# Order Estimation via Matrix Completion for Multi-Switch Antenna Selection

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***Abstract*—This letter addresses the problem of order estimation for uniform linear arrays (ULAs) with multi-switch antenna selection in the small-sample regime. Multi-switch antenna selection results in a data matrix with missing entries, a scenario for which existing order estimation methods that build on the eigenvalues of the sample covariance matrix do not perform well. A direct application of the Davis-Kahan theorem allows us to show that the signal subspace is quite robust in the presence of missing entries. Based on this finding, this letter proposes a matrix completion (MC) subspace-based order estimation criterion that exploits the shift-invariance property of ULAs. A recently proposed shift-invariant matrix completion (SIMC) method is used for reconstructing the data matrix, and the proposed order estimation criterion is based on the chordal subspace distance between two submatrices extracted from the reconstructed matrix for increasing values of the dimension of the signal subspace. Our simulation results show that the method provides accurate order estimates with percentages of missing entries higher than 50%.**

***Index Terms*—Order estimation, Uniform linear array, MIMO, Multi-switch antenna selection, Matrix completion**

## I. Introduction

The problem of estimating the number of signals received by an array of sensors, also known as source enumeration or order estimation problem is a classical problem in array signal processing. It has applications in numerous fields such as wireless communications, radar, biomedical, and geophysical signal processing [1], [2]. Classical approaches to solving this problem are based on information-theoretic criteria [3]–[6], which use order fitting rules based on functions of the eigenvalues of the sample covariance matrix (SCM) penalized by the model complexity. The performance of these methods drastically degrades in the so-called small-sample regime [7] in which the number of antennas and samples are within the same order of magnitude.

Motivated by the recent use of large-scale arrays, different order estimation methods based on random matrix theory are proposed in the literature for the small-sample regime [7]–[9]. These methods, however, usually provide poor results when the data matrix has missing entries, which is the problem considered in this letter.

A data matrix with missing entries might occur when i) one or more sensors are damaged, or ii) only a few sensors are intentionally sampled to reduce the overall hardware cost. The latter is the case when using a multi-switch antenna selection architecture [10], [11] in which, at every time instant, a random switch selects a subset of antennas whose RF signals are downconverted and further processed. The data matrix with multi-switch antenna selection has therefore missing entries. A low-rank matrix completion (MC) approach [12] that exploits the shift-invariance property (SIP) of ULAs has recently been proposed in [11] to recover the complete data matrix as if it had been received by the full array. Once the data matrix has been reconstructed, the direction-of-arrival (DOA) of the received signals can readily be estimated. A similar idea has been applied in [13] for multiple-input and multiple-output (MIMO) radar. The shift-invariant matrix completion (SIMC) method proposed in [11], however, assumes that the number of sources is known.

This letter addresses the order estimation problem in the missing data scenario for uniform linear arrays (ULAs) in the presence of spatially white noise. The eigenvalue-based order estimation methods such as LSMDL [8] and BIC [9] do not perform well with missing entries. In addition, as we have already pointed out, existing MC reconstruction methods require the order to be known. In this letter, based on the Davis-Kahan theorem [14], [15] we first show that the signal subspace changes gradually with missing data. Motivated by this result, we propose a subspace-based order estimation criterion, which exploits the shift-invariance property of ULAs. The proposed approach uses the SIMC algorithm [11] for increasing values of dimension and the order is estimated by evaluating chordal subspace distance (CSD) [16] between two submatrices extracted from the reconstructed matrix.

