Pre- and Post-FFT Interference Leakage Minimization for MIMO OFDM Networks

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***Abstract*—Interference alignment (IA) has been shown to achieve the maximum degrees of freedom in the multiple-input multiple-output (MIMO) K-user interference channel (IFC). In the presence of frequency-selective channels, orthogonal frequency-division multiplexing (OFDM) is typically used to deal with the multipath nature of the channel. While IA techniques can be applied in a per-subcarrier basis (post-FFT), the existence of symbol timing offsets (STOs) between the desired and the interfering OFDM symbols decreases the system performance dramatically. To solve this problem, we design pre-FFT precoders and decoders for single-beam MIMO IFCs for OFDM transmissions. Since the IA decoders operate before the FFT, they mitigate the interference before synchronization takes place. We show that our proposed scheme improves the system performance when STOs occur, in comparison with traditional post-FFT IA techniques. We provide simulation results to compare post- and pre-FFT beamforming techniques and to illustrate the performance of the proposed method.**

***Index Terms*—Channel shortening, interference alignment, interference MIMO channel, OFDM, pre-FFT, post-FFT.**

I. INTRODUCTION

Orthogonal frequency-division multiplexing (OFDM) is used often to deal with frequency-selective wireless channels. In the K-user MIMO interference channel (IFC) with OFDM transmissions, IA algorithms can be applied in a per-subcarrier basis (which is henceforth referred to as post-FFT IA).[1] For this technique to work in practice, all transmitted OFDM symbols must be time-aligned. If uncoordinated, the asynchronous interferences impairs the detection of the desired OFDM symbols, and thus the performance of the post-FFT IA techniques is significantly degraded. Therefore, the interference must be mitigated before synchronization takes place. In addition to this problem, post-FFT schemes require larger system and computational complexity than their pre-FFT counterparts, especially when the number of antennas and/or subcarriers is large. In beamforming problems, pre-FFT techniques have been proposed as a low-complexity approach for MIMO-OFDM systems [1]–[3].

In [4], the alternating minimization algorithm for the K-user MIMO IFC [5] was extended to the case of convolutive MIMO channels under single-carrier transmissions. Space-time precoding and decoding was applied showing that multiuser interference (MUI) cannot be completely nullified. In [4] it was also shown that other issues not present in the flat-fading case come up when users transmit over frequency-selective channels.

In this paper, we propose a pre-FFT interference leakage (IL) minimization algorithm for the K-user single-beam MIMO IFC with OFDM transmissions, which is an extension of the work in [4] to the multicarrier case. With the proposed algorithm, the interference can be minimized before the synchronization process, and hence it is not affected by symbol timing offsets (STOs) between users. The proposed pre-FFT scheme takes a more general form than the aforementioned pre-FFT beamforming, as we consider space-time precoders and decoders of length L, which allow us to trade off performance for computational complexity. Our contributions with respect to [4] are as follows: We extend the algorithm in [4] to OFDM signals. We incorporate a penalty term in the IL cost function that penalizes the length of the equivalent channel, based on the sum-squared autocorrelation minimization (SAM) [6], and hence takes the ISI and the intercarrier interference (ICI) into account. In other words, we jointly minimize the IL and perform channel shortening. We use the positive real lemma [7] to reformulate the spectral mask constraints required by the problem as linear matrix inequality (LMI) in time domain, thereby reducing considerably the computational complexity of the original method in [4].

The remainder of the paper is organized as follows: Section II describes the system model. The proposed algorithm is derived in Section III. Section IV provides simulations to evaluate the performance of the algorithm and to compare it with post-FFT IA. Finally, Section V concludes the paper.

II. MIMO IFC WITH UNCOORDINATED OFDM TRANSMISSIONS

We consider a single-beam K-user MIMO IFC where the MIMO channels are frequency-selective. To deal with the multipath nature of the channel, users send their data using OFDM signals with N subcarriers. We use a cyclic prefix (CP) of length N_{CP} assumed to be larger than the channel delay spread. Thus, each OFDM symbol has a total of

[1] Notice that we use the terms post-FFT and pre-FFT to refer to frequency and time domain transmit-receive processing, respectively. Likewise, and following the standard nomenclature in IFC, the transmit and receive filters are denoted as precoders and decoders, respectively.

