DETECTION OF GAUSSIAN SIGNALS IN UNKNOWN TIME-VARYING CHANNELS

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ABSTRACT

Detecting the presence of a white Gaussian signal distorted by a noisy time-varying channel is addressed by means of three different detectors. First, the generalized likelihood ratio test (GLRT) is found for the case where the channel has no temporal structure, resulting in the well-known Bartlett's test. Then it is shown that, under the transformation group given by scaling factors, a locally most powerful invariant test (LMPIT) does not exist. Two alternative approaches are explored in the low signal-to-noise ratio (SNR) regime: the first assigns a prior probability density function (pdf) to the channel (hence modeled as random), whereas the second assumes an underlying basis expansion model (BEM) for the (now deterministic) channel and obtains the maximum likelihood (ML) estimates of the parameters relevant for the detection problem. The performance of these detectors is evaluated via Monte Carlo simulation.

Index Terms— Detection theory, time-varying channels, basis expansion model, generalized likelihood ratio, locally most powerful invariant.

1. INTRODUCTION

The detection of signals in noisy channels is an important problem arising in a wide variety of applications such as sonar [1], radar [2] or spectrum sensing for Cognitive Radio [3]. Although most existing detection rules assume that the channel is time-invariant, such assumption may be unrealistic in many scenarios. For example, in narrowband communications the symbol period can be comparable to the coherence time of the channel. In acoustic communications, such as those for underwater environments, transmissions are affected by large Doppler spreads [1]. Other applications where detectors must operate at very low Signal-to-Noise Ratio (SNR) conditions, e.g. spectrum sensing for Cognitive Radio, require long observation windows within which the channel may change significantly, especially in environments affected by mobility.

Previous work on time-varying channels has addressed estimation, coding, prediction, etc. (see e.g. [4]); however, to the best of our knowledge, little effort has been devoted to activity detection. Works of this kind include [5], where a known constant-magnitude signal is to be detected after propagating through a channel that follows a basis expansion model (BEM) [6]. Nevertheless, the fact that the signal is seldom known in practice calls for different approaches not relying on this prior knowledge.

This work was supported by the the European Regional Development Fund (ERDF) and the Spanish Government under projects DYNACS (TEC2010-21245-C02-02/TCM), COSIMA (TEC2010-19545-C04-03) and COMONSENS (CONSOLIDER-INGENIO 2010 CSD2008-00010), FPU grant AP2010-0149, and by the Galician Regional Government under projects "Consolidation of Research Units" 2009/62 and 2010/85.

To sidestep this issue, we adopt a white Gaussian signal model and present three detectors based on three alternative formulations of the problem. Firstly, the generalized likelihood ratio test (GLRT) [7] is derived for the case where the flat-fading channel is regarded as deterministic with no temporal structure. Secondly, it is shown that, even in the low SNR scenario, the optimal invariant (under scaling operations) detector depends on unknown parameters, and therefore we can conclude that a locally most powerful invariant test (LMPIT) does not exist. In order to avoid this dependency, we propose two different approaches. The first one assigns a prior probability density function (pdf) to the channel, resulting in a test following the Bayesian philosophy [7]. The second approach adopts a BEM for the deterministic channel with unknown parameters, whose invariant component is estimated in a GLRT-like fashion.

The paper is structured as follows: Sec. 2 introduces the data model and states the problem. Sec. 3 derives the GLRT for temporally unstructured (i.e., fast fading) channels. Sec. 4 shows that the LMPIT does not exist and presents two alternative detectors. Finally, some numerical examples illustrate the performance of the proposed detectors in Sec. 5, whereas conclusions are summarized in Sec. 6.