## II. Problem Statement

Let us consider $K$ narrowband signals impinging on a large half-wavelength ULA with $M$ antennas. For a fully digital receiver with $M$ RF-branches, the received signal at time instant or snapshot $n$ is

$$\mathbf{z}[n] = [\mathbf{a}(\theta_1), \cdots, \mathbf{a}(\theta_K)]\,\mathbf{s}[n] + \mathbf{e}[n] = \mathbf{A}\,\mathbf{s}[n] + \mathbf{e}[n], \quad (1)$$

where $\mathbf{e}[n]$ is the noise vector, $\mathbf{s}[n] = [s_1[n], \ldots, s_K[n]]^T$ is the signal vector with complex gains $s_k[n]$, $\mathbf{A} = [\mathbf{a}(\theta_1), \cdots, \mathbf{a}(\theta_K)]$ is the steering matrix and $\mathbf{a}(\theta_k) = \begin{bmatrix} 1 & e^{-j\theta_k} & \cdots & e^{-j\theta_k(M-1)} \end{bmatrix}^T$ is the $M \times 1$ complex array response to the $k$th source with electrical angle $\theta_k$,

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which is unknown. The signals are assumed to be uncorrelated, and the noise is spatially white: they are modeled as $\mathbf{s}[n] \sim \mathcal{CN}_K(\mathbf{0}, \mathbf{\Sigma})$ with $\mathbf{\Sigma} = \text{diag}(\sigma_1^2, \ldots, \sigma_K^2)$, and $\mathbf{e}[n] \sim \mathcal{CN}_M(\mathbf{0}, \sigma^2\mathbf{I})$, respectively. Using the signal model in (1), the full $M \times M$ covariance matrix is

$$\mathbf{R} = E\left[\mathbf{z}[n]\mathbf{z}^H[n]\right] = \mathbf{R}_s + \sigma^2\mathbf{I} \tag{2}$$

where $\mathbf{R}_s = \mathbf{A}\mathbf{\Sigma}\mathbf{A}^H$. After collecting $N$ snapshots, the full data matrix $\mathbf{Z}=\left[\mathbf{z}[1], \ldots, \mathbf{z}[N]\right]$ can be written as $\mathbf{Z}=\mathbf{X}+\mathbf{E}$, where $\mathbf{E}=\left[\mathbf{e}[1], \ldots, \mathbf{e}[N]\right]$, and $\mathbf{X}=\mathbf{A}\mathbf{S}$ is the noiseless signal component with $\mathbf{S}=\left[\mathbf{s}[1], \ldots, \mathbf{s}[N]\right]$.

We consider a multi-switch antenna selection receiver [10], [11] in which $L$ out of the $M$ antennas are randomly selected at each time instant or snapshot to be downconverted and sampled at baseband. The resulting $L \times N$ samples are arranged in a matrix $\mathbf{Z}_d \in \mathbb{C}^{M \times N}$ with the missing entries replaced by zeros. The sampling process can be compactly expressed as $\mathbf{Z}_d = P_\Omega(\mathbf{Z})$, where $\Omega \subseteq \{1, \ldots, M\} \times \{1, \ldots, N\}$ is the set of observed (antenna, time) indexes, and $P_\Omega$ is a projection operator that sets to zero the missing entries and leaves the observed ones unchanged.

The problem addressed in this letter is, given the observed data matrix $\mathbf{Z}_d$, to estimate the number of sources $K$. We assume that $K$ satisfies $K \ll L < M$.

## III. Subspace Perturbation with Missing Data

Order estimation methods such as LSMDL [8] and BIC [9], which are based on the eigenvalues of the sample covariance matrix (SCM) given by $\hat{\mathbf{R}} = \frac{1}{N}\mathbf{Z}\mathbf{Z}^H$, do not perform well with missing entries, i.e. when $\mathbf{Z}=\mathbf{Z}_d$. This motivates to the study of subspace-based order estimation methods [17]–[20]. In this section, we study the impact of missing data on the principal angles of the signal subspace of the covariance matrix in (2). To do so, we analyze the problem from a matrix perturbation standpoint, noting that the eigenvectors of the signal subspace of $\hat{\mathbf{R}}$ are also left singular vectors of $\mathbf{Z}$ associated with the $K$ largest singular values. A perturbed matrix is a matrix altered after the addition of a second matrix. Here, we regard $\mathbf{Z}_d$ as a perturbed version of $\mathbf{Z}$, with the perturbation being caused by the missing entries. The Davis-Kahan theorem [14], [15], adapted to our setup in Theorem 1, is a useful tool to measure the angular difference between singular vectors of two matrices.