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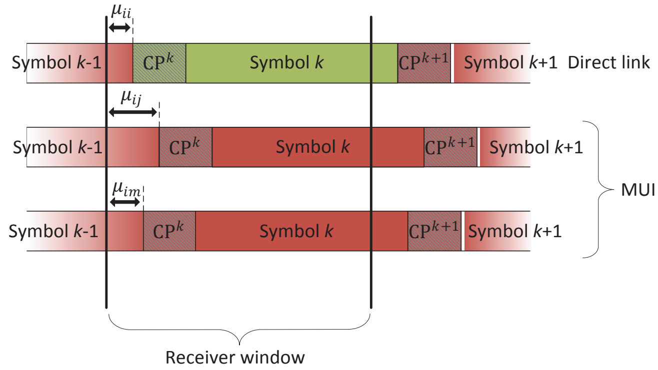

Fig. 1. STOs at the ith receiver in a 3-user scenario.

$N_B = N + N_{CP}$ time domain samples. The kth OFDM symbol transmitted by user i can be expressed as the N-point IDFT of the data symbols, i.e.,

$$x_i^k[n] = \frac{1}{\sqrt{N}} \sum_{\ell=0}^{N-1} s_i^k[\omega_\ell] e^{\frac{2\pi \jmath}{N}\ell n}\ , \quad n = 0, \ldots, N-1\ , \qquad (1)$$

where $s_j^k[\omega_\ell] \in \mathbb{C}$ is the kth symbol transmitted by user j on the ℓth subcarrier. Let N_t and N_r be the number of transmit and receive antennas, respectively.[2] Then, the convolutive time-domain MIMO channel from transmitter j to receiver i is represented as $\mathbf{H}_{ij}[n] \in \mathbb{C}^{N_r \times N_t}$, $n = 0, \ldots, L_h - 1$; where the MIMO channel order is taken as the maximum among those of the different pairwise channels.

When users transmit in a totally uncoordinated fashion and post-FFT IA techniques are applied, the receivers will not be able to synchronize properly to the desired OFDM symbol due to the MUI. Therefore, interference must be eliminated (or at least sufficiently reduced) before the synchronization stage takes place. To this end, we apply in this paper space-time or pre-FFT precoders and decoders at the transmitter and receiver side, respectively. Denoting the pre-FFT precoder of transmitter j as $\mathbf{v}_j[n] \in \mathbb{C}^{N_t \times 1}$, and the pre-FFT decoder of receiver i as $\mathbf{u}_i[n] \in \mathbb{C}^{N_r \times 1}$, $n = 1, \ldots, L-1$, the output signal at receiver i is given by

$$\begin{aligned} z_i[n] = & \underbrace{\mathbf{u}_i^H[-n] * \mathbf{H}_{ii}[n] * \mathbf{v}_i[n] * \tilde{x}_i[n-\mu_{ii}]}_{\text{direct link}} + \\ & \underbrace{\sum_{j \neq i} \mathbf{u}_i^H[-n] * \mathbf{H}_{ij}[n] * \mathbf{v}_j[n] * \tilde{x}_j[n-\mu_{ij}]}_{\text{MUI}} + \\ & \underbrace{\mathbf{u}_i^H[-n] * \mathbf{n}_i[n]}_{\text{noise}}\ , \end{aligned} \qquad (2)$$

where we have dropped the OFDM symbol index to indicate a stream of OFDM symbols. In (2), μ_{ij} is the integer STO in samples between transmitter j and receiver i, which is illustrated in Fig. 1 for a 3-user scenario, $\mathbf{n}_i[n] \sim \mathcal{N}(\mathbf{0}, \sigma^2\mathbf{I})$ is the additive spatially and temporally white Gaussian noise at receiver i, and $\tilde{x}_j[n]$ is the stream of OFDM symbols transmitted by user j, whose kth symbol can be expressed as

$$\tilde{x}_j^k[n] = \begin{cases} x_j^k[N - N_{CP} + n] & 0 \leq n \leq N_{CP} - 1 \\ x_j^k[n - N_{CP}] & N_{CP} \leq n \leq N - 1 \end{cases}. \qquad (3)$$