2. DATA MODEL AND PROBLEM FORMULATION

Let us consider an analog waveform, which is bandpass filtered and downconverted to baseband. At the ADC output, N samples are gathered in the vector $\boldsymbol{y} \in \mathbb{C}^N$. Under the null hypothesis $\mathcal{H}_0$, only noise is present: $\boldsymbol{y} = \boldsymbol{w}$, where $\boldsymbol{w} \sim \mathcal{CN}(\boldsymbol{0}, \sigma^2\boldsymbol{I})$ represents a circularly complex white Gaussian noise with zero mean and variance σ^2. Under the alternative hypothesis $\mathcal{H}_1$, a signal passing a frequency-flat time-varying (TV) channel is also received: $\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{w}$, where $\boldsymbol{H} \doteq \operatorname{diag}\{\boldsymbol{h}\}$, $\boldsymbol{h} = [\begin{array}{cccc} h_0 & h_1 & \cdots & h_{N-1} \end{array}]^T$ collects the TV channel gains, and a Gaussian signal model is adopted, so that $\boldsymbol{x} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{I})$.

Given $\boldsymbol{h}$ and σ^2, the data $\boldsymbol{y}$ follow a circular complex Gaussian distribution with zero mean and covariance matrix $\mathbb{E}[\boldsymbol{y}\boldsymbol{y}^H|\boldsymbol{g}, \sigma^2] = \operatorname{diag}(\boldsymbol{g}) + \sigma^2\boldsymbol{I}$, where $\boldsymbol{g} = [\begin{array}{cccc} g_0 & g_1 & \cdots & g_{N-1} \end{array}]^T$ with $g_k \doteq |h_k|^2$ is the vector of instantaneous channel power gains. Thus, the pdf of the observations is

$$p(\boldsymbol{y}|\boldsymbol{g}, \sigma^2) = \frac{1}{\pi^N \prod_{n=0}^{N-1}(g_n + \sigma^2)} e^{-\sum_{n=0}^{N-1} \frac{|y_n|^2}{g_n + \sigma^2}}. \qquad (1)$$

We assume that both the channel $\boldsymbol{h}$ and the noise power σ^2 are unknown. The detection problem can be stated as:

$$\mathcal{H}_0 : \boldsymbol{g} = \boldsymbol{0}, \qquad \mathcal{H}_1 : \boldsymbol{g} \neq \boldsymbol{0}. \qquad (2)$$

In the sequel, in order to take into account any potential structure in the time variation of the channel, we will follow two different approaches. On the one hand, a BEM [6] is used when the channel

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is modeled as an unknown deterministic vector, which allows us to represent the channel as $\boldsymbol{h} = \boldsymbol{F}\boldsymbol{c}$, where $\boldsymbol{F} \in \mathbb{C}^{N \times K}$ is known and has $K \leq N$ orthonormal columns (basis functions), and the vector of coefficients $\boldsymbol{c} \in \mathbb{C}^K$ is unknown. On the other hand, when the channel is modeled as a stochastic process, the only assumption about the sequence of instantaneous channel power gains $\{g_n\}$ is that it is a wide-sense stationary process, i.e. $\bar{\boldsymbol{g}} = \mathbb{E}[\boldsymbol{g}] \propto \boldsymbol{1}$, and $\boldsymbol{R} = \mathbb{E}[(\boldsymbol{g} - \bar{\boldsymbol{g}})(\boldsymbol{g} - \bar{\boldsymbol{g}})^T]$ is a Toeplitz matrix[1]. Note that for time-invariant channels, one has $\boldsymbol{g} \propto \boldsymbol{1}$ and the two hypotheses become indistinguishable, due to the fact that both $\boldsymbol{g}$ and σ^2 are unknown.

3. GENERALIZED LIKELIHOOD RATIO TEST

The GLRT is a well-known approach in which the unknown parameters are replaced by their maximum likelihood (ML) estimates [7]. For the hypothesis testing problem (2), the GLRT is given by

$$\mathcal{L}(\boldsymbol{y}) = \frac{\max\limits_{\sigma^2} \, p(\boldsymbol{y}|\boldsymbol{g} = \boldsymbol{0}, \sigma^2)}{\max\limits_{\boldsymbol{g} \in \mathbb{G}, \sigma^2} \, p(\boldsymbol{y}|\boldsymbol{g}, \sigma^2)} \underset{\mathcal{H}_1}{\overset{\mathcal{H}_0}{\gtrless}} \gamma, \tag{3}$$

where $\mathbb{G}$ denotes the set of feasible channel power gain vectors $\boldsymbol{g}$ (such as those which can be achieved from the BEM $\boldsymbol{h} = \boldsymbol{F}\boldsymbol{c}$).