*Theorem 1:* [15] Let $\mathbf{Z}$, $\mathbf{Z}_d \in \mathbb{C}^{M \times N}$ have singular values $\sigma_1 \geq \ldots \geq \sigma_{\min(M,N)}$ and $\hat{\sigma}_1 \geq \ldots \geq \hat{\sigma}_{\min(M,N)}$ respectively. Fix $1 \leq r \leq s \leq \text{rank}(\mathbf{Z})$ and assume that $\min(\sigma_{r-1}^2 - \sigma_r^2, \sigma_s^2 - \sigma_{s+1}^2) > 0$, where $\sigma_0^2 := \infty$ and $\sigma_{\text{rank}(\mathbf{Z})+1}^2 := -\infty$. Let $d = s - r + 1$, and let $\mathbf{U} = [\mathbf{u}_r, \mathbf{u}_{r+1}, \ldots, \mathbf{u}_s] \in \mathbb{C}^{M \times d}$ and $\hat{\mathbf{U}} = [\hat{\mathbf{u}}_r, \hat{\mathbf{u}}_{r+1}, \ldots, \hat{\mathbf{u}}_s] \in \mathbb{C}^{M \times d}$, where $\mathbf{u}_r$ and $\hat{\mathbf{u}}_r$ are the $r$th left singular vectors of $\mathbf{Z}$ and $\mathbf{Z}_d$, respectively. Then

$$||\sin\Theta(\hat{\mathbf{U}}, \mathbf{U})||_F \leq 2\,(2\sigma_1 + \|\mathbf{Z}_d - \mathbf{Z}\|_2) \frac{\min\left(d^{1/2}\|\mathbf{Z}_d - \mathbf{Z}\|_2, \|\mathbf{Z}_d - \mathbf{Z}\|_F\right)}{\min(\sigma_{r-1}^2 - \sigma_r^2, \sigma_s^2 - \sigma_{s+1}^2)} \tag{3}$$

In the Davis-Kahan theorem, $\sin\Theta(\hat{\mathbf{U}}, \mathbf{U})$ is defined entrywise, and $\Theta(\hat{\mathbf{U}}, \mathbf{U})$ is a $d \times d$ diagonal matrix whose $j$th diagonal entry is the principal angle $\cos^{-1}\rho_j$, being $\rho_j$ the $j$th singular value of $\hat{\mathbf{U}}^H\mathbf{U}$. Then, the lower $||\sin\Theta(\hat{\mathbf{U}}, \mathbf{U})||_F$ is, the more similar the subspaces spanned by the columns of $\hat{\mathbf{U}}$ and $\mathbf{U}$ are. Since the singular vectors corresponding to the largest singular values span the signal subspace, we set $r = 1$ and $s = t$ in (3) and obtain the following upper bound

$$||\sin\Theta(\hat{\mathbf{U}}_t, \mathbf{U}_t)||_F \leq \frac{2(2\sigma_1 + \|\mathbf{Z}_d - \mathbf{Z}\|_2)\|\mathbf{Z}_d - \mathbf{Z}\|_F}{\sigma_t^2 - \sigma_{t+1}^2} \tag{4}$$