Following the matrix notation in [8], (3) can be written as

$$\tilde{\mathbf{x}}_j^k = \mathbf{P}_T \mathbf{F}^H \mathbf{s}_j^k\ , \qquad (4)$$

where $\mathbf{P}_T = [[\mathbf{0}\ \mathbf{I}_{N_{CP}}]^T \mathbf{I}_N]^T \in \mathbb{N}^{N_B \times N}$ is the CP adding matrix, $\mathbf{F} \in \mathbb{C}^{N \times N}$ is the DFT matrix of size N and $\mathbf{s}_j^k \in \mathbb{C}^{N \times 1}$ is the data symbol vector transmitted by user j in the kth OFDM symbol. The equivalent SISO channel from transmitter j to receiver i is $\bar{h}_{ij}[n] = \mathbf{u}_i^H[-n] * \mathbf{H}_{ij}[n] * \mathbf{v}_j[n]$, which has a length of $L_{\text{eq}} = 2L + L_h - 2$, and $\breve{n}_i[n] = \mathbf{u}_i^H[-n] * \mathbf{n}_i[n]$ is now a colored Gaussian noise. Using (4), (2) can be rewritten in matrix notation as

$$\begin{aligned} \mathbf{z}_i^k = & \underbrace{\left[\mathbf{F}\mathbf{P}_R \boldsymbol{\Delta}_{ii} \bar{\mathbf{H}}_{ii} \mathbf{P}_T \mathbf{F}^H\right]_{\text{diagonal}} \mathbf{s}_i^k}_{\text{desired signal}} + \\ & \underbrace{\left[\mathbf{F}\mathbf{P}_R \boldsymbol{\Delta}_{ii} \bar{\mathbf{H}}_{ii} \mathbf{P}_T \mathbf{F}^H\right]_{\text{off-diagonal}} \mathbf{s}_i^k}_{\text{ICI}} + \\ & \underbrace{\mathbf{F}\mathbf{P}_R \boldsymbol{\Delta}_{ii}^{\text{prev}} \bar{\mathbf{H}}_{ii} \mathbf{P}_T \mathbf{F}^H \mathbf{s}_i^{k-1}}_{\text{ISI}} + \\ & \underbrace{\mathbf{F}\mathbf{P}_R \sum_{j \neq i} \left(\boldsymbol{\Delta}_{ij} \bar{\mathbf{H}}_{ij} \mathbf{P}_T \mathbf{F}^H \mathbf{s}_j^k + \boldsymbol{\Delta}_{ij}^{\text{prev}} \bar{\mathbf{H}}_{ij} \mathbf{P}_T \mathbf{F}^H \mathbf{s}_j^{k-1}\right)}_{\text{MUI}} + \\ & \underbrace{\mathbf{F}\mathbf{P}_R \breve{\mathbf{n}}_i^k}_{\text{noise}}, \end{aligned} \qquad (5)$$

where $\mathbf{P}_R = [\mathbf{0}\ \mathbf{I}_N] \in \mathbb{N}^{N \times N_B}$ is the CP removing matrix and $\bar{\mathbf{H}}_{ij} \in \mathbb{C}^{N_B + L_{\text{eq}} - 1 \times N_B}$ is the equivalent SISO channel with convolutional (Toeplitz) structure between transmitter j and receiver i, which is a function of the corresponding precoders and decoders. The STOs are modeled using the matrices $\boldsymbol{\Delta}_{ij} = [\mathbf{I}_{N_B}\ \mathbf{0}]_{0\downarrow\mu_{ij}} \in \mathbb{N}^{N_B \times (N_B + L_{\text{eq}} - 1)}$ and $\boldsymbol{\Delta}_{ij}^{\text{prev}} = [\mathbf{I}_{N_B}\ \mathbf{0}]_{0\uparrow(N_B - \mu_{ij})} + [\mathbf{0}\ [\mathbf{I}_{L_{\text{eq}}-1}\ \mathbf{0}]^T]_{0\downarrow\mu_{ij}} \in \mathbb{N}^{N_B \times (N_B + L_{\text{eq}} - 1)}$, where the operators $[\cdot]_{0\downarrow a}$ and $[\cdot]_{0\uparrow a}$ denote a vertical downshift and upshift of length a, respectively, with zero insertion. Finally, $\breve{\mathbf{n}}_i \in \mathbb{C}^{N_B \times 1}$ is the colored noise in vector form. Note that, if $L_{\text{eq}} \leq N_{CP}$, the CP adding and removing operations make the equivalent channel circulant. Notice also that, if the synchronizer for user i works properly, the corresponding STO is equal to zero (i.e., $\mu_{ii} = 0$).