The ML estimate of σ^2 under the null hypothesis $\mathcal{H}_0$ is given by $\hat{\sigma}^2 = \frac{1}{N}\sum_{n=0}^{N-1} |y_n|^2$. The ML estimation problem under the alternative hypothesis $\mathcal{H}_1$ can be written as

$$\underset{\boldsymbol{g} \in \mathbb{G}, \sigma^2}{\text{minimize}} \sum_{n=0}^{N-1} \left[\log(g_n + \sigma^2) + \frac{|y_n|^2}{g_n + \sigma^2} \right], \tag{4}$$

which is not convex and therefore difficult to solve in general. Although we could resort to iterative numerical methods, possibly affected by local minima, we will not follow that approach here. Instead, we will focus on the limiting case without any particular temporal structure, i.e., $\mathbb{G} = \mathbb{R}_+^N$. (This can be obtained from the BEM if we choose $K = N$, thus modeling fast time variations in the channel). In that scenario, the ML estimates of $\boldsymbol{g}$ and σ^2 satisfy

$$\hat{g}_n + \hat{\sigma}^2 = |y_n|^2, \qquad n = 0, \ldots, N-1, \tag{5}$$

which results in the following GLRT statistic[2]

$$\mathcal{L}_{\text{GLRT}}(\boldsymbol{z}) = \frac{\left(\prod_{n=0}^{N-1} |y_n|^2\right)^{1/N}}{\frac{1}{N}\sum_{n=0}^{N-1} |y_n|^2} \propto \prod_{n=0}^{N-1} z_n, \tag{6}$$

where $z_n \doteq |y_n|^2 / \sum_{n=0}^{N-1} |y_n|^2$, and $\boldsymbol{z} = [\, z_0 \quad \cdots \quad z_{N-1} \,]^T$. That is, the GLRT statistic is given by the geometric over arithmetic mean of the *instantaneous power* of the observations. This test is the well-know Bartlett's test [8] for homoscedasticity (equality of variances). Ratios of geometric to arithmetic means (of periodogram samples) also appear in other contexts under the names of spectral flatness measure or Wiener entropy [9]. Note that the GLRT rejects the null hypothesis $\mathcal{H}_0$ for low values of $\mathcal{L}_{\text{GLRT}}$ in (6), which satisfies $0 \leq \mathcal{L}_{\text{GLRT}} \leq 1$. Moreover, the exact and asymptotic distributions of $\mathcal{L}_{\text{GLRT}}$ can be obtained as particular cases of the results in [10].

[1] Clearly, these conditions are satisfied under the realistic assumption of a wide-sense stationary TV channel $\{h_n\}$.

[2] From now on, we will use $\propto$ to denote equality up to monotone increasing transformations not depending on the observations.

4. TOWARDS LOCALLY BEST INVARIANT TESTS

Although the GLRT usually results in simple and intuitive detection rules with good performance, it is well known that it is not optimal in the Neyman-Pearson sense [7, 11, 12]. In order to explore the possibility of deriving optimal tests, we focus now on the class of tests preserving the invariance of our testing problem under scaling operations, and we consider the challenging case of very close hypotheses (that is, very low SNR). In particular, we obtain the density ratio of the maximal invariant statistics[3] [11,12] with the help of Wijsman's theorem [13], which ensures that (under mild assumptions) this ratio can be obtained by integrating over the group of transformations defining the problem invariances. Thus, the density ratio of the maximal invariant statistic ($\boldsymbol{z}$) for our particular problem is

$$\mathcal{L}(\boldsymbol{z}, \boldsymbol{g}) = \frac{\int_0^\infty p(a\boldsymbol{y}|\boldsymbol{g}, \sigma^2) a^{2N} da}{\int_0^\infty p(a\boldsymbol{y}|\boldsymbol{g} = \boldsymbol{0}, \sigma^2) a^{2N} da}, \tag{7}$$

