NONCOHERENT MULTIUSER GRASSMANNIAN CONSTELLATIONS FOR THE MIMO MULTIPLE ACCESS CHANNEL

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ABSTRACT

We consider the design of multiuser constellations for a multiple access channel (MAC) with K users, with M antennas each, that transmit simultaneously to a receiver equipped with N antennas through a Rayleigh block-fading channel, when no channel state information (CSI) is available to either the transmitter or the receiver. In full-diversity scenarios where the coherence time is at least $T \geq (K+1)M$, the proposed constellation design criterion is based on the asymptotic expression of the multiuser pairwise error probability (PEP) derived by Brehler and Varanasi in [1]. Although this PEP expression was previously considered intractable for optimization, in this work we derive a closed-form formula for its unconstrained gradient and perform Riemannian optimization in the Grassmannian manifold to design multiuser constellations for the MIMO MAC with state-of-the-art performance in terms of symbol error rate (SER).

***Index Terms*—** Noncoherent communications, MIMO, multiple access channel (MAC), Grassmannian, Riemannian gradient

1. INTRODUCTION

In multiple-input multiple-output (MIMO) noncoherent wireless communications over fast fading channels, the channel state information (CSI) is assumed to be unknown at both the transmitter and receiver. In the single-user case and under additive Gaussian noise, it was proved by Hochwald and Marzetta [2, 3] that the $T \times M$ space-time transmit matrices that achieve the ergodic noncoherent capacity for the MIMO block-fading model can be factored as the product of an isotropically distributed $T \times M$ truncated unitary matrix, also called Stiefel matrix, and a diagonal $M \times M$ matrix with real non-negative entries. Further, when $T \gg M$ the nonzero entries of the diagonal matrix take the same value, showing that in this regime it is optimal to transmit unitary space-time codewords $\mathbf{X}^H\mathbf{X} = \mathbf{I}_M$. Using the same signal model, Zheng and Tse [4] proved that at high signal-to-noise ratio (SNR) and when $T \geq 2M$, ergodic capacity can be achieved by transmitting isotropically distributed unitary matrices. Motivated by these information-theoretic results, numerous methods for the design of single-user constellations formed by truncated unitary signal matrices, called unitary space-time modulations (USTM), have been investigated and proposed over the last decades [5–15]. In MIMO noncoherent constellations, information is carried by the column span (i.e., a subspace) of the transmitted $T \times M$ matrix, $\mathbf{X}$. The problem of designing single-user noncoherent codebooks is thus closely related to finding optimal packings in Grassmann manifolds [4, 16], and the resulting constellations are referred to as Grassmannian constellations.

In the multiuser case, the design of noncoherent constellations is significantly more complex, as many of the theoretical results that exist for the single-user case (as well as the insights gained from them), such as the optimality of unitary space-time or Grassmannian constellations at high-SNR, are no longer true. In this work, we consider the design of noncoherent constellations for the MIMO multiple access channel (MAC), a problem for which there is no satisfactory solution yet. In the MAC several users transmit information simultaneously over the same bandwidth and at the same channel use or time slot to a common receiver. A common example is the uplink channel in broadband cellular communications, where several users communicate with a base station (BS). In the case of coherent communications with perfect channel state information (CSI) at the receiver or BS, capacity results for the MIMO MAC can be found in [17].

For noncoherent communications, however, the full capacity region of the MIMO-MAC is unknown. For the K-user single-input multiple-output (SIMO) MAC, it was conjectured by Shamai and Marzetta in [18] that for block-fading channels with coherence time $T > 1$ the sum capacity can be achieved by no more than $K = T$ users, which is supported by asymptotic analysis and simulation results. For the two-user MIMO MAC an achievable DoF (degrees of freedom) region has been proposed in [19]. The optimal DoF region for a two-user SIMO MAC has been derived in [20]. Existing theoretical results however do not provide clear insights regarding the structure of the transmit space-time matrices for the MIMO MAC.

