PASSIVE DETECTION OF RANK-ONE GAUSSIAN SIGNALS FOR KNOWN CHANNEL SUBSPACES AND ARBITRARY NOISE

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ABSTRACT

This paper addresses the passive detection of a common signal in two multi-sensor arrays. For this problem, we derive a detector based on likelihood theory for the case of one-antenna transmitters, independent Gaussian noises with arbitrary spatial structure, Gaussian signals, and known channel subspaces. The detector uses a likelihood ratio where all but one of the unknown parameters are replaced by their maximum likelihood (ML) estimates. The ML estimation of the remaining parameter requires a numerical search, and it is therefore estimated using a sample-based estimator. The performance of the proposed detector is illustrated by means of Monte Carlo simulations and compared with that of the detector for unknown channels, showing the advantage of this knowledge.

***Index Terms*—** Generalized likelihood ratio (GLR), hypothesis test, multi-sensor array, passive radar.

1. INTRODUCTION

In recent decades, passive radar systems [1] have gained a lot of attention. This is a type of bistatic radar [2] where the transmitter is non-cooperative, that is, there is no control over the transmitted signal. There are many systems that can be used as non-cooperative transmitters, a.k.a. illuminators of opportunity, such as terrestrial TV and FM broadcast transmitters, mobile phone base transceiver stations (i.e., 4G and 5G systems), communication or navigation satellites [2]. The use of illuminators of opportunity is the basis for the appeal of passive radar: 1) the system does not disclose its location, allowing for covert operation; 2) it is energy efficient; and 3) it is simple and cheap to deploy.

The lack of control over the transmitted signal affects the detection performance, as it has not been optimized for detection. Thus, to achieve passive radar systems with satisfactory performance, it is common to use an additional channel, the reference channel. This second channel improves the performance, but requires more advanced detectors. These two channels are either obtained through beamforming or directional antennas [3]: The reference channel always receives the signal transmitted by the illuminator of opportunity and the surveillance channel only measures this signal when there is a target that reflects it.

Passive detection with a reference channel has been studied over the last several years and many detection algorithms have been proposed. In the case of single-input single-output (SISO) channels, the standard detector is based on the cross-correlation between the signal received by the surveillance and reference channels. Although it resembles the matched filter, this detector is not optimal because the signal of the reference channel is contaminated by noise [4]. To overcome this problem, [5] derives a generalized likelihood ratio test (GLRT). There are also many papers that consider multiple-input multiple-output (MIMO) channels. For instance, [6] used generalized coherence [7] to propose an ad-hoc detector for passive detection. However, there are also many papers that consider principled detectors. The work in [3] derived the GLRT for an unknown deterministic (first-order model) transmitted signal in spatially and temporally white noise, and known/unknown channels. The GLRT for stochastic (second-order model) waveforms was derived in [8–11], considering different models for the spatial correlation of the noise. A systematic review of GLRTs under different assumptions are summarized in [12].

In this work, we consider the detection of a signal transmitted by a one-antenna illuminator of opportunity, which is assumed Gaussian distributed (second-order model), and the noises at the surveillance and reference arrays are independent and with arbitrary spatial structure. However, contrary

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to [8–11], we consider that the channel subspaces are known, which is a reasonable assumption when the array geometry is known (e.g., for uniform linear arrays). For this detection problem, we derive a detector based on likelihood theory. In particular, the proposed detector is given by the likelihood ratio test where all but one of the unknown parameters under both hypotheses are replaced by their maximum likelihood (ML) estimates. The remaining parameter is not estimated using the ML framework as the maximization problem does not have a closed-form solution and numerical techniques would, therefore, be necessary. Alternatively, this parameter is obtained using a sample-based estimator. Finally, we illustrate by means of Monte Carlo simulations the performance of the proposed detector and compare it with that of [8, 9], which consider unknown channels.