III. JOINT MUI-ISI-ICI MINIMIZATION

In this section, we present an interference minimization algorithm which takes into account the ISI and ICI introduced by the equivalent channel. As in the single-carrier case [4], a spectral mask constraint must be imposed in order to avoid FDMA-like solutions, which do not achieve the maximum DoF of the wideband IFC. We will also show that the design

[2]For notation simplicity, we assume that the network is symmetric, i.e., all users have the same number of transmit and receive antennas.

of pre-FFT decoders and precoders implies a tradeoff between the MUI and the ISI/ICI, because applying precoding and decoding in time domain results in longer equivalent channels.

A. Autocorrelation-based design

Using (5), the MUI power at receiver i can be expressed as the sum of the energies of the interference equivalent channels, i.e.,

$$P_i^{\text{MUI}} = \sum_{j \neq i} \sum_{n=0}^{L_{\text{eq}}-1} \left| \bar{h}_{ij}[n] \right|^2 = \frac{1}{N_B} \sum_{j \neq i} \left\| \bar{\mathbf{H}}_{ij} \right\|_F^2 \ . \quad (6)$$

Notice that, although the received signal (5) depends on the STOs, the MUI power for pre-FFT precoding and decoding depends only on the energy of the equivalent channels, and it is therefore independent of the STOs. For this reason, the assumption of integer STOs in (2), can be made without any impact on the algorithm.

If the equivalent channel length exceeds the CP, i.e., $L_{\text{eq}} > N_{CP}$, ISI as well as ICI will appear in the current OFDM symbol. The ISI and ICI powers at receiver i are respectively given by

$$P_i^{\text{ISI}} = \left\| \mathbf{P}_R \boldsymbol{\Delta}_{ii}^{\text{prev}} \bar{\mathbf{H}}_{ii} \mathbf{P}_T \right\|_F^2 \ , \quad (7)$$

$$P_i^{\text{ICI}} = \left\| \left[\mathbf{F} \mathbf{P}_R \boldsymbol{\Delta}_{ii} \bar{\mathbf{H}}_{ii} \mathbf{P}_T \mathbf{F}^H \right]_{\text{off-diagonal}} \right\|_F^2 \ . \quad (8)$$

Since the length of the equivalent channel is increased by the pre-FFT scheme, shorter decoders and precoders are needed to reduce the ISI and ICI. Reducing the length of the precoders and decoders, however, will increase the MUI. Thus, the pre-FFT scheme implies a tradeoff between the ISI/ICI and the MUI. With these considerations, the precoders and decoders that minimize the total interference (MUI+ISI+ICI) can be found by solving the following optimization problem

$$\underset{\mathbf{u}_i[n], \mathbf{v}_j[n]}{\text{minimize}} \quad \sum_{i=1}^{K} \left(P_i^{\text{MUI}} + P_i^{\text{ISI}} + P_i^{\text{ICI}} \right) \ , \quad (9)$$

$$\text{subject to} \quad \left| \bar{\mathfrak{h}}_{ii}[\omega_\ell] \right|^2 > 0, \ \ell = 0, \ldots, N-1 \ , \quad (10)$$

where $\bar{\mathfrak{h}}_{ij}[\omega_\ell]$ is the N-point DFT of $\bar{h}_{ij}[n]$. Note that (10) is the spectral mask needed in the asymptotic SNR regime to avoid FDMA-like solutions. Notice also that in the flat-fading case a constraint in the norm of the precoders and decoders is enough to ensure that the direct links do not vanish with probability one. With frequency-selective MIMO channels, however, the norm constraint must be imposed at every subcarrier to satisfy the same condition on the direct links. Therefore, the norm of the frequency response of the precoders and decoders must be greater than $\alpha > 0$ (i.e., $\|\mathfrak{v}_i[\omega_\ell]\|^2 \geq \alpha$ and $\|\mathfrak{u}_i[\omega_\ell]\|^2 \geq \alpha$, where we have introduced a parameter, α, strictly larger than zero, to achieve a practical solution in the finite SNR regime) at any subcarrier, hence ensuring that the direct link does not vanish.