Of particular importance for the noncoherent K-user MIMO MAC is the work of Brehler and Varanasi in [1], where the authors derived an asymptotic expression of the joint pairwise probability of error (PEP) of the optimum receiver. Moreover, they showed that the coherence time must be at least $T \geq (K+1)M$ to ensure full diversity of NM for each user. However, the PEP expression in [1] was considered to be intractable for optimization, so none of the subsequent studies have used it as a criterion to design multiuser constellations. Most of the proposed criteria in the literature either optimize single-user Grassmannian designs with or without partitioning; that is, using independently designed single-user codebooks, or designing a large single-user codebook that is then partitioned according

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to some subspace distance measure into K smaller single-user codebooks [1, 21–23]. In this work, we derive the gradient of the PEP expression [1] in the Grassmannian manifold. Using this expression, we design noncoherent multiuser constellations for the MIMO MAC in full-diversity scenarios with improved SER performance.

Notation: In this paper, matrices are denoted by bold-faced upper case letters, column vectors are denoted by bold-faced lower case letters, and scalars are denoted by light-faced lower case letters. The superscript $(\cdot)^H$ denotes Hermitian conjugate. The trace and determinant of a matrix $\mathbf{A}$ will be denoted, respectively, as $\mathrm{tr}(\mathbf{A})$ and $\det(\mathbf{A})$. We let $\mathbf{I}_n$ denote the identity matrix of size n, $\mathcal{CN}(0,1)$ denote a complex proper Gaussian distribution with zero mean and unit variance, and $\mathbf{x} \sim \mathcal{CN}_n(\mathbf{0},\mathbf{R})$ denote a complex Gaussian vector in $\mathbb{C}^n$ with zero mean and covariance matrix $\mathbf{R}$.

2. SYSTEM MODEL AND PEP

2.1. System model

We consider K users in a noncoherent MIMO MAC simultaneously transmitting to a common base station. Let the receiver have N antennas and assume for simplicity that all users have the same number of transmit antennas M. The channel of user k is $\mathbf{H}_k \in \mathbb{C}^{M\times N}$, is assumed to remain constant over T symbol periods, over which communication occurs, and has a Rayleigh fading distribution ($\mathbf{H}_k(i,j) \sim \mathcal{CN}(0,1)$). The channels of all users change to an independent realization in the next transmission block (block-fading channel). User k transmits at rate R_k (bits/channel use), so within a coherence block sends a matrix equiprobably chosen from a codebook $\mathcal{C}_k = \{\mathbf{X}_{k,1},\dots,\mathbf{X}_{k,L_k}\}$ with $L_k = 2^{R_k T}$. We will assume that the transmitted matrices are semi-unitary or Stiefel ($\mathbf{X}_{k,i}^{\mathrm{H}}\mathbf{X}_{k,i} = \mathbf{I}_M$), although this is not necessarily optimal in the MAC. We address in this paper the design of the joint codebook $\mathcal{C} = \{\mathcal{C}_1,\dots,\mathcal{C}_K\}$.

Let us focus on a two-user MIMO MAC and define the $T\times 2M$ matrix of transmitted codewords $\mathbf{F}_i = [\mathbf{X}_{1,i_1},\mathbf{X}_{2,i_2}]$ adhering to the notation of [1, 24]. The multiuser codeword $\mathbf{F}_i$ is not a Stiefel matrix anymore ($\mathbf{F}_i^{\mathrm{H}}\mathbf{F}_i \neq \mathbf{I}_{2M}$) even if $\mathbf{X}_{k,i_k}$ are for $k = 1,2$. The set of 2-user codewords is

$$\mathcal{F} = \{\mathbf{F}_i = [\mathbf{X}_{1,i_1},\mathbf{X}_{2,i_2}],\ \mathbf{X}_{1,i_1}\in\mathcal{C}_1, \mathbf{X}_{2,i_2}\in\mathcal{C}_2\},$$