where $\mathbf{U}_t=[\mathbf{u}_1, \ldots, \mathbf{u}_t]$ and $\hat{\mathbf{U}}_t=[\hat{\mathbf{u}}_1, \ldots, \hat{\mathbf{u}}_t]$. The bound on the proximity of the $t$ largest left singular vectors of $\mathbf{Z}$ and $\mathbf{Z}_d$ in (4) increases with the Frobenius norm of the difference between the perturbed matrix $\mathbf{Z}_d$ and the complete matrix $\mathbf{Z}$ as $\mathcal{O}\left((M-L)^{1/2}\right)$, but also with $(\sigma_t^2 - \sigma_{t+1}^2)^{-1}$, which depends on the difference between consecutive singular values of $\mathbf{Z}$. Since, in general, $(\sigma_K^2 - \sigma_{K+1}^2) >> (\sigma_t^2 - \sigma_{t+1}^2)$ for $t > K$, we expect the signal subspaces of $\mathbf{Z}$ and $\mathbf{Z}_d$ to be similar, as shown in expression (4) when particularized to $t=K$, but much more dissimilar subspaces when some singular vectors of the noise subspace are included in $\mathbf{U}_t$ and $\hat{\mathbf{U}}_t$, i.e. when $t>K$ in (4).

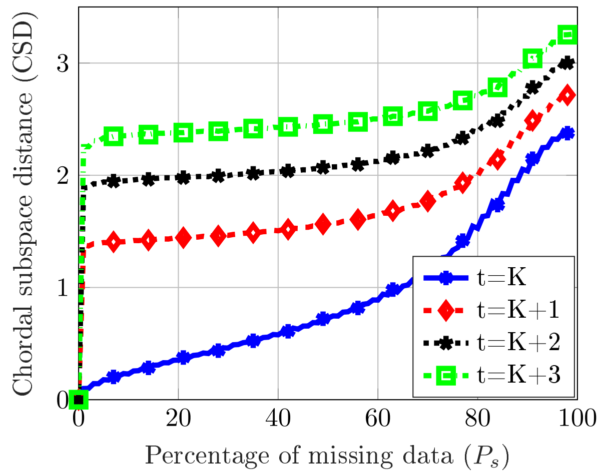


Fig. 1: CSD vs. $P_s$ for $M = 50$, $N = 50$, $K = 3$, $\Delta_\theta = 10°$ and SNR $= 20$ dB for different values of $t$.

This analysis is illustrated in Fig. 1, which shows the CSD between $\mathbf{U}_t$ and $\hat{\mathbf{U}}_t$, defined as

$$d_{cs} = \frac{1}{\sqrt{2}}\|\mathbf{U}_t\mathbf{U}_t^H - \hat{\mathbf{U}}_t\hat{\mathbf{U}}_t^H\|_F, \tag{5}$$

versus the percentage of missing data ($P_s$) for different values of $t \geq K$, and for $M = 50$, $N = 50$, $K = 3$, $\Delta_\theta = 10°$ and SNR=20 dB, where $\Delta_\theta$ is the source separation and SNR is the signal-to-noise-ratio defined as $\text{SNR}=10\log\frac{\text{tr}(\mathbf{R}_s)}{M\sigma^2}$, where $\text{tr}(\cdot)$ denotes the trace. Here, the number of sampled sensors is $L = \lfloor\frac{M(100-P_s)}{100}\rfloor$, where $\lfloor\cdot\rfloor$ is the floor function. Clearly the signal subspace of $\mathbf{Z}_d$ changes gradually compared to the signal subspace of $\mathbf{Z}$ as the number of missing entries increases, whereas for $t > K$ the subspace distance increases abruptly with $P_s$ due to the addition of noise eigenvectors.

## IV. Order Estimation using Matrix Completion

This section discusses a novel approach for order estimation with missing data that exploits the SIP of ULAs. The proposed approach uses the recently proposed SIMC algorithm [11] for increasing values of the signal subspace dimension, and the order is estimated by evaluating the CSD between two different estimates of the signal subspace extracted from the reconstructed matrix.