In [4], a similar problem has been solved for the single-carrier case, where the precoders and decoders that minimize the MUI are designed through their autocorrelation function and resorting to an alternating minimization algorithm similar to that used in the flat fading case [5], [9], [10]. To use a similar alternating minimization approach to solve (9), we need to express (or approximate) the ISI and ICI in terms of the autocorrelation function of the precoders and decoders. To this end, we use the approximation proposed in [6] which is given by

$$P_i^{\text{ISI}} + P_i^{\text{ICI}} \simeq \sum_{|n|=N_{CP}+1}^{L_{\text{eq}}-1} |r_{ii}[n]|^2 \ , \quad (11)$$

where $r_{ii}[n] = \bar{h}_{ii}[n] * \bar{h}_{ii}^*[-n]$ is the autocorrelation of the equivalent SISO channel between transmitter i and receiver i. Using the approximation, each step of the alternating minimization procedure can be written as a convex optimization problem, similar to [4]. For this purpose, let us define

$$\mathbf{R}_{v_i}[n] = \mathbf{v}_i[n] * \mathbf{v}_i^H[-n] \ , \quad (12)$$

$$\mathbf{S}_{v_i}[\omega_\ell] = \sum_{n=-L+1}^{L-1} \mathbf{R}_{v_i}[n] e^{-\frac{2\pi \jmath}{N} \ell n} \ . \quad (13)$$

Then,

$$P_i^{\text{MUI}} = \sum_{i \neq j} r_{ij}[0] \ , \quad (14)$$

where the autocorrelation function can be written as

$$r_{ij}[n] = \text{Tr} \left(\mathbf{R}_{v_j}[n] * \mathbf{H}_{ij}^H[-n] * \mathbf{R}_{u_i}[n] * \mathbf{H}_{ij}[n] \right) \ . \quad (15)$$

In (15), the definition of the autocorrelation function of the ith decoder, $\mathbf{R}_{u_i}[n]$, is completely analogous to that of the ith precoder (12). Notice that the MUI power in OFDM systems is the same as in the single-carrier case [4].

Directly solving in the autocorrelation function of the precoders (for fixed decoders), leads to a convex optimization problem (that can be efficiently solved):

$$\underset{\mathbf{R}_{v_j}[n]}{\text{minimize}} \quad \sum_{i \neq j} r_{ij}[0] + \sum_{|n|=N_{CP}+1}^{L_{\text{eq}}-1} |r_{jj}[n]|^2 \ , \quad (16)$$

$$\begin{aligned}
\text{subject to} \quad & \text{Tr} \left(\mathbf{S}_{v_j}[\omega_\ell] \right) \geq \alpha \ , \ \ell = 0, \ldots, N-1 \ , \\
& \mathbf{S}_{v_j}[\omega_\ell] \succeq 0 \ , \ \ell = 0, \ldots, N-1 \ , \\
& \mathbf{S}_{v_j}[\omega_\ell] = \sum_{n=-L+1}^{L-1} \mathbf{R}_{v_j}[n] e^{-\frac{2\pi \jmath}{N} \ell n}, \ell = 0, \ldots, N-1, \\
& \mathbf{R}_{v_j}[n] = \mathbf{R}_{v_j}^H[-n] \ , \ n = 0, \ldots, L-1 \\
& \text{Tr} \left(\mathbf{R}_{v_j}[0] \right) = 1 \ .
\end{aligned}$$

The optimization problem for the decoders (for fixed precoders) is analogous to (16), but exchanging the role of transmitters and receivers [5].