and has cardinality $|\mathcal{F}| = |\mathcal{C}_1\times\mathcal{C}_2| = L_1L_2$. Due to different path losses the two users can have different SNRs. Without loss of generality, we take the SNR of user 1 as a reference and denote it as ρ, while the SNR of user 2 is $\beta_2\rho$. The model generalizes to K users. When the 2-user codeword $\mathbf{F}_i$ is transmitted, the received signal is

$$\mathbf{Y} = \mathbf{X}_{1,i_1}\mathbf{H}_1 + \sqrt{\beta_2}\mathbf{X}_{2,i_2}\mathbf{H}_2 + \sqrt{\frac{M}{T\rho}}\mathbf{W}, \tag{1}$$

where $\mathbf{W}\in\mathbb{C}^{T\times N}$ represents the additive Gaussian noise, modeled as $w_{ij} \sim \mathcal{CN}(0,1)$.

Conditioned on the transmitted signal, each column of $\mathbf{Y}$ follows a zero-mean complex normal distribution with covariance matrix $\mathbf{R}_i = \mathbf{X}_{1,i_1}\mathbf{X}_{1,i_1}^{\mathrm{H}} + \beta_2\mathbf{X}_{2,i_2}\mathbf{X}_{2,i_2}^{\mathrm{H}} + \frac{M}{T\rho}\mathbf{I}_T$, so the density of $\mathbf{Y}|\mathbf{F}_i$ is

$$p(\mathbf{Y}|\mathbf{F}_i) = \frac{1}{\pi^{TN}\det(\mathbf{R}_i)^N}\exp\mathrm{tr}\left(-\mathbf{R}_i^{-1}\mathbf{Y}\mathbf{Y}^{\mathrm{H}}\right). \tag{2}$$

The optimum Maximum Likelihood (ML) detector when codewords are chosen with equal probability is

$$\hat{\mathbf{F}}_i = \arg\min_{\mathbf{F}_i\in\mathcal{F}}\ \mathrm{tr}\left(\mathbf{Y}^{\mathrm{H}}\mathbf{R}_i^{-1}\mathbf{Y}\right) + N\log\det(\mathbf{R}_i). \tag{3}$$

Notice that the ML detector at the BS needs to know the SNR of all users. The SNR depends primarily on the path loss and therefore varies on a much slower temporal scale than the multipath fading. It is therefore feasible to have this long-term CSI available at the BS.

2.2. Noncoherent pairwise error probability (PEP)

Assuming full-diversity scenarios where the coherence time is at least $T \geq (K+1)M$, Brehler and Varanasi derived in [1] the asymptotic joint pairwise error probability of the ML detector in the noncoherent case. Since this result forms the starting point of the proposed design criterion, we summarize it in this subsection.

Let us introduce the following notation for the orthogonal projection matrix onto the orthogonal complement of the subspace spanned by the columns of $\mathbf{M}$:

$$\mathbf{P}_{\mathbf{M}}^{\perp} = \mathbf{I} - \mathbf{M}(\mathbf{M}^{\mathrm{H}}\mathbf{M})^{-1}\mathbf{M}^{\mathrm{H}}.$$

Following [1], when comparing two joint hypotheses $\mathbf{F}_i$ vs. $\mathbf{F}_j$, the single-user codewords are to be reordered within the multiuser codeword so that the terms in error appear first, i.e., $\mathbf{F}_i = [\mathbf{F}_i^{(e)}\ \mathbf{F}^{(c)}]$ and $\mathbf{F}_j = [\mathbf{F}_j^{(e)}\ \mathbf{F}^{(c)}]$, where $\mathbf{F}^{(c)}$ are the codewords common to the two hypotheses or multiuser codewords, and $\mathbf{F}_i^{(e)},\ \mathbf{F}_j^{(e)}$ the codewords of the users in error between the two different hypotheses. With these conventions in place, the following proposition shows the expression derived in [1] for the asymptotic PEP $\mathcal{P}(\mathbf{F}_i \to \mathbf{F}_j)$.