### A. Shift Invariance property (SIP)

When ULAs are used, a property called shift-invariance holds, which forms the basis of the ESPRIT method [21], [22] and its many variants. According to this property, if we extract two subarrays of size $M-1$ by keeping the first and the last $M-1$ sensors of an $M$-sensor array, the steering matrices $\mathbf{A}_1$ and $\mathbf{A}_2$ of these subarrays are related by a unitary diagonal matrix

$$\mathbf{A}_1 \text{diag}(e^{-j\theta_1}, \ldots, e^{-j\theta_K}) = \mathbf{A}_2$$

and consequently they span the same subspace of dimension $K$. In a noiseless case without missing data, if we extract $K$-dimensional signal subspaces from $\hat{\mathbf{R}}^{\uparrow}$ and $\hat{\mathbf{R}}^{\downarrow}$, which denote the sample covariance matrices built from the first and the last $M-1$ sensors of the array, the CSD between these two subspaces is zero. In a noisy situation, the CSD will not be exactly zero, but will have a small value for sufficiently high SNR values. A large value of the CSD implies that the $K$-dimensional subspaces extracted from $\hat{\mathbf{R}}^{\uparrow}$ and $\hat{\mathbf{R}}^{\downarrow}$ are far apart from each other and, consequently, the shift-invariance property does not hold.

### B. Shift-invariant Matrix Completion (SIMC)

A technique to reconstruct the signal matrix from the sparse $\mathbf{Z}_d$ that exploits the SIP of ULA and the low-rank structure of the noiseless data matrix has been proposed in [11]. The problem of estimating the noiseless low-rank signal matrix $\mathbf{X}$ from $\mathbf{Z}_d \in \mathbb{C}^{M\times N}$, where $\mathbf{Z}_d = P_\Omega(\mathbf{Z})$ and $\mathbf{Z} = \mathbf{X} + \mathbf{E}$, amounts to solving [12]

$$\begin{aligned} \min_{\mathbf{X}\in\mathbb{C}^{M\times N}} \quad & ||\mathbf{X}||_* \\ \text{subject to} \quad & ||P_\Omega(\mathbf{X}-\mathbf{Z}_d)||_F \leq \eta \end{aligned} \tag{6}$$

where $||\mathbf{X}||_*$ denotes the nuclear norm, and $\eta > 0$ is a tolerance parameter that limits the fitting error.

Matrix $\mathbf{X}$ can be factored as $\mathbf{X} = \mathbf{W}\mathbf{H}^H$, where $\mathbf{W} \in \mathbb{C}^{M\times p}$ and $\mathbf{H} \in \mathbb{C}^{N\times p}$, and $p$ limits the rank of the reconstructed matrix. Then, using the identity $||\mathbf{X}||_* = \min_{\mathbf{X}=\mathbf{W}\mathbf{H}^H} \frac{1}{2}\left(\left\|\mathbf{W}\right\|_F^2 + \left\|\mathbf{H}\right\|_F^2\right)$, $\mathbf{X}$ can be estimated by solving the optimization problem [23]

$$\begin{aligned} \{\hat{\mathbf{W}}, \hat{\mathbf{H}}\} = \underset{\substack{\mathbf{W}\in\mathbb{C}^{M\times p}\\ \mathbf{H}\in\mathbb{C}^{N\times p}}}{\operatorname{argmin}} & \left\|P_\Omega\left(\mathbf{Z}_d - \mathbf{W}\mathbf{H}^H\right)\right\|_F^2 \\ & + \mu\left(\left\|\mathbf{W}\right\|_F^2 + \left\|\mathbf{H}\right\|_F^2\right), \end{aligned} \tag{7}$$

where $\mu$ is a regularization parameter. The SIMC method in [11] searches for a solution that satisfies the SIP by enforcing the relation

$$\mathbf{w}_i^H \mathbf{T} = \mathbf{w}_{i-1}^H \qquad i = 2, \ldots, M \tag{8}$$

where $\mathbf{w}_i^H$ is the $i$th row of $\mathbf{W}$, and $\mathbf{T} \in \mathbb{D}$, where $\mathbb{D}$ is the set of $p\times p$ diagonal complex matrices not necessarily unitary. To enforce (8), the SIMC cost function includes an additional regularization term as follows

