_{p,i} = \quad & -\mathbf{G}_i^H(\sigma^2\mathbf{I} + \mathbf{G}_i\mathbf{Q}_i\mathbf{G}_i^H)^{-1}\mathbf{H}_i\mathbf{P}\mathbf{H}_i^H \\
& \times \left(\mathbf{I} + (\sigma^2\mathbf{I} + \mathbf{G}_i\mathbf{Q}_i\mathbf{G}_i^H)^{-1}\mathbf{H}_i\mathbf{P}\mathbf{H}_i^H\right)^{-1} \\
& \times (\sigma^2\mathbf{I} + \mathbf{G}_i\mathbf{Q}_i\mathbf{G}_i^H)^{-1}\mathbf{G}_i.
\end{aligned} \quad (5)$$

By this approximation, the optimization problem at each iteration turns into:

$$\begin{aligned}
\underset{\{\mathbf{Q}_i\}_{i=1}^N}{\text{maximize}} \quad & R_s = \sum_{i=1}^N R_{s,i} && (6a)\\
\text{s.t.} \quad & \sum_{i=1}^N \text{Tr}\left(\mathbf{A}_i^{l-1}\mathbf{Q}_i\right) \leqslant B^{l-1}, && (6b)\\
& \sum_{i=1}^N \text{Tr}(\mathbf{Q}_i) \leqslant 2Q_{\max}, && (6c)\\
& \mathbf{Q}_i \succeq 0, \quad \text{for} \quad i = 1,2,...,N. && (6d)
\end{aligned}$$

In (6), the coefficients $\mathbf{A}_i^{l-1}$ and B^{l-1} can be obtained as

$$\mathbf{A}_i^{l-1} = -\nabla_{\mathbf{Q}_i} R_{p,i}(\mathbf{Q}_i^{l-1}), \quad (7a)$$

$$B^{l-1} = \sum_{i=1}^N R_{p,i}(\mathbf{Q}_i^{l-1}) - \overline{R} - \sum_{i=1}^N \text{Tr}\Big(\nabla_{\mathbf{Q}_i} R_{p,i}(\mathbf{Q}_i^{l-1})\mathbf{Q}_i^{l-1}\Big). \quad (7b)$$

The solution of (6) can be derived by using the dual function and KKT approach. The Lagrangian for the optimization problem (6) can be written as

$$\begin{aligned}
\mathcal{L}(\{\mathbf{U}_i\}_{i=1}^N, \lambda, \mu) = \quad & \mu(\textstyle\sum_{i=1}^N \text{Tr}(\mathbf{Q}_i) - 2Q_{\max}) \\
& - \textstyle\sum_{i=1}^N \frac{1}{2}\log_2 |\mathbf{I} + (\sigma^2\mathbf{I} + \mathbf{D}_i\mathbf{P}_i\mathbf{D}_i^H)^{-1}\mathbf{F}_i\mathbf{Q}_i\mathbf{F}_i^H| \\
& - \textstyle\sum_{i=1}^N \text{Tr}(\mathbf{U}_i\mathbf{Q}_i) + \lambda(\sum_{i=1}^N \text{Tr}(\mathbf{A}_i^{l-1}\mathbf{Q}_i) - B^{l-1}),
\end{aligned} \quad (8)$$

where λ, μ, and $\{\mathbf{U}_i\}_{i=1}^N$ are the Lagrangian multipliers of the constraints (6b), (6c), and (6d), respectively [21]. Equating the derivative of the Lagrangian to zero we obtain

$$\begin{aligned}
\tfrac{\partial\mathcal{L}}{\partial\mathbf{Q}_i} = \mathbf{0} \Rightarrow & -\mathbf{F}_i^H\left[\mathbf{I} + (\sigma^2\mathbf{I} + \mathbf{D}_i\mathbf{P}_i\mathbf{D}_i^H)^{-1}\mathbf{F}_i\mathbf{Q}_i\mathbf{F}_i^H\right]^{-1}\mathbf{F}_i \\
& \times (\sigma^2\mathbf{I} + \mathbf{D}_i\mathbf{P}_i\mathbf{D}_i^H)^{-1} - \mathbf{U}_i + \lambda\mathbf{A}_i^{l-1} + \mu\mathbf{I} = \mathbf{0}.
\end{aligned} \quad (9)$$