B. Spectral mask constraint as LMI

Using the positive real lemma [7], we are able to express the spectral mask constraint in (16) in time domain, as a linear function of the autocorrelation of the precoders and decoders,

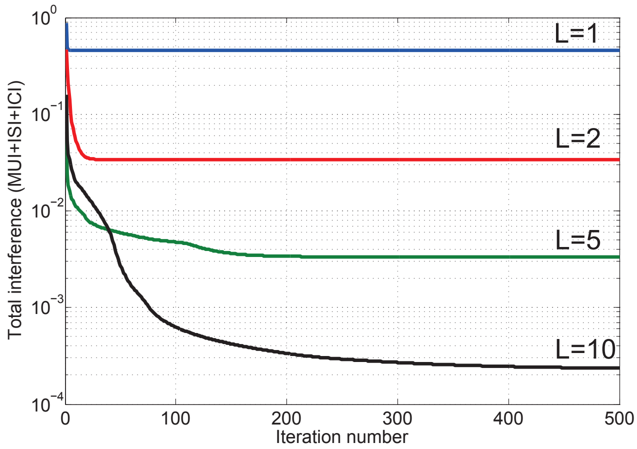

Fig. 2. Total interference (MUI+ISI+ICI) vs. number of iterations for different filter lengths, L.

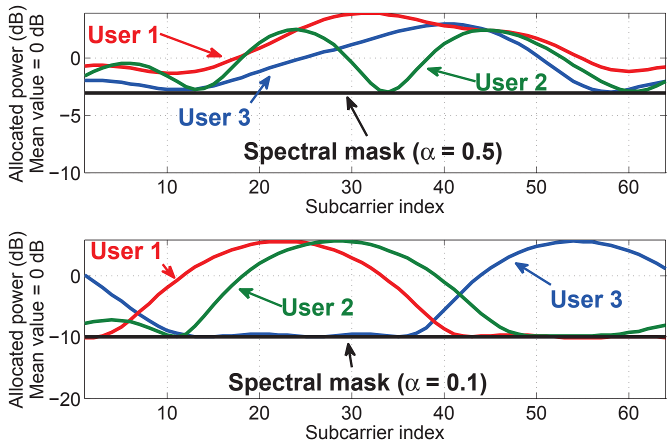

Fig. 3. Power allocation for a particular channel realization with $L = 5$. The spectral mask is set to $\alpha = 0.5$ (top) and $\alpha = 0.1$ (bottom)

requiring less computational complexity. According to [11], the spectral mask constraints in (16) are equivalent to

$$\mathrm{Tr}\left(\mathbf{S}_{v_j}[\omega_\ell]\right) \geq \alpha\,,\ \forall \ell \ \Rightarrow\ \tilde{\mathbf{r}}_{v_j} - \alpha\boldsymbol{\delta} \ = \mathrm{L}_1^*\left(\mathbf{X}\right)\,, \quad (17)$$

$$\mathbf{S}_{v_j}[\omega_\ell] \succeq 0\,,\ \forall \ell \ \Rightarrow\ \tilde{\mathbf{R}}_{v_j} \ = \mathrm{L}_{N_t}^*\left(\mathbf{Y}\right)\,, \quad (18)$$

where $\tilde{\mathbf{r}}_{v_j} = [\mathrm{Tr}(\mathbf{R}_{v_j}[0]), \ldots, \mathrm{Tr}(\mathbf{R}_{v_j}[L-1])]^T$, $\tilde{\mathbf{R}}_{v_j} = [\mathbf{R}_{v_j}^T[0], \ldots, \mathbf{R}_{v_j}^T[L-1]]^T$, $\boldsymbol{\delta}$ is the first column of the $L \times L$ identity matrix, $\mathbf{X} \in \mathbb{S}^L$ and $\mathbf{Y} \in \mathbb{S}^{LN_t}$, where $\mathbb{S}^M$ denotes the set of $M \times M$ positive semidefinite Hermitian matrices. The linear operator $\mathrm{L}_n^*(\mathbf{A})$ is defined as [11]

$$\mathrm{L}_n^*(\mathbf{A}) = \left[\mathrm{Tr}_{0,n}\left(\mathbf{A}\right)^T, \ldots, \mathrm{Tr}_{L-1,n}\left(\mathbf{A}\right)^T\right]^T\,, \quad (19)$$

where the operator $\mathrm{Tr}_{k,n}(\mathbf{A})$ denotes the sum of the n-size blocks on the kth lower off-block-diagonal of $\mathbf{A}$.