where $a \in \mathbb{R}_+$ is a scale factor and a^{2N} represents the Jacobian of the transformation. Since we are considering scale invariant tests, the density ratio will not depend on σ^2, and we can assume $\sigma^2 = 1$ without loss of generality. Moreover, noting that the density ratio does not depend on the total power $\sum_{n=0}^{N-1} |y_n|^2$, we can get rid of the denominator in the above expression and write

$$\mathcal{L}(\boldsymbol{z}, \boldsymbol{g}) \propto \int_0^\infty \prod_{n=0}^{N-1} \frac{a^2}{1 + g_n} e^{-\frac{z_n a^2}{1 + g_n}} da. \tag{8}$$

Unfortunately, the right-hand side of (8) still depends on the unknown parameters $\boldsymbol{g}$, and therefore we can conclude that there does not exist a uniformly most powerful invariant test (UMPIT) [11,12]. In order to check whether an LMPIT exists, we focus now on the case of close hypotheses. A second-order Taylor expansion with respect to $\boldsymbol{g}$ of the integral in (8) yields

$$\mathcal{L}(\boldsymbol{z}, \boldsymbol{g}) \simeq \int_0^\infty a^{2N} e^{-a^2} \Psi(a, \boldsymbol{z}, \boldsymbol{g}) da, \tag{9}$$

where

$$\begin{aligned} \Psi(a, \boldsymbol{z}, \boldsymbol{g}) \doteq\, & 1 + (a^2 \boldsymbol{z} - \boldsymbol{1})^T \boldsymbol{g} \\ & + \frac{1}{2} \boldsymbol{g}^T \left[(a^2 \boldsymbol{z} - \boldsymbol{1})(a^2 \boldsymbol{z} - \boldsymbol{1})^T - \text{diag}\left\{ a^2 \boldsymbol{z} - \boldsymbol{1} \right\} \right] \boldsymbol{g}. \end{aligned} \tag{10}$$

Since this expression still depends on unknown parameters, we conclude that an LMPIT does not exist for this problem. Next we propose two alternative ways to overcome this difficulty and obtain useful detectors, namely the introduction of a prior pdf on $\boldsymbol{g}$, and the ML estimation of the relevant parameters for our testing problem.

4.1. Bayesian Approach (Prior on $\boldsymbol{g}$)

The introduction of a prior pdf on the channel coefficients $\boldsymbol{h}$ obviously induces a prior pdf on $\boldsymbol{g}$. Direct application of Wijsman's theorem to this case yields the density ratio

$$\mathcal{L}(\boldsymbol{z}) \propto \int_0^\infty a^{2N} e^{-a^2} \mathbb{E}_{\boldsymbol{g}}[\Psi|\boldsymbol{z}] da. \tag{11}$$

[3] A maximal invariant statistic can be informally defined as an invariant function of the observations containing all the relevant statistical information for the class of invariant detectors.

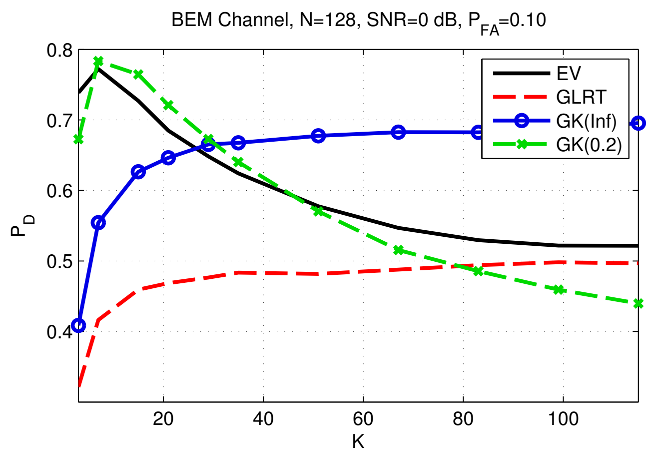

Fig. 1. Probability of detection *vs.* the number of basis functions used in the BEM model.