Proposition 1 (Asymptotic Pairwise Error Probability [1]) *Let us assume no correlation between the channel fading coefficients, equal SNR users [1], and that $\mathbf{F}_i^{(e)H}\mathbf{P}_{\mathbf{F}_j}^{\perp}\mathbf{F}_i^{(e)}$ has full rank (i.e. $T \geq (e+K)M$, with e the number of symbols in error). Then, the total pairwise error probability of the optimal detector, for detecting $\mathbf{F}_j$ when receiving $\mathbf{F}_i$, approaches (when the SNR grows) arbitrarily closely to*

$$\mathcal{P}(\mathbf{F}_i \to \mathbf{F}_j) = \frac{\sigma^{2eNM}\sum_{n=0}^{eNM}\binom{2eNM-n}{eNM}(n!)^{-1}(\hat{c}_{ij})^n}{\det(\mathbf{F}_i^{(e)H}\mathbf{P}_{\mathbf{F}_j}^{\perp}\mathbf{F}_i^{(e)})^N}, \tag{4}$$

where $\hat{c}_{ij} = N\log\frac{\mathbf{F}_i^{\mathrm{H}}\mathbf{F}_i}{\mathbf{F}_j^{\mathrm{H}}\mathbf{F}_j} \geq 0$, condition that can always be guaranteed by relabeling the hypothesis accordingly; σ^2 is the noise variance.

3. PROPOSED JOINT CONSTELLATION DESIGN

The PEP expression in (4) has not been used as an optimization criterion so far as it was considered untractable for optimization. Further, it was thought not to give clear insights for constellation design, as discussed in [22] and [23]. In this section, we propose a simplified union-bound criterion for the design of noncoherent MAC constellations based on this expression.

Notice that the denominator in $\mathcal{P}(\mathbf{F}_i \to \mathbf{F}_j)$ is the factor that encodes for the distance between joint codewords in error, since for $K = 1$ it reduces to the coherence design criterium of [15]. Hence, for simplicity, we focus on optimizing these factors by neglecting the sum term in the numerator. This leads us to consider a multiuser union-bound cost function for the design of noncoherent multiuser constellations:

$$f(\mathcal{C}) = \sum_{i\neq j}\sigma^{2eNM}\det(\mathbf{F}_i^{(e)H}\mathbf{P}_{\mathbf{F}_j}^{\perp}\mathbf{F}_i^{(e)})^{-N}, \tag{5}$$

[1]To consider users with different SNRs simply requires introducing a fixed diagonal matrix in the cost function.

where the sum is over all the joint multiuser codewords in $\mathcal{C}$. Single-user noncoherent constellations using the union-bound criterion have been obtained in our previous works [15, 25].

Notice that e in (5) can take values from 1 symbol in error to all the K users in error, which makes the number of terms in the sum increasingly large: as the size of $|\mathcal{C}| = L_1 \cdots L_K$ grows, the number of pairs of hypothesis i, j, i.e. number of terms in the sum (5), grows as $\sim |\mathcal{C}|^2$. For example, for two users $K = 2$, and $e = 1$, there are $L_1(L_1-1)L_2 + L_1L_2(L_2-1)$ terms in Eq. (5), whereas for $e = 2$ there are $L_1L_2(L_1-1)(L_2-1)$ terms, that is, the number of terms with two symbols in error grows with one order higher. This would make the multiuser optimization problem computationally unfeasible as the number of users and codewords grows. However, the contribution of the factor σ^{2eNM} is σ^{2NM} for the less numerous one-error terms and σ^{4NM} for the more numerous two-error terms. Since σ is inversely proportional to the SNR, the two-error terms are weighed two orders of magnitude less than the one-error terms. Because of this, and in order for the optimization to become feasible computationally, we propose to consider only the one-symbol-in-error terms, that is

$$F(\mathcal{C}) = \sum_{\substack{\mathbf{F}_i \neq \mathbf{F}_j \in \mathcal{C} \\ e=1}} \det(\mathbf{F}_i^{(e)\mathrm{H}} \mathbf{P}_{\mathbf{F}_j}^{\perp} \mathbf{F}_i^{(e)})^{-N}. \tag{6}$$