The remainder of this paper is organized as follows. Section 2 formulates the passive detection problem in the case of Gaussian signals as a test for the covariance structure of the observations. The proposed detector is derived in Section 3 and its performance is evaluated in Section 4. Finally, Section 5 presents the main conclusions of this work and future research lines.

Notation: In this paper, matrices are denoted by bold-faced upper case letters, column vectors are denoted by bold-faced lower case letters, and scalars are denoted by light-face lower case letters. The superscripts $(\cdot)^T$ and $(\cdot)^H$ denote transpose and Hermitian, respectively. The trace and determinant of a matrix $\mathbf{A}$ will be denoted, respectively, as $\mathrm{tr}(\mathbf{A})$ and $\det(\mathbf{A})$. The set $\mathbb{C}^L$ is the complex Euclidean space with the standard inner product. The matrix $\mathrm{blkdiag}_L(\mathbf{A})$ denotes a block-diagonal matrix built from the $L \times L$ blocks in the diagonal of $\mathbf{A}$. The notation $\mathbf{x} \sim \mathcal{CN}_L(\mathbf{0}, \mathbf{R})$ denotes a complex Gaussian vector in $\mathbb{C}^L$ with zero mean and covariance $\mathbf{R}$.

2. PROBLEM FORMULATION

This work considers the passive detection of a common signal in two multi-sensor arrays. One *reference* array always observes the transmitted signal, whereas one *surveillance* array only observes the signal when the target is present. In particular, we consider one transmitting antenna, L receiving antennas in both arrays, known channel subspaces, and independent noises with arbitrary covariance matrices. Thus, the signal model is

$$\begin{aligned} \mathcal{H}_0 &: \mathbf{y} = \begin{bmatrix} \mathbf{0} \\ \mathbf{h}_r \end{bmatrix} x + \mathbf{n}, \\ \mathcal{H}_1 &: \mathbf{y} = \begin{bmatrix} \mathbf{h}_s \\ \mathbf{h}_r \end{bmatrix} x + \mathbf{n}, \end{aligned} \tag{1}$$

where $\mathbf{y} = [\mathbf{y}_s^T \, \mathbf{y}_r^T]^T$ is the stack of the received signal at the surveillance channel, $\mathbf{y}_s \in \mathbb{C}^L$, and at the reference channel, $\mathbf{y}_r \in \mathbb{C}^L$; $\mathbf{h}_s \in \mathbb{C}^L$ and $\mathbf{h}_r \in \mathbb{C}^L$ are the channels between the transmitter and the surveillance and reference arrays, respectively; and x is the transmitted signal. Moreover, $\mathbf{n} = [\mathbf{n}_s^T \, \mathbf{n}_r^T]^T$ contains the independent noises at the surveillance array, $\mathbf{n}_s \in \mathbb{C}^L$, and at the reference array, $\mathbf{n}_r \in \mathbb{C}^L$, which are distributed as $\mathbf{n}_s \sim \mathcal{CN}_L(\mathbf{0}, \mathbf{\Sigma}_{ss})$ and $\mathbf{n}_r \sim \mathcal{CN}_L(\mathbf{0}, \mathbf{\Sigma}_{rr})$, with $\mathbf{\Sigma}_{ss}, \mathbf{\Sigma}_{rr}$ positive definite covariance matrices without further structure.

A more precise definition of the hypotheses depends on the assumptions on the transmitted signals. We consider that $x \sim \mathcal{CN}(0, \sigma_x^2)$, with unknown variance σ_x^2. Then, the covariance matrix under $\mathcal{H}_1$ is

$$\mathbf{R}_1 = \begin{bmatrix} \sigma_x^2 \mathbf{h}_s \mathbf{h}_s^H + \mathbf{\Sigma}_{ss} & \sigma_x^2 \mathbf{h}_s \mathbf{h}_r^H \\ \sigma_x^2 \mathbf{h}_r \mathbf{h}_s^H & \sigma_x^2 \mathbf{h}_r \mathbf{h}_r^H + \mathbf{\Sigma}_{rr} \end{bmatrix}.$$