$$\begin{aligned} \{\hat{\mathbf{W}}, \hat{\mathbf{H}}, \hat{\mathbf{T}}\} = \underset{\substack{\mathbf{W}\in\mathbb{C}^{M\times p}\\ \mathbf{H}\in\mathbb{C}^{N\times p}\\ \mathbf{T}\in\mathbb{D}}}{\operatorname{argmin}} & \sum_{(i,j)\in\Omega} \left|\mathbf{Z}_d(i,j) - \mathbf{w}_i^H\mathbf{h}_j\right|^2 \\ & + \mu\Big(\sum_{i=1}^{M} \|\mathbf{w}_i\|_2^2 + \sum_{j=1}^{N} \|\mathbf{h}_j\|_2^2\Big) \\ & + \alpha \sum_{i=2}^{M} \|\mathbf{w}_i^H\mathbf{T} - \mathbf{w}_{i-1}^H\|_2^2 \end{aligned} \tag{9}$$

where $\alpha$ is an additional regularization parameter. The solution $\hat{\mathbf{X}}_{SIMC} = \hat{\mathbf{W}}\hat{\mathbf{H}}^H$ can be obtained by iteratively optimizing (9) over each $\mathbf{w}_i^H$, $\mathbf{h}_j^H$ and $\mathbf{T}$ until convergence.

Finally, the SIMC method applies as a last post-processing stage the Optimal Subspace Estimation (OSE) technique [24]–[26], which takes $\hat{\mathbf{X}}_{SIMC}$ as input and provides a rank-$p$ covariance matrix $\hat{\mathbf{R}}_{SIMC}$ with the required shift-invariant structure as output. For a full account of the SIMC method, the reader is referred to [11].

### C. Order Estimation via SIMC (OE-SIMC)

Since $\hat{\mathbf{R}}_{SIMC}$ satisfies the shift-invariance property, the CSD between the $p$-dimensional subspaces extracted from $\hat{\mathbf{R}}_{SIMC}^{\uparrow} := \hat{\mathbf{R}}_{SIMC}(1:M-1, 1:M-1)$ and $\hat{\mathbf{R}}_{SIMC}^{\downarrow} := \hat{\mathbf{R}}_{SIMC}(2:M, 2:M)$ should take a small value when $p = K$ and a large value otherwise. The explanation of this behavior is that the MC algorithm does not provide an accurate signal subspace estimate for $p < K$, whereas for $p > K$ the extracted signal subspace will include some noise directions. In both situations, the CSD increases. In summary, this reasoning shows that the CSD reaches its minimum for p = K, this minimum value being zero in a noiseless situation. Based on this observation, an order estimation criterion is proposed, which first reconstructs $\hat{\mathbf{R}}_{SIMC}$ for increasing values of $p$, and then estimates the CSD between two $p$-dimensional subspaces extracted from $\hat{\mathbf{R}}_{SIMC}^{\uparrow}$ and $\hat{\mathbf{R}}_{SIMC}^{\downarrow}$ as follows.

Let $\mathbf{U}^{\uparrow} \in \mathbb{C}^{(M-1)\times p}$ and $\mathbf{U}^{\downarrow} \in \mathbb{C}^{(M-1)\times p}$ be the $p$ largest eigenvectors of $\hat{\mathbf{R}}_{SIMC}^{\uparrow}$ and $\hat{\mathbf{R}}_{SIMC}^{\downarrow}$, respectively. The order estimation criterion using SIMC, denoted as OE-SIMC, is

$$\hat{K}_{OE-SIMC} = \underset{1\leq p\leq p_{max}}{\operatorname{argmin}} \frac{\|\mathbf{U}^{\uparrow}\mathbf{U}^{\uparrow H} - \mathbf{U}^{\downarrow}\mathbf{U}^{\downarrow H}\|_F}{p} \tag{10}$$

where $p_{max}$ is an overestimation of $K$. Since $\mathbf{U}^{\uparrow}$ and $\mathbf{U}^{\downarrow}$ are extracted for an increasing number of dimensions, the order estimation criterion in (10) is normalized with the number of eigenvectors used in each case. A summary of the proposed method is shown in Algorithm 1.