Since the coefficient matrices are scaled identity matrices, we can simplify (9) as

$$-\frac{|f_i|^2}{\sigma^2 + |d_i|^2 p_i}\left[\mathbf{I} + \frac{|f_i|^2}{\sigma^2 + |d_i|^2 p_i}\mathbf{Q}_i\right]^{-1} - \mathbf{U}_i + \lambda\mathbf{A}_i^{l-1} + \mu\mathbf{I} = \mathbf{0}. \quad (10)$$

Algorithm I Proposed solution for constraint on the total rate

Initialize $\mathbf{Q}_i^0$ in the feasible set of the problem (3)
Compute $\mathbf{A}_i^0$ and B^0 using (7)
Set $l = 1$

While $||\mathbf{Q}_i^l - \mathbf{Q}_i^{l-1}||_2 \geq \epsilon$ for at least one subband **do**

$$\mathbf{Q}_i^l = \left[(\lambda \mathbf{A}_i^{l-1} + \mu \mathbf{I})^{-1} - \frac{\sigma^2 + |d_i|^2 p_i}{|f_i|^2}\mathbf{I}\right]^+$$

where $\text{Tr}(\mathbf{A}_i^{l-1}\mathbf{Q}_i) = B^{l-1}$ and $\sum_{i=1}^N \text{Tr}(\mathbf{Q}_i) = 2Q_{\max}$
Update $\mathbf{A}_i^l$ and B^l using (7)
Set $l = l + 1$
End (While)

Note that if a constraint is not active, its corresponding Lagrangian multiplier is zero. Thus, if the solution of (10), $\mathbf{Q}_i$, is positive semidefinite for $\mathbf{U}_i = \mathbf{0}$, (6d) is not active, and consequently, its corresponding Lagrangian multiplier, $\mathbf{U}_i$, is zero. Otherwise, $\mathbf{U}_i$ should be determine in a way that the eigenvalues of $\mathbf{Q}_i$ are non-negative. The eigenvectors of $\mathbf{Q}_i$ affect only constraint (6b), which implies that the eigenvectors of $\mathbf{Q}_i$ are equal to the eigenvectors of $\mathbf{A}_i^{l-1}$ (see [19, Lemma 2]). Hence, the optimal solution can be obtained by solving (10) for $\mathbf{U}_i = \mathbf{0}$ and replacing the negative eigenvalues of $\mathbf{Q}_i$, if there are any, by zero. Finally, the closed-form solution for $\mathbf{Q}_i$ is

$$\mathbf{Q}_i = \left[(\lambda \mathbf{A}_i^{l-1} + \mu \mathbf{I})^{-1} - \frac{\sigma^2 + |d_i|^2 p_i}{|f_i|^2}\mathbf{I}\right]^+, \qquad (11)$$

where $\mathbf{Q} = [\mathbf{X}]^+$ denotes an operator that replaces negative eigenvalues of $\mathbf{X}$ with zeros. The Lagrangian multipliers μ and λ can be derived by solving $\sum_{i=1}^N \text{Tr}(\mathbf{Q}_i) = 2Q_{\max}$ and $\sum_{i=1}^N \text{Tr}\left(\mathbf{A}_i^{l-1}\mathbf{Q}_i\right) = B^{l-1}$. Through (11), it can be easily observed that $\mathbf{Q}_i$ follows the feasibility structure (3e) if $\mathbf{A}_i^{l-1}$, and consequently the initial point, satisfies (3e). This solution is iterated until convergence. We summarize the solution in Algorithm I.

When the power constraint is the only active constraint, the optimal solution is proper signaling, which can be obtained by the well-known water-filling approach. As a result, improper signaling can only be beneficial if the rate constraint is active. This is due to the fact that the rate of the SU is a decreasing function of the circularity coefficients; hence, a proper signal maximizes the rate when there is only a power budget constraint. However, when the transmitted power is restricted by interference, the allowed transmission power can be increased by using improper signaling, which may result in a higher achievable rate [7]. But even in this case, the optimal solution may still turn out to be proper, depending on the channel coefficients.