Using this new formulation, problem (16) is equivalent to

$$\begin{aligned} \underset{\mathbf{R}_{v_j}[n],\mathbf{X},\mathbf{Y}}{\text{minimize}} \quad & \sum_{i \neq j} r_{ij}[0] + \sum_{|n|=N_{CP}+1}^{L_{\mathrm{eq}}-1} |r_{jj}[n]|^2\,, \\ \text{subject to} \quad & \tilde{\mathbf{r}}_{v_j} - \alpha\boldsymbol{\delta} = \mathrm{L}_1^*\left(\mathbf{X}\right)\,, \\ & \tilde{\mathbf{R}}_{v_j} = \mathrm{L}_{N_t}^*\left(\mathbf{Y}\right)\,, \\ & \mathrm{Tr}\left(\mathbf{R}_{v_j}[0]\right) = 1\,, \\ & \mathbf{X} \succeq 0\,, \\ & \mathbf{Y} \succeq 0\,. \end{aligned} \quad (20)$$

Once the autocorrelations have been obtained, the actual precoders (analogously for the decoders) can be computed in two steps: first, a SISO spectral factorization algorithm is applied for each antenna in order to obtain N_t single-antenna filters whose autocorrelations match those of the diagonal elements of $\mathbf{R}_{v_j}[n]$. Second, the phase of each filter is modified such that the cross autocorrelation terms match those of $\mathbf{R}_{v_j}[n]$. A detailed description of this procedure can be found in [4].

IV. SIMULATION RESULTS

In this section we present some numerical examples to illustrate the performance of the proposed algorithm. Specifically, we consider a scenario with $K = 3$ users (transmitter-receiver pairs) equipped with 2 antennas each, and one data stream per user. This system is denoted as $(2 \times 2, 1)^3$. All nine pairwise MIMO channels follow a Rayleigh fading model with a power delay profile (PDP) given by

$$\mathrm{PDP}[n] = (1-\rho)\,\rho^n\,,\ n = 0, \ldots, L_h - 1\,, \quad (21)$$

with $0 < \rho < 1$. The root mean square (rms) delay spread is the second central moment of the PDP, and is given by [12, Chapter 4]

$$\sigma_\tau = \sqrt{\sum_{n=0}^{L_h-1} \mathrm{PDP}[n] n^2 - \left(\sum_{n=0}^{L_h-1} \mathrm{PDP}[n] n\right)^2} \ \text{samples}\,. \quad (22)$$

The rms delay spread is a measure of the frequency selectivity of the channel. In the simulations we use $\sigma_\tau = 0.35$ for low frequency selectivity and $\sigma_\tau = 1.95$ for high frequency selectivity. We set $N = 64$ subcarriers and a CP length of $N_{CP} = 16$ samples, which are typical parameters in indoor scenarios. Without loss of generality, we consider unit transmit power and define the signal to noise ratio (SNR) as $10\log_{10}(\frac{1}{\sigma^2})$. Finally, we consider $\alpha = 0.5$. A study of the system performance for different values of α can be found in [4]. In Fig. 2 we show the convergence of the total interference (i.e., MUI+ISI+ICI) for different filter lengths, L, and averaged over 50 channel realizations. The total interference decreases when the filter order increases, but the computational complexity increases as well.

Fig. 3 shows the power allocation over the different subcarriers for $\alpha = 0.5$ (top) and $\alpha = 0.1$ (bottom), and a filter length of $L = 5$. As stated in Section III, when α is low, users try to transmit over different frequency bands, hence decreasing the overall sum-rate. Concretely, the sum-rate achieved for $\alpha = 0.5$ is equal to 14.93 bps/Hz when the SNR is 20 dB; while 11.30 bps/Hz are achieved with the same SNR for $\alpha = 0.1$.