With regards to the computational cost of the method, SIMC requires $\mathcal{O}((M+N)p^3)$ multiplications per iteration to solve (9), plus the cost of the OSE step which is $\mathcal{O}(M^2N) + 2\mathcal{O}((Mp)^3)$. In addition, OE-SIMC requires a compact SVD with a cost of $\mathcal{O}(Mp^2)$ to obtain the eigenvectors required to compute the CSD. The overall cost of OE-SIMC is obtained after multiplying these quantities by $p_{max}$.

**Algorithm 1:** Order Estimation via SIMC (OE-SIMC)

**Input:** $\mathbf{Z}_d$, $p_{max}$
**Output:** Order estimate $\hat{K}_{OE-SIMC}$
**for** $p = 1, \ldots, p_{max}$ **do**
  Find $\hat{\mathbf{R}}_{SIMC}$ using SIMC and extract $\hat{\mathbf{R}}^{\uparrow}_{SIMC}$ and $\hat{\mathbf{R}}^{\downarrow}_{SIMC}$
  Find $\mathbf{U}^{\uparrow}$ and $\mathbf{U}^{\downarrow}$ as the $p$ largest eigenvectors of $\hat{\mathbf{R}}^{\uparrow}_{SIMC}$ and $\hat{\mathbf{R}}^{\downarrow}_{SIMC}$
Estimate the number of sources using (10)

## V. Simulation Results

In this section, the performance of the proposed order estimation criterion is illustrated by means of Monte Carlo simulations. For all examples, we assume that $K$ equal-power uncorrelated narrowband signals with a separation of $\Delta_\theta$ are impinging on a ULA with $M$ half-wavelength separated antennas. We consider $p_{max}=\lfloor M/5 \rfloor$, $L$ denotes the number of randomly sampled sensors per snapshot, and $P_s$ denotes the percentage of missing entries in the data matrix.

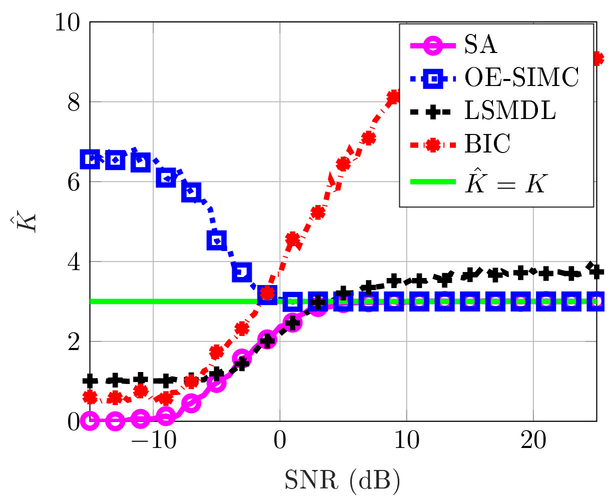


Fig. 2: $\hat{K}$ vs. SNR for $M=50$, $N=50$, $P_s=75\%$, $L=12$, $K=3$ and $\Delta_\theta=10°$.

The first experiment shows the estimated number of sources $\hat{K}$ (mean value) vs. SNR for $M=50$, $N=50$, $P_s=75\%$, $L=12$, $K=3$ and $\Delta_\theta=10°$. For comparison, we select LSMDL [8] and BIC [9] as eigenvalue-based methods, and the subspace averaging (SA) method proposed in [19]. Fig. 2 shows that OE-SIMC and SA provide good results. On the contrary, LSMDL and BIC perform poorly with missing data and, therefore, they are not included in the next experiments.