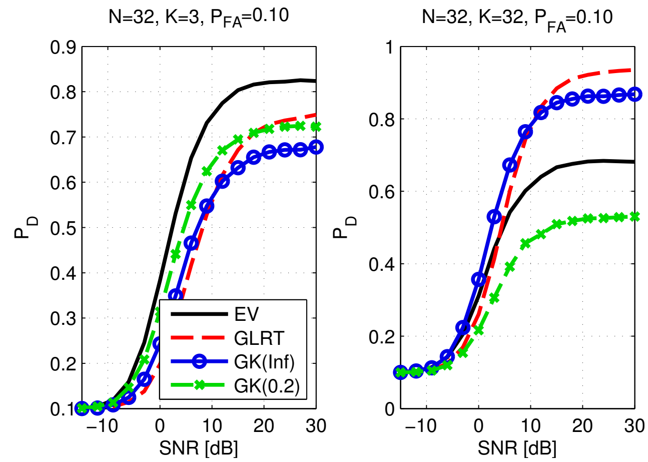

Fig. 2. Probability of detection *vs.* SNR for two values of K (number of basis in the BEM).

Moreover, taking into account that $\mathbb{E}[\boldsymbol{g}] \propto \mathbf{1}$, $\|\boldsymbol{z}\|_1 = 1$, and that the covariance matrix $\boldsymbol{R}$ is a symmetric Toeplitz matrix, this test statistic can be rewritten as

$$\mathcal{L}_{\text{GK}}(\boldsymbol{z}) \propto \boldsymbol{z}^T \boldsymbol{R} \boldsymbol{z}. \tag{12}$$

where "GK" stands for *generalized kurtosis,* whose meaning will become clear later. Thus, the test rejects the null hypothesis $\mathcal{H}_0$ for large values of $\mathcal{L}_{\text{GK}}$. Note that in the limiting case of independent channel gains h_n, the matrix $\boldsymbol{R}$ has the form $\boldsymbol{R} = c\boldsymbol{I}$, where c is a positive constant, and the test statistic reduces to

$$\mathcal{L}_{\text{GK}}(\boldsymbol{z}) \propto \|\boldsymbol{z}\|_2^2 = \frac{\|\boldsymbol{y}\|_4^4}{\|\boldsymbol{y}\|_2^4}, \tag{13}$$

which is always bounded between N^{-1} and 1. Interestingly, this particular test was obtained in a different context (testing for homogeneity of covariance matrices) in [14] following a completely different approach. The test statistic can be seen as a monotone function of the sample excess kurtosis [15]. This is as a direct consequence of the fact that the observations y_n follow a Gaussian distribution under the null hypothesis $\mathcal{H}_0$ and a leptokurtic distribution (with kurtosis $\gamma = 3\text{Var}[g_n]/\mathbb{E}^2[g_n] > 0$) under the alternative $\mathcal{H}_1$.

4.2. Deterministic Approach (Basis Expansion Model)

Another means to avoid the dependency of (9) with the unknown vector $\boldsymbol{g}$ is to follow a GLRT-like approach. Let us start by decomposing the vector $\boldsymbol{h}$ as $\boldsymbol{h} = \|\boldsymbol{h}\|_2 \tilde{\boldsymbol{h}}$, or equivalently, $\boldsymbol{g} = \|\boldsymbol{g}\|_1 \tilde{\boldsymbol{g}}$, where obviously $\|\tilde{\boldsymbol{g}}\|_1 = \tilde{\boldsymbol{g}}^T \mathbf{1} = 1$. Thus, in the low SNR scenario ($\|\boldsymbol{g}\|_1 \ll 1$), the quadratic term in Ψ is negligible, and we can write

$$\Psi(a, \boldsymbol{z}, \|\boldsymbol{g}\|_1, \tilde{\boldsymbol{g}}) = 1 + \|\boldsymbol{g}\|_1 (a^2 \boldsymbol{z} - \mathbf{1})^T \tilde{\boldsymbol{g}}. \tag{14}$$