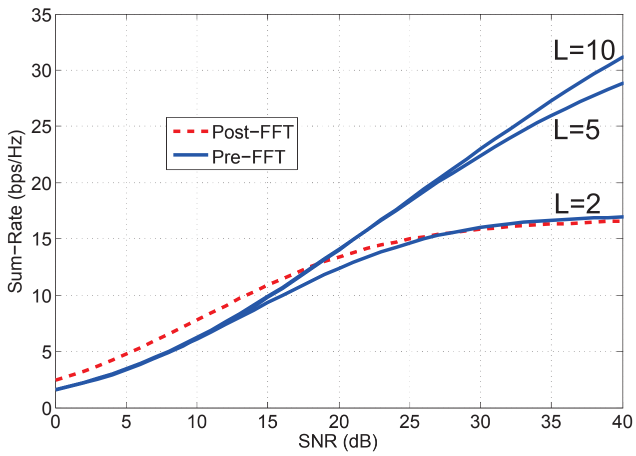

Fig. 4. Comparison of the sum-rate performance between pre-FFT and post-FFT IA for $\sigma_\tau = 0.35$, averaged over 50 channel realizations

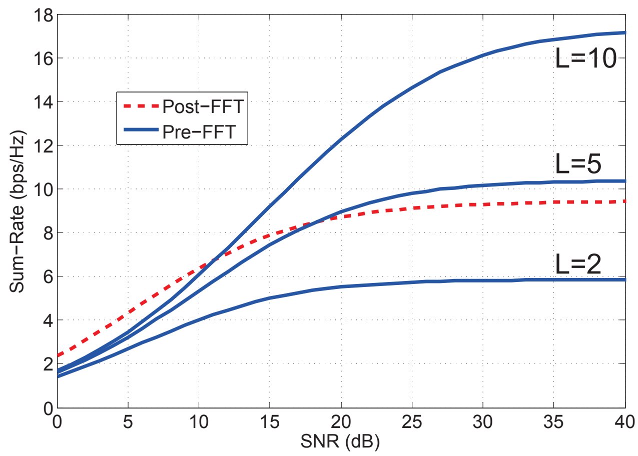

Fig. 5. Comparison of the sum-rate performance between pre-FFT and post-FFT IA for $\sigma_\tau = 1.95$, averaged over 50 channel realizations.

Now suppose that there exists some synchronization mechanism between the desired transmitter-receiver pairs, i.e., the receivers are able to synchronize to the desired frame even in the presence of asynchronous interferences. With this ideal setting, post-FFT IA techniques could be applied and compared with our proposed pre-FFT scheme. To simulate the STOs between users, we consider the starting point of each user frame to be uniformly distributed between 0 and $N + N_{CP}$. Alternatively, the sum-rate performance (averaged over 50 channel realizations and 1000 different STOs) for both pre-FFT and post-FFT IA are shown in Fig. 4 and Fig. 5 for $\sigma_\tau = 0.35$ and for $\sigma_\tau = 1.95$, respectively. Even in the ideal situation in which the receivers are able to detect the desired frame, the pre-FFT performs better than post-FFT in the medium and high SNR regime, when $L \geq 5$. We also observe that, when the sum-rate is limited by noise, the filtered noise in the pre-FFT scheme decreases the system performance. The sum-rate degradation of the post-FFT IA is due to the time domain windowing of the interfering OFDM symbols. As these OFDM symbols are not time-aligned with the receiver window, they will introduce ISI and ICI which cannot be suppressed with the IA decoder.

V. CONCLUSIONS

In this paper, we have addressed the problem of designing pre-FFT precoders and decoders for the K-user MIMO IFC with OFDM transmissions. We have proposed a joint MUI-ISI-ICI minimization algorithm which mitigates the interference before the synchronization procedure, enabling the frame detection in asynchronous networks and reducing the complexity with respect to post-FFT beamforming. As for single-carrier transmissions, we have introduced a spectral mask constraint to maximize the achievable DoF, which has been rewritten as LMI in time domain. We have shown through simulations that the proposed pre-FFT scheme outperforms post-FFT IA even when the desired transmitter-receiver pairs are perfectly synchronized. An interesting further line of research would be to consider other cost functions such as maximum signal to interference plus noise ratio (SINR) or minimum mean square error (MMSE), which will improve the system performance, especially in the low SNR regime.

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