Moreover, since we are assuming that the channel subspaces are known, we can decompose them as $\mathbf{h}_s = a_s \mathbf{u}_s$ and $\mathbf{h}_r = a_r \mathbf{u}_r$, where $\mathbf{u}_s, \mathbf{u}_r$ are the basis vectors for the surveillance and reference channel subspaces, with $\|\mathbf{u}_s\| = \|\mathbf{u}_r\| = 1$, and a_s, a_r are unknown. Hence, we can rewrite $\mathbf{R}_1$ as

$$\mathbf{R}_1 = \begin{bmatrix} q_{ss} \mathbf{u}_s \mathbf{u}_s^H + \mathbf{\Sigma}_{ss} & q_{sr} \mathbf{u}_s \mathbf{u}_r^H \\ q_{sr}^* \mathbf{u}_r \mathbf{u}_s^H & q_{rr} \mathbf{u}_r \mathbf{u}_r^H + \mathbf{\Sigma}_{rr} \end{bmatrix},$$

where $q_{ss} = \sigma_x^2 |a_s|^2, q_{rr} = \sigma_x^2 |a_r|^2$, and $q_{sr} = \sigma_x^2 a_s a_r^*$, are unknown parameters. Here, $q_{ss}, q_{rr} > 0$, and $q_{ss} q_{rr} = |q_{sr}|^2$. Under $\mathcal{H}_0$, the covariance matrix is easily obtained by setting $\mathbf{h}_s = \mathbf{0}$, i.e.,

$$\mathbf{R}_0 = \begin{bmatrix} \mathbf{\Sigma}_{ss} & \mathbf{0} \\ \mathbf{0} & q_{rr} \mathbf{u}_r \mathbf{u}_r^H + \mathbf{\Sigma}_{rr} \end{bmatrix}.$$

To conclude, the detection problem comes down to the following detection for the covariance structure of $\mathbf{y}$:

$$\begin{aligned} \mathcal{H}_0 &: \mathbf{y} \sim \mathcal{CN}_L(\mathbf{0}, \mathbf{R}_0), \\ \mathcal{H}_1 &: \mathbf{y} \sim \mathcal{CN}_L(\mathbf{0}, \mathbf{R}_1). \end{aligned} \tag{2}$$

3. DERIVATION OF THE DETECTOR

In this section, we will derive a detector based on likelihood theory to solve (2) assuming that we have access to N independent and identically distributed observations of $\mathbf{y}$, $\mathbf{Y} = [\mathbf{y}_1 \cdots \mathbf{y}_N]$. The detector is based on the likelihood ratio, where all but one of the unknown parameters are replaced by their maximum likelihood estimates, and the remaining estimate is based on a sample-based estimator.

Under $\mathcal{H}_0$, since $\mathbf{\Sigma}_{ss}, \mathbf{\Sigma}_{rr}$ are positive definite covariance matrices without further structure, $\mathbf{R}_0$ is a block-diagonal matrix whose blocks are only positive definite. Then, the ML estimate is

$$\hat{\mathbf{R}}_0 = \mathrm{blkdiag}_L(\mathbf{S}) = \begin{bmatrix} \mathbf{S}_{ss} & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_{rr} \end{bmatrix},$$

where the sample covariance matrix is

$$\mathbf{S} = \frac{1}{N} \mathbf{Y} \mathbf{Y}^H = \begin{bmatrix} \mathbf{S}_{ss} & \mathbf{S}_{sr} \\ \mathbf{S}_{sr}^H & \mathbf{S}_{rr} \end{bmatrix}.$$

The compressed log-likelihood is[1]

$$\log \ell(\hat{\mathbf{R}}_0; \mathbf{Y}) = -\log\det(\mathbf{S}_{ss}) - \log\det(\mathbf{S}_{rr}).$$