The proposed design criterion for full-diversity scenarios finally is:

$$\underset{\mathcal{C}_1, \ldots, \mathcal{C}_K}{\operatorname{argmin}} \; F(\mathcal{C}). \tag{7}$$

3.1. Gradient formula

We propose to perform a gradient descent algorithm over the packing $\mathcal{C}$ to minimize the cost function (6), which is an approximation to the PEP union bound. Since the users transmit Grassmannian constellations (USTM codes), we need the Riemannian gradient vector of $F(\mathcal{C})$ in the Grassmannian product manifold. First, the following theorem gives the unconstrained gradient of $F\mathcal{C})$.

Theorem 1 *Let $\mathbf{M}_j := \mathbf{F}_j^{\mathrm{H}}\mathbf{F}_j$, $\mathbf{G}_{ij} := \mathbf{F}_i^{(e)\mathrm{H}} \mathbf{P}_{\mathbf{F}_j}^{\perp} \mathbf{F}_i^{(e)}$, and $\mathcal{F}_{ij} = \det(\mathbf{G}_{ij})^{-N}$. The unconstrained Euclidean gradient of $F(\mathcal{C})$ with respect to codeword $\mathbf{X}$ is:*

$$D_{\mathbf{X}}F(\mathcal{C}) = \sum_{\substack{\mathbf{F}_i \neq \mathbf{F}_j \in \mathcal{C} \\ e=1}} D_{\mathbf{X}}\mathcal{F}_{ij}(\mathbf{F}_i^{(e)}, \mathbf{F}_j), \tag{8}$$

where for a codeword in error, $\mathbf{X} = \mathbf{X}^{(e)}$, the gradient matrix is given by the corresponding block of M columns in the following expression

$$\begin{aligned} &D_{\mathbf{X}^{(e)}}\mathcal{F}_{ij}(\mathbf{F}_i^{(e)}, \mathbf{F}_j) = \\ &-2N\mathcal{F}_{ij} \left[\mathbf{P}_{\mathbf{F}_j}^{\perp}\mathbf{F}_i^{(e)}\mathbf{G}_{ij}^{-1}\right][i_1(\mathbf{X}^{(e)}) : i_M(\mathbf{X}^{(e)})], \end{aligned} \tag{9}$$

where the notation $\mathbf{F}[i_1(\mathbf{X}) : i_M(\mathbf{X})]$ has been used to denote the extraction of the M columns in $\mathbf{F}$ corresponding to the position of the codeword $\mathbf{X}$ inside the concatenated matrix.

Similarly, for $\mathbf{X} = \mathbf{X}^{(c)}$, a codeword not in error, we have:

$$\begin{aligned} &D_{\mathbf{X}^{(c)}}\mathcal{F}_{ij}(\mathbf{F}_i^{(e)}, \mathbf{F}_j) = \\ &2N\mathcal{F}_{ij} \Big[(\mathbf{I}_T - \mathbf{F}_j\mathbf{M}_j^{-1}\mathbf{F}_j^{\mathrm{H}})\mathbf{F}_i^{(e)} \cdot \\ &\quad \cdot \mathbf{G}_{ij}^{-1}\mathbf{F}_i^{(e)\mathrm{H}}\mathbf{F}_j\mathbf{M}_j^{-1}\Big][j_1(\mathbf{X}^{(c)}) : j_M(\mathbf{X}^{(c)})]. \end{aligned} \tag{10}$$