3.2. Constraint on the rate of each subband

We now solve problem (3) assuming a constraint on the rate of the PU in each subband. We employ the analytical results in [7] to implement an algorithm for this optimization problem. In [7], a similar optimization problem for a single-carrier scenario is considered. Here, we extend the results to the OFDM scenario.

Lemma 1. *The signal transmitted on subband i is improper if and only if the following two conditions are met:*

1. *The corresponding channel coefficients satisfy* $\frac{|g_i|^2(\sigma^2+p_i|d_i|^2)}{|f_i|^2\sigma^2} > 1 - \frac{p_i|h_i|^2}{\sigma^2(2^{2\overline{R}_i}-1)}$.
2. *The power allocated to subband i is greater than a given threshold $q_{i,0}^{thr}$, which we derive further below.*

Proof. According to Theorem 1 in [7], improper signaling in subband i is only beneficial if $\frac{|g_i|^2(\sigma^2+p_i|d_i|^2)}{|f_i|^2\sigma^2}$ is greater than a threshold that does not depend on the channel coefficients. When improper signaling is rate-maximizing on a subband, this is due to the fact the maximum allowed power on that subband can be increased by using improper signaling. If the power allocated to this subband is greater than a threshold, which depends on the channel coefficients, the optimal transmission is improper. According to (11) in [7], the maximum allowed power $q_i^{\max}$ is a function of the circularity coefficient, hence we can write $q_i^{\max}(k_i)$. If the power allocated to the subband is greater than $q_{i,0}^{\text{thr}} = q_i^{\max}(0)$, improper signaling is optimal. □

Algorithm II Proposed solution for the rate constraint on each subband

Obtain C_i and D_i, for $i = 1, ..., N$, by (13),
For $i = 1, \ldots, N$
If $D_i > 0$
Improper is beneficial in this subband, and thus
$R_{s,i} = \frac{1}{2}\log_2(C_i + D_i q_i)$,
$q_i = q_{i,l}^{\text{thr}} = \frac{1}{|g_i|^2}\left(\frac{p_i|h_i|^2}{2^{\overline{R}_i}-1} - \sigma^2\right)$, and
$q_{i,u}^{\text{thr}} = \frac{1}{2|g_i|^2}\left(\frac{p_i|h_i|^2}{2^{2\overline{R}_i - R_{p,i}^{\max}}-1} - \sigma^2\right)$
Else
Proper is beneficial in this subband, and thus
$R_{s,i} = \log_2(1 + \frac{|f_i|^2}{\sigma^2+|d_i|^2 p_i} q_i)$, $q_i = q_{i,l}^{\text{thr}} = 0$, and
$q_{i,u}^{\text{thr}} = \frac{1}{|g_i|^2}\left(\frac{p_i|h_i|^2}{2^{\overline{R}_i}-1} - \sigma^2\right)$
End (If)
End (For)

While $q_i = q_{i,l}^{\text{thr}}$ for at least one improper subband **do**

$$q_i = \begin{cases} \left[\frac{1}{\mu} - \frac{C_i}{D_i}\right]_{q_{i,l}^{\text{thr}}}^{q_{i,u}^{\text{thr}}} & \text{if subband is improper} \\ \left[\frac{1}{\mu} - \frac{\sigma^2+|d_i|^2 p_i}{|f_i|^2}\right]_0^{q_{i,u}^{\text{thr}}} & \text{if subband is proper} \end{cases}$$

for $i = 1, \ldots, N$, where μ is such that $\sum_{i=1}^N q_i = Q_{\max}$
For $i = 1, \ldots, N$
If subband is improper and $q_i = q_{i,l}^{\text{thr}}$
Make subband proper, and thus
$R_{s,i} = \log_2(1 + \frac{|f_i|^2}{\sigma^2+|d_i|^2 p_i} q_i)$
$q_{i,l}^{\text{thr}} = 0$, and $q_{i,u}^{\text{thr}} = \frac{1}{|g_i|^2}\left(\frac{p_i|h_i|^2}{2^{\overline{R}_i}-1} - \sigma^2\right)$
End (If)
End (For)
End (While)

According to Lemma 1, improper signaling may be beneficial on a subband depending on the channel coefficients. However, the power allocated to a subband plays an important role as well. Since the SU can allocate power to different subbands, proper transmissions may be the optimal solution even in subbands where improper signaling is potentially beneficial.