Under $\mathcal{H}_1$, the problem is much more challenging. In fact, as will be seen, it is not possible to obtain the ML estimates of all unknown parameters. First, let us rewrite $\mathbf{R}_1$ as

$$\mathbf{R}_1 = \begin{bmatrix} \mathbf{R}_{ss} & q_{sr}\mathbf{u}_s\mathbf{u}_r^H \\ q_{sr}^*\mathbf{u}_r\mathbf{u}_s^H & \mathbf{R}_{rr} \end{bmatrix}, \tag{3}$$

where $\mathbf{R}_{ss}$ and $\mathbf{R}_{rr}$ are $L \times L$ positive definite matrices without further structure and $\mathbf{R}_{sr} = q_{sr}\mathbf{u}_s\mathbf{u}_r^H$ is a rank-one matrix. To simplify the estimation, we define the following transformed parameters [13]: $\mathbf{\Theta}_a = \mathbf{R}_{rr}$ and

$$\mathbf{\Theta}_b = \mathbf{R}_{ss} - |q_{sr}|^2\mathbf{u}_s(\mathbf{u}_r^H\mathbf{R}_{rr}^{-1}\mathbf{u}_r)\mathbf{u}_s^H.$$

This defines a one-to-one mapping between the transformed parameters $\{\mathbf{\Theta}_a, \mathbf{\Theta}_b, q_{sr}\}$ and the original (or natural) parameters $\{\mathbf{R}_{ss}, \mathbf{R}_{rr}, q_{sr}\}$. With these definitions, the log-likelihood under $\mathcal{H}_1$ can be rewritten as

$$\log \ell(\mathbf{\Theta}_a, \mathbf{\Theta}_b, q_{sr}; \mathbf{Y}) = -\log\det(\mathbf{\Theta}_a) - \operatorname{tr}\left(\mathbf{\Theta}_a^{-1}\mathbf{S}_{rr}\right) \\ - \log\det(\mathbf{\Theta}_b) - \operatorname{tr}\left(\mathbf{\Theta}_b^{-1}\mathbf{M}(q_{sr}, \mathbf{\Theta}_a)\right),$$

where

$$\mathbf{M}(q_{sr}, \mathbf{\Theta}_a) = \mathbf{S}_{ss} + |q_{sr}|^2\eta_r(\mathbf{\Theta}_a)\mathbf{u}_s\mathbf{u}_s^H \\ - q_{sr}\mathbf{u}_s\mathbf{u}_r^H\mathbf{\Theta}_a^{-1}\mathbf{S}_{sr}^H - q_{sr}^*\mathbf{S}_{sr}\mathbf{\Theta}_a^{-1}\mathbf{u}_r\mathbf{u}_s^H,$$

and

$$\eta_r(\mathbf{\Theta}_a) = \mathbf{u}_r^H\mathbf{\Theta}_a^{-1}\mathbf{S}_{rr}\mathbf{\Theta}_a^{-1}\mathbf{u}_r.$$

It is easy to show that the ML estimate of $\mathbf{\Theta}_b$ is $\hat{\mathbf{\Theta}}_b = \mathbf{M}(q_{sr}, \mathbf{\Theta}_a)$, which yields

$$\log \ell(\mathbf{\Theta}_a, \hat{\mathbf{\Theta}}_b, q_{sr}; \mathbf{Y}) = -\log\det(\mathbf{\Theta}_a) - \operatorname{tr}\left(\mathbf{\Theta}_a^{-1}\mathbf{S}_{rr}\right) \\ - \log\det(\mathbf{M}(q_{sr}, \mathbf{\Theta}_a)).$$

The ML estimate of q_{sr} can be obtained as

$$\hat{q}_{sr} = \arg\min_{q_{sr}} \quad \det(\mathbf{M}(q_{sr}, \mathbf{\Theta}_a)).$$