Proof: Detailed proof of this result can be found in [26, Theorem 1]. □

3.2. Grassmannian optimization

Since USTM optimization must be performed on the Grassmannian manifold, the unconstrained gradient given in Theorem 1 must be projected in the tangent space. The tangent space projector for the Grassmann manifold is $\mathrm{P}_{\mathbf{X}}(\dot{\mathbf{Z}}) = (\mathbf{I}_T - \mathbf{X}\mathbf{X}^{\mathrm{H}})\dot{\mathbf{Z}}$, for any tangent matrix $\dot{\mathbf{Z}}$ at point $\mathbf{X}$, and the retraction function $\mathrm{R}_{\mathbf{X}}$ from the tangent space to the manifold is the QR decomposition. Algorithm 1 shows the general Riemannian manifold optimization method for designing noncoherent MIMO MAC constellations for K users, using the proposed cost function $F(\mathcal{C})$.

Algorithm 1 Grassmannian optimization for the K-user MAC

Input: $\sum_{k=1}^{K} L_k$ uniformly distributed points in $\mathbb{C}^{T \times M}$
Output: Optimized joint constellation $\mathcal{C}$

1. Compute unconstrained gradient $D_{\mathbf{X}}F(\mathcal{C})$ for every codeword $\mathbf{X} = \mathbf{X}_{k,i}$ in $\mathcal{C}_k$, $(k = 1, \ldots, K, i = 1, \ldots, L_k)$.
2. Project down to the chosen manifold tangent space at every $\mathbf{X}$:
$$\nabla_{\mathbf{X}}F(\mathcal{C}) = \mathrm{P}_{\mathbf{X}}(D_{\mathbf{X}}F(\mathcal{C})).$$
3. Compute the norm of the full gradient:
$$||\nabla F(\mathcal{C})|| = \sqrt{\sum_{k=1}^{K}\sum_{i=1}^{L_k} ||\nabla_{\mathbf{X}_{k,i}}F(\mathcal{C})||_F^2}.$$
4. Move every codeword a step h in the direction of steepest ascent (descent) retracting back onto the manifold:
$$\mathbf{X}_{new} = \mathrm{R}_{\mathbf{X}}\left(\pm h \frac{\nabla_{\mathbf{X}}F(\mathcal{C})}{||\nabla F(\mathcal{C})||}\right).$$
5. Evaluate $F(\mathcal{C}_{new})$ and repeat step 4 with smaller h until cost function improves its value with respect to $F(\mathcal{C})$.
6. Update constellation by substituting $\mathbf{X} \mapsto \mathbf{X}_{new}$ for every codeword.
7. Repeat $1-6$ until the number of iterations or improvement in $F(\mathcal{C})$ reach a threshold.
8. Return constellation $\mathcal{C}_k = \{\mathbf{X}_{k,i}\}_{i=1}^{L_k}$, for every user $k = 1, \ldots, K$.

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4. RESULTS

We study the symbol-error-rate (SER) performance of 2-user MIMO MAC designs obtained using the proposed joint optimization method (7). Moreover, we assume that: i) the two users have the same average SNR, ii) there is no correlation between the channel fading coefficients, and iii) the scenarios allow for full diversity, which for a 2-user MAC means $T \geq 3M$.

In Fig. 1 we study the case of $T = 3$ symbol periods, $M = 1$ transmit antennas, $N = 5$ receive antennas, and $B = 5$ bits per codeword. We compare the joint multiuser design obtained from the proposed criterion (labeled as "Min-UB' in the figure) versus single-user designs obtained from the chordal distance optimization [14] (labeled as "Chordal") and the coherence criterion of [15] (labeled as "Coherence"), along with the multiuser designs obtained from the criteria proposed in [27] (labeled as "$J_{1/2}$", "β", and "δ", which are proxy functions for the PEP). The design labeled "MinMax-PEP" is a simplified version of (7) where only the worst PEP term of (6) is improved at every iteration, instead of the union bound. The proposed design Min-UB shows a very significant improvement in performance over the rest of the designs in this scenario.