We propose an iterative algorithm to solve this problem. In general, if subband i is potentially beneficial for improper signaling, the rate of the SU in this subband can be written as [7]

$$R_{s,i} = \begin{cases} \log_2\left(1 + \frac{|f_i|^2 q_i}{\sigma^2+|d_i|^2 p_i}\right) & \text{if} \quad 0 \leqslant q_i \leqslant q_{i,0}^{\text{thr}}, \\ \frac{1}{2}\log_2\left(C_i + D_i q_i\right) & \text{if} \quad q_{i,0}^{\text{thr}} \leqslant q_i \leqslant q_{i,1}^{\text{thr}}, \end{cases} \qquad (12)$$

where C_i and D_i are constant coefficients, which can be obtained from [7]:

$$C_i = 1 + \frac{|f_i|^4}{|g_i|^4\left(p_i|d_i|^2 + \sigma^2\right)}\left(\frac{p_i^2|h_i|^4}{2^{2\overline{R}_i} - 1} - \sigma^4\right), \qquad (13a)$$

$$D_i = \frac{2|f_i|^2}{p_i|d_i|^2+\sigma^2} + \frac{2|f_i|^4}{|g_i|^2(p_i|d_i|^2+\sigma^2)^2}\left(\frac{p_i|h_i|^2}{2^{2\overline{R}_i}-1} - \sigma^2\right). \qquad (13b)$$

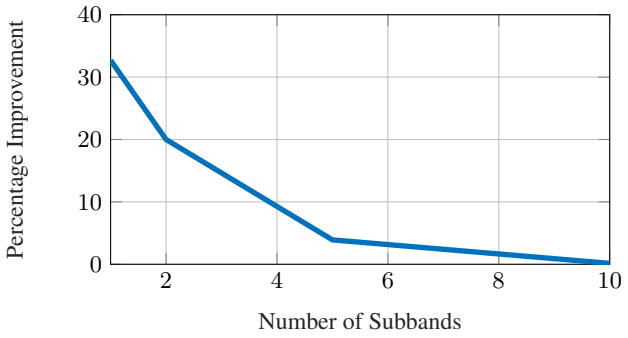

Fig. 2. Improvement by employing improper signaling versus the number of subbands for $\alpha = 70\%$. The results are for the constraint on total rate.

Note that the first condition in Lemma 1 is equal to $D_i > 0$. Finally, the thresholds in (12) are

$$q_{i,0}^{\text{thr}} = \frac{1}{|g_i|^2}\left(\frac{p_i|h_i|^2}{2^{\overline{R}_i}-1} - \sigma^2\right), \tag{14a}$$

$$q_{i,1}^{\text{thr}} = \frac{1}{2|g_i|^2}\left(\frac{p_i|h_i|^2}{2^{2\overline{R}_i - R_{p,i}^{\max}}-1} - \sigma^2\right), \tag{14b}$$

where $R_{p,i}^{\max} = R_{p,i}(q_i = 0)$ refers to the maximum achievable rate of the PU on subband i. The thresholds in (14a) and (14b) are the maximum allowed power of the SU in proper, $k_i = 0$, and maximally improper, $k_i = 1$, cases, respectively.

In our proposed iterative algorithm, it is assumed in the first iteration that an improper signal is transmitted on a subband if an improper signal is potentially beneficial on that subband, i.e., the first condition in Lemma 1 is fulfilled. As a result, the allocated power is constrained to be in the interval $[q_{i,0}^{\text{thr}}, q_{i,1}^{\text{thr}}]$. The rate of the SU is computed based on (12). The solution of this optimization problem is the well-known water-filling approach [16]. If, after solving the optimization in (3), the power allocated to an improper subband is equal to $q_{i,0}^{\text{thr}}$, proper signaling is the optimal solution for this subband. Thus, we update the set of proper and improper subbands and solve again the problem in (3). This procedure is iterated until all the powers allocated to all improper subbands are greater than $q_{i,0}^{\text{thr}}$. The proposed algorithm is summarized in Algorithm II. Note that $q_{i,l}^{\text{thr}}$ and $q_{i,u}^{\text{thr}}$ are lower and upper bounds of the power allocated to each subband in Algorithm II, respectively. Moreover, $[X]_a^b = \max\left(\min(X, b), a\right)$.