In the next experiment, we consider a scenario with $M=50$, $N=50$, SNR$=10$ dB, $K=5$ and $\Delta_\theta=10°$. Fig. 3 depicts the probability of correct detection ($P_d$) when $P_s$ is varied between $0\%$ and $95\%$. As observed, SA performs well for a wide range of $P_s$ values, but OE-SIMC outperforms SA and provides good results with percentages of missing entries higher than $75\%$.

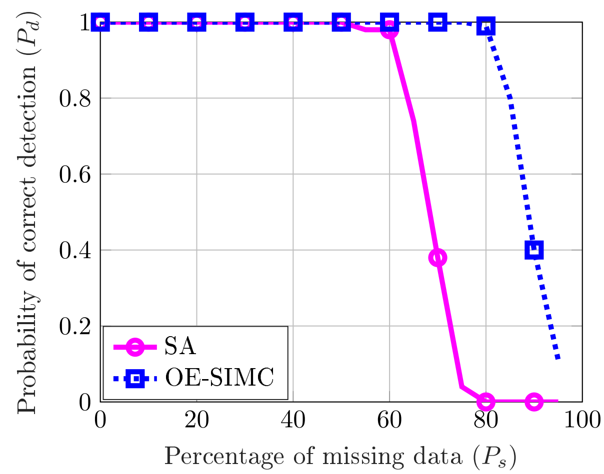


Fig. 3: $P_d$ vs. $P_s$ for $M=50$, $N=50$, SNR$=10$ dB, $K=5$ and $\Delta_\theta=10°$.

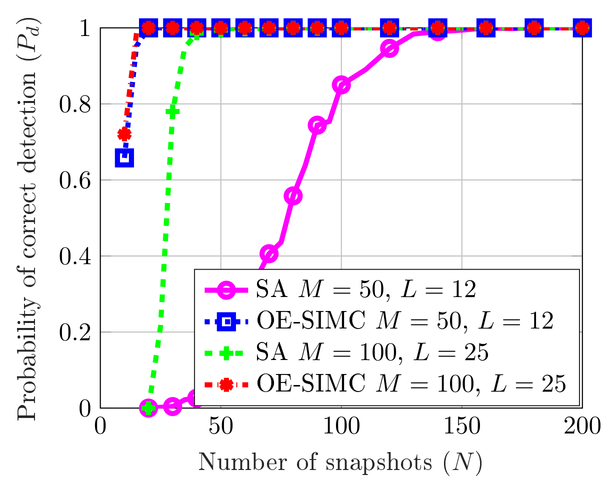


Fig. 4: $P_d$ vs. $N$ for $P_s=75\%$, SNR$=20$ dB, $K=5$ and $\Delta_\theta=10°$ for i) $M=50$ and $L=12$, and ii) $M=100$ and $L=25$.

In the last experiment, $P_d$ vs. $N$ is evaluated for two scenarios: i) $M=50$ and $L=12$, and ii) $M=100$ and $L=25$, when other parameters are fixed to $P_s=75\%$, SNR$=20$ dB, $K=5$ and $\Delta_\theta=10°$. As Fig. 4 shows, for $M=50$ SA requires around 150 snapshots to perform well but OE-SIMC starts providing good results when $N$ is as small as 20. For $M=100$, SA starts performing well in the small-sample regime, but the performance of OE-SIMC is almost unaffected by $M$.

## VI. Conclusions

In this letter, we have studied the order estimation problem with missing entries for multi-switch antenna selection receivers. We showed that the signal subspace changes gradually with missing entries and then proposed a subspace-based order estimation criterion based on the chordal subspace distance between two submatrices extracted from the reconstructed data matrix after matrix completion. The simulation results indicate that the proposed method performs well in the small-sample regime, even with a small percentage of sampled antennas.

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