A few lines of algebra show that the determinant can be expressed as

$$\det(\mathbf{M}(q_{sr}, \mathbf{\Theta}_a)) = [1 - \eta_{sr}^*(\mathbf{\Theta}_a)q_{sr} - \eta_{sr}(\mathbf{\Theta}_a)q_{sr}^* \\ + |q_{sr}|^2\left(\eta_s\eta_r(\mathbf{\Theta}_a) + |\eta_{sr}(\mathbf{\Theta}_a)|^2 - \eta_s\alpha(\mathbf{\Theta}_a)\right)]\det(\mathbf{S}_{ss}),$$

where $\eta_s = \mathbf{u}_s^H\mathbf{S}_{ss}^{-1}\mathbf{u}_s$,

$$\eta_{sr}(\mathbf{\Theta}_a) = \mathbf{u}_s^H\mathbf{S}_{ss}^{-1/2}\mathbf{C}\mathbf{S}_{rr}^{1/2}\mathbf{\Theta}_a^{-1}\mathbf{u}_r,$$

and

$$\alpha(\mathbf{\Theta}_a) = \mathbf{u}_r^H\mathbf{\Theta}_a^{-1}\mathbf{S}_{rr}^{1/2}\mathbf{C}^H\mathbf{C}\mathbf{S}_{rr}^{1/2}\mathbf{\Theta}_a^{-1}\mathbf{u}_r,$$

with the coherence matrix defined as $\mathbf{C} = \mathbf{S}_{ss}^{-1/2}\mathbf{S}_{sr}\mathbf{S}_{rr}^{-1/2}$. Setting the derivative of $\det(\mathbf{M}(q_{sr}, \mathbf{\Theta}_a))$ to zero, we get

$$\hat{q}_{sr}(\mathbf{\Theta}_a) = \frac{\eta_{sr}(\mathbf{\Theta}_a)}{|\eta_{sr}(\mathbf{\Theta}_a)|^2 + \eta_s(\eta_r(\mathbf{\Theta}_a) - \alpha(\mathbf{\Theta}_a))},$$

and the compressed log-likelihood is therefore

$$\log \ell(\mathbf{\Theta}_a, \hat{\mathbf{\Theta}}_b, \hat{q}_{sr}(\mathbf{\Theta}_a); \mathbf{Y}) = -\log\det(\mathbf{S}_{ss}) \\ - \log\det(\mathbf{\Theta}_a) - \operatorname{tr}\left(\mathbf{\Theta}_a^{-1}\mathbf{S}_{rr}\right) \\ + \log\left(1 + \frac{|\eta_{sr}(\mathbf{\Theta}_a)|^2}{\eta_s(\eta_r(\mathbf{\Theta}_a) - \alpha(\mathbf{\Theta}_a))}\right). \tag{4}$$

The maximization of (4) with respect to $\mathbf{\Theta}_a$ is extremely difficult and would rely on numerical procedures. To avoid them, taking into account that $\mathbf{\Theta}_a = \mathbf{R}_{rr}$, we propose to use a sample-based estimate: $\hat{\mathbf{\Theta}}_a = \mathbf{S}_{rr}$. Then, the compressed log-likelihood is

$$\log \ell(\hat{\mathbf{\Theta}}_a, \hat{\mathbf{\Theta}}_b, \hat{q}_{sr}; \mathbf{Y}) = -\log\det(\mathbf{S}_{ss}) - \log\det(\mathbf{S}_{rr}) \\ + \log\left(1 + \frac{|\eta_{sr}|^2}{\eta_s(\eta_r - \alpha)}\right),$$

where $\hat{q}_{sr} = \hat{q}_{sr}(\mathbf{S}_{rr})$, $\eta_r = \eta_r(\mathbf{S}_{rr}) = \mathbf{u}_r^H\mathbf{S}_{rr}^{-1}\mathbf{u}_r$,

$$\eta_{sr} = \eta_{sr}(\mathbf{S}_{rr}) = \mathbf{u}_s^H\mathbf{S}_{ss}^{-1}\mathbf{S}_{sr}\mathbf{S}_{rr}^{-1}\mathbf{u}_r,$$

and

$$\alpha = \alpha(\mathbf{S}_{rr}) = \mathbf{u}_r^H\mathbf{S}_{rr}^{-1}\mathbf{S}_{sr}^H\mathbf{S}_{ss}^{-1}\mathbf{S}_{sr}\mathbf{S}_{rr}^{-1}\mathbf{u}_r.$$