4. NUMERICAL RESULTS

In this section, we provide some numerical results. We consider Rayleigh fading channels, in which the real and imaginary parts of the channel coefficients are independent Gaussian random variables $\mathcal{N}(0, 1)$. For the sake of simplicity, it is assumed that the PU transmits with a fixed power P on each subband. The transmitted power of the PU on each subband, P, and variance of the additive Gaussian noise, σ^2 are equal to 1. The results are obtained by averaging over 100 independent channel realizations.

Let us first consider the problem where there is a constraint on the total PU rate. In Fig. 2, the improvement in rate obtained by employing improper signaling is shown for $\alpha = 70\%$, where $\alpha = \frac{\overline{R}}{R_p(\mathbf{q}=0)}$ is the loading factor. In order to consider the effect of identical parallel channels, we assumed that $Q_{\max} = PN$. In other words, the power budget of the SU is increased at the same rate as the transmission power of the PU. The rate improvement by employing improper signaling decreases rapidly with increasing number of subbands, from 30% for $N = 1$ to less than 1% for $N = 10$. This is due to the fact that the SU can allocate power more flexibly when there are more subbands.

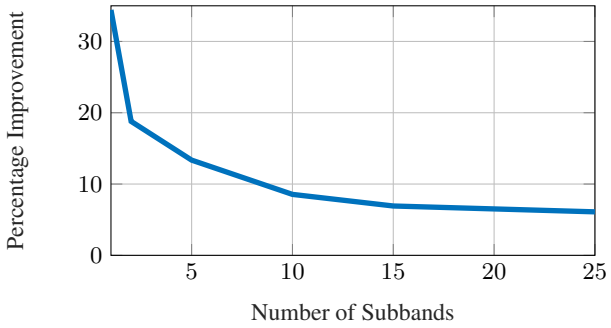

Fig. 3. Improvement by employing improper signaling versus the number of subbands for $\alpha = 70\%$. The results are for the rate constraint on each subband.

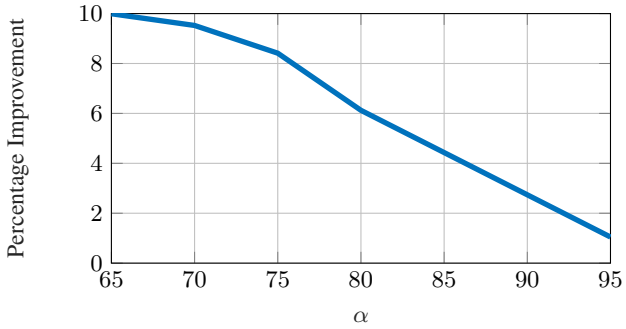

Fig. 4. Improvement by employing improper signals versus α for $Q_{\max} = 20$ and $N = 10$. The results are for the rate constraint on each subband.

Let us now consider the problem where there is a rate constraint on each subband. Figure 3 shows the improvement obtained by employing improper signaling versus the number of subbands for $\alpha = 70\%$ and $Q_{\max} = PN$. As before, the improvements rapidly decrease with an increasing number of subbands.

In Fig. 4, the improvements of improper signaling over proper signaling are shown as a function of α. The improvement is a decreasing function of α. When $N = 10$, which is a relatively low number, the improvement is less than 10% for $\alpha > 65\%$. Moreover, as shown in Fig. 3, this gain decreases with increasing number of subbands.

We notice that improper signaling is more beneficial in the scenario where the rate threshold is on each subband. The reason is that, for example, when $\alpha = 80\%$ it is possible for the PU to neglect one subband out of five without violating the rate constraint. Thus, the SU can transmit proper signals with maximum power on that subband. As we observe in Fig. 3, even though improper signaling is more effective in the second scenario, the benefits also rapidly decrease with increasing number of subbands N.

5. CONCLUSION

Improper signaling has received quite a bit of attention lately as a means to improve achievable rates in interference channels. In this paper, we investigated whether improper signaling is also beneficial in an UCR OFDM system. While there are indeed some benefits, these are minor and mainly apply to scenarios with small number of OFDM subbands. Such a rather negative result may be surprising, but it shows that improper signaling is not a magic tool that works in every case. Rather, its use needs to be justified on a case-by-case basis.

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