Now, we propose to find the vector $\tilde{\boldsymbol{h}}$ (or equivalently $\tilde{\boldsymbol{g}}$) maximizing $\Psi(a, \boldsymbol{z}, \|\boldsymbol{g}\|_1, \tilde{\boldsymbol{g}})$, where $\tilde{\boldsymbol{h}} = \boldsymbol{F}\tilde{\boldsymbol{c}}$ and $\|\tilde{\boldsymbol{c}}\|_2 = 1$. To this end, write

$$\Psi(a, \boldsymbol{z}, \|\boldsymbol{g}\|_1, \tilde{\boldsymbol{g}}) \propto \boldsymbol{z}^T \tilde{\boldsymbol{g}} = \tilde{\boldsymbol{c}}^H \boldsymbol{F}^H \boldsymbol{\Lambda}_{\boldsymbol{z}} \boldsymbol{F} \tilde{\boldsymbol{c}}, \tag{15}$$

where $\boldsymbol{\Lambda}_{\boldsymbol{z}} = \text{diag}(\boldsymbol{z})$. Thus, we can see that our problem reduces to the maximization of the correlation between the sequences of *instantaneous power* observations z_n and the instantaneous SNRs g_n, which resembles the idea of subspace matched detectors [12]. Clearly, the solution is given by the principal eigenvector of $\boldsymbol{F}^H \boldsymbol{\Lambda}_{\boldsymbol{z}} \boldsymbol{F}$, and this results in the test statistic

$$\mathcal{L}_{\text{EV}}(\boldsymbol{z}) = \lambda_{\max}\left(\boldsymbol{F}^H \boldsymbol{\Lambda}_{\boldsymbol{z}} \boldsymbol{F}\right), \tag{16}$$

which is bounded between N^{-1} and 1. Note that this test rejects the null hypothesis $\mathcal{H}_0$ for large values of $\mathcal{L}_{\text{EV}}$ and, in the limiting case of lack of temporal channel structure (with $\boldsymbol{F}$ a complete basis), the test statistic reduces to the largest element in $\boldsymbol{z}$.

5. SIMULATION RESULTS

The performance of the proposed detectors is illustrated in this section by means of some numerical examples. In all the experiments, the signal is generated as a circular complex white Gaussian process with zero mean and unit variance, whereas the channel follows a BEM model whose vector of coefficients $\boldsymbol{c}$ is generated as $\mathcal{CN}(\mathbf{0}, \frac{N}{K}\boldsymbol{I}_K)$. This ensures that $\mathbb{E}[\|\boldsymbol{h}\|_2^2] = N$, and the SNR can be defined as $1/\sigma^2$. The columns in $\boldsymbol{F}$ are the K columns with the lowest frequencies in the unitary IDFT matrix. Note that this setting benefits the EV detector, which always uses the actual value of K.

The matrix $\boldsymbol{R}$ used in the GK detector is chosen as $[\boldsymbol{R}]_{ij} = \exp\{-\rho|i-j|\}$. For this reason, we will refer to this test as the GK(ρ) detector. In particular, when ρ is small, $\boldsymbol{R}$ will approach the all-ones matrix (slow fading), whereas for high ρ, $\boldsymbol{R}$ will resemble the identity (fast fading). We will denote the latter case as GK(Inf).

Fig. 1 shows the probability of detection (P_{D}) for fixed false alarm rate $P_{\text{FA}} = 0.1$ *vs.* the number of basis functions K in the BEM, which is proportional[4] to the channel Doppler spread. The EV test is seen to perform better for low values of K, whereas GK(Inf) has the advantage for larger K. The parameter ρ allows to tune how quickly the GK test assumes the channel to vary. As it could be expected, with small ρ the performance is better for slow channels (low K), whereas a larger ρ improves detection for faster channels.

The dependence of P_{D} with respect to the SNR is depicted in Fig. 2, with a low K to the left and with a higher one to the right. Like in Fig. 1, the EV detector exhibits the best performance for $K = 3$. However, as seen for $K = 32$, the GLRT beats the other detectors for sufficiently high SNR. There are two explanations for this behavior: first, in the derivation of the GLRT it was assumed

[4]Due to the choice of $\boldsymbol{F}$, it is be given by $\omega_D = \frac{K-1}{N}\pi$.