Finally, the likelihood ratio is

$$\Lambda = \frac{\ell(\hat{\mathbf{\Theta}}_a, \hat{\mathbf{\Theta}}_b, \hat{q}_{sr}; \mathbf{Y})}{\ell(\hat{\mathbf{R}}_0; \mathbf{Y})} = \left(1 + \frac{|\eta_{sr}|^2}{\eta_s(\eta_r - \alpha)}\right)^N,$$

and the proposed detector is

$$\lambda = \Lambda^{1/N} - 1 = \frac{|\eta_{sr}|^2}{\eta_s(\eta_r - \alpha)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta, \tag{5}$$

where η is a properly selected threshold.

4. NUMERICAL RESULTS

In this section, we evaluate, by means of Monte Carlo simulations, the performance of the detector in (5). Additionally, we compare this detector with the detector that does not know the channel subspace [8], which is given by the largest canonical correlation (i.e., the largest singular value of $\mathbf{C}$). For each Monte Carlo simulation, we generate different channels and noise covariance matrices. The channel gains are Rayleigh distributed and the noise covariance matrices are

[1] In the following, we will omit terms in the log-likelihood that do not depend on data.

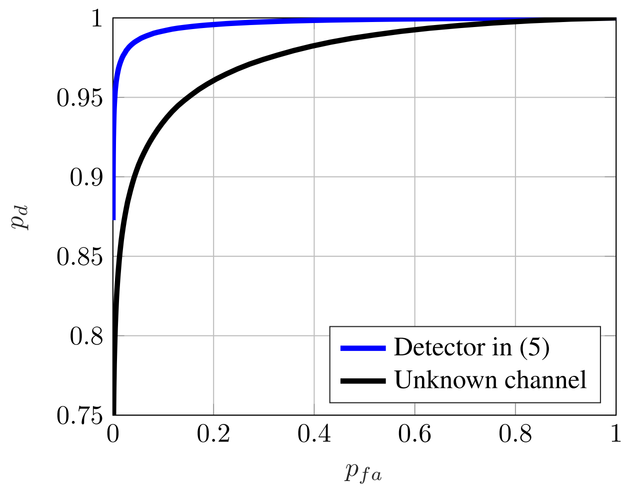

Fig. 1: ROC curves for an experiment with $N = 64$, $L = 4$ and $\text{SNR}_s = \text{SNR}_r = -12$ dBs

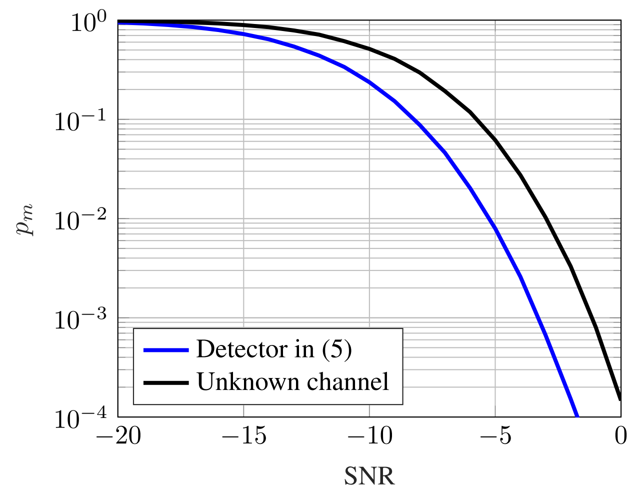

Fig. 2: Probability of missed detection vs. $\text{SNR}_s = \text{SNR}_r = \text{SNR}$ for an experiment with $N = 32$, $L = 4$ and $p_{fa} = 10^{-4}$

Wishart distributed, scaled to achieve a desired SNR, defined as $\text{SNR}_i = 10 \log_{10}(r_{xx} \|\mathbf{h}_i\|^2 / \operatorname{tr}(\mathbf{\Sigma}_{ii}))$, where $i = \{s, r\}$.