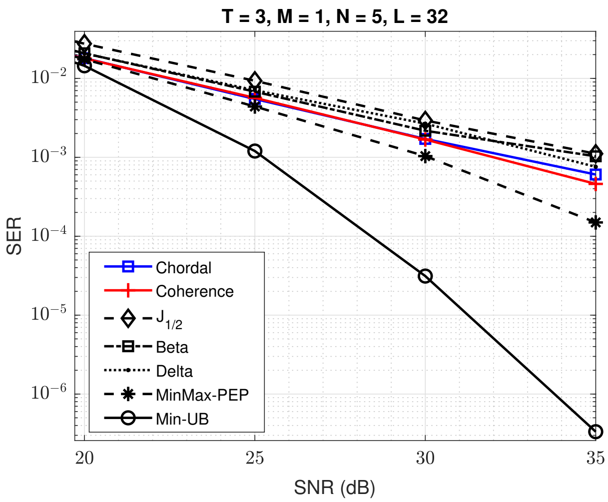

Fig. 1. Comparison of different multiuser noncoherent constellation designs for a 2-user MAC with $T = 3$, $M = 1$, $N = 5$ and $L = 32$ codewords.

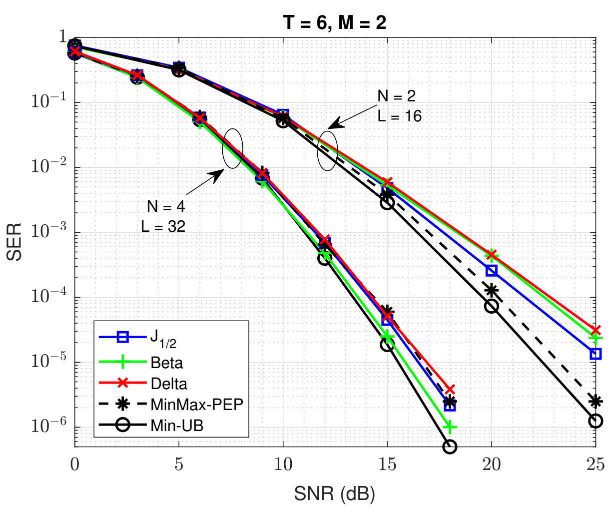

Fig. 2. Comparison of different multiuser noncoherent constellation designs for a 2-user MAC with $T = 6$, $M = 2$, $N \in \{2, 4\}$ and $L \in \{16, 32\}$.

In Fig. 2 we compare the different multiuser design criteria in a scenario with $T = 6$, $M = 2$, and a different number of receive antennas and joint codewords. The gap in performance between the proposed criterion and the proxy functions of [27] seems to be reduced when increasing the number of codewords but still, the former outperforms all designs studied so far. It is important to point out that the performance of a given optimized constellation in the MAC depends on many parameters: number of users, number of antennas, coherence time, etc. In particular, the number of antennas affects the diversity (the slope of the SER vs. SNR curve) and thus can lead to important differences in performance. For example, Fig. 1 shows the performance of constellations in projective space whereas Fig. 2 shows codebooks in a proper Grassmannian manifold. This may explain the differences observed between the two figures. Overall, we may conclude that our proposed design method provides state-of-the-art multiuser constellations for the MAC in the full-diversity case.

5. CONCLUSION

We have proposed a Riemmanian optimization criterium for designing noncoherent constellations for the MIMO MAC in full-diversity scenarios, i.e. $T \geq (K + 1)M$, for any number of users. The cost function is a union bound of the dominant terms of the asymptotic joint PEP, corresponding to those where only one of the users of the MAC channel is in error. Moreover, we have provided, for the first time in the literature, explicit closed-form formulas for the gradient of the dominant term of this function. Finally, we have shown that constellations optimized with this criterium outperform existing single-user and multi-user design methods.

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