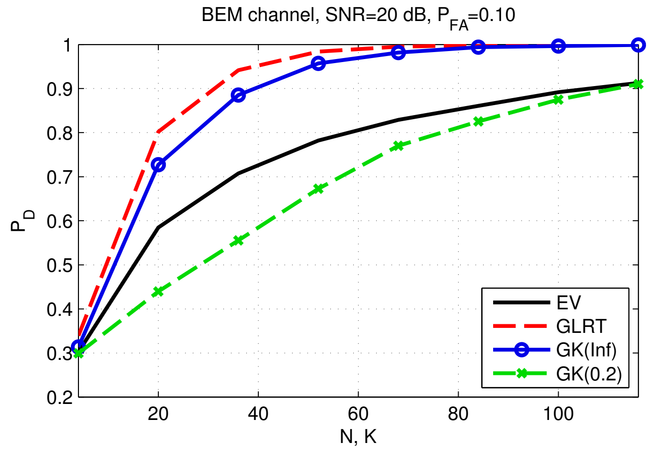

Fig. 3. Performance of the proposed detectors versus the number of samples N.

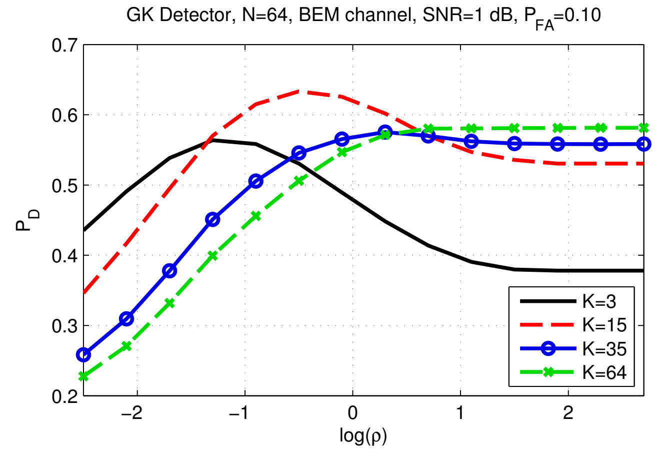

Fig. 4. Influence of the parameter ρ on the performance of the GK detector. Higher values of K require higher values of ρ.

$K = N$ (and in fact, this test is not the true GLRT if $K < N$); and second, the other detectors assume the SNR is low.

It is also noted that $P_{\rm D}$ is bounded away from 1 even as the SNR approaches infinity. In order to elucidate this effect, in Fig. 3 we represent $P_{\rm D}$ for fixed $P_{\rm FA}$ *vs.* N and K, which take on the same values. The SNR was chosen to be high so that the curves are close to their asymptotic values when the SNR approaches infinity. In view of this figure, we can conclude that $P_{\rm D}$ will not approach 1 unless the data record is long enough. For example, with $K = N = 20$, $P_{\rm D}$ remains bounded away from 1 for all of the detectors considered, regardless of the SNR. The GLRT exhibits the highest ceiling due to the same reasons as in the previous paragraph.

Finally, Fig. 4 highlights the importance of correctly setting the parameter ρ in the GK detector. Each curve corresponds to a different channel model. High values of K demand high values of ρ and *vice-versa*. We must also point out that when channel variations are fast (high K) the sensitivity of $P_{\rm D}$ to the value of ρ is low, provided that ρ is high enough. Hence, for fast-fading channels, knowledge of the exact Doppler spread is not as critical as for slow channels.

6. CONCLUSION

Three detectors of Gaussian signals in time-varying channels have been proposed. The first one follows the GLRT approach, whereas the other two focus on the low-SNR scenario and are derived from Wijsman's theorem introducing a prior pdf or an ML estimate. Whereas the first test assumes that the channel has no structure, the other two exploit the available *a priori* information about the Doppler spread of the channel by means of a parameter that has to be properly tuned. Simulation results show that good performance can be achieved for a wide variety of Doppler spreads. Future work will be pointed to the theoretical analysis of the detectors as well as to the derivation of alternative tests.

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