Figure 1 displays the receiver operating characteristic (ROC) curves of the two aforementioned detectors for an experiment with $N = 64$, $L = 4$ and $\text{SNR}_s = \text{SNR}_r = -12$ dBs. As this figure shows, knowing the channel subspaces provide a significant performance gain. Figure 2 depicts the probability of missed detection, p_m, for a fixed probability of false alarm $p_{fa} = 10^{-4}$, for varying equal $\text{SNR}_s = \text{SNR}_r = \text{SNR}$, $N = 32$, and $L = 4$. This figure also shows the advantage provided by the knowledge of the channel subspace. Finally, Figure 3 shows the probability of missed detection, for a fixed probability of false alarm $p_{fa} = 5 \cdot 10^{-3}$, for varying N, and $\text{SNR}_s = \text{SNR}_r = -10$ dB and $L = 4$. This figure also shows that (5) outperforms the GLR for unknown channels, and the performance advantage seems to increase as N increases.

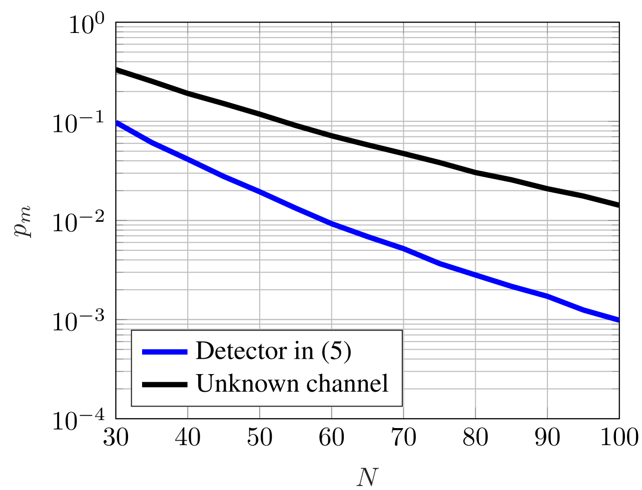

Fig. 3: Probability of missed detection vs. N for an experiment with $\text{SNR}_s = \text{SNR}_r = -10$ dB, $L = 4$ and $p_{fa} = 5 \cdot 10^{-3}$

5. CONCLUSIONS

This paper has studied passive detection for one-antenna transmitters when the channel subspaces of the surveillance and reference arrays are known, and the noises at each array are independent with an unstructured positive definite covariance matrix. We have also assumed that the transmitted signal is Gaussian distributed. The proposed detector is based on likelihood theory where all but one of the unknown parameters are replaced by their maximum likelihood (ML) estimates. Under $\mathcal{H}_1$, it is not possible to derive the ML estimate of one parameter without resorting to numerical methods. This parameter is the covariance matrix of the reference channel and, therefore, an estimator based on the sample covariance matrix of the channel is used. The proposed detector is compared with the generalized likelihood ratio test for unknown channel subspaces, which illustrates the advantage of knowing the channel subspaces.

In the future, this work will be extended to consider transmitters with more than one antenna, in which case the scalar q_{sr} becomes the matrix $\mathbf{Q}_{sr}$. Additionally, we also plan to derive a numerical method to obtain the ML estimate of $\mathbf{\Theta}_a$ and evaluate if the proposed sample-based estimator achieves a performance close to that of the numerical search.

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