POWER MINIMIZATION IN MULTI-TIER NETWORKS WITH FLEXIBLE DUPLEXING

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ABSTRACT

In this paper we present an algorithm to minimize transmit power in multiple-input multiple-output (MIMO) heterogeneous networks (HetNets) with flexible duplexing, a promising strategy that allows the coexistence of uplink and downlink cells within the same time and frequency resource block. First, the proposed algorithm minimizes transmit power for a given uplink/downlink (UL/DL) combination, and afterwards, the optimal solution out of the explored UL/DL combinations is selected. To reduce the computational cost of exploring all the UL/DL settings, we propose a hierarchical switching (HS) approach that considers a reduced subset of transmit directions. By means of Monte Carlo simulations, we show that the proposed technique provides significant power savings with respect to a conventional time-division duplex (TDD) scheme.

Index Terms— Convex optimization, discrete optimization, flexible duplexing, heterogeneous networks, power efficiency.

1. INTRODUCTION

Over the last years, the diversity of the different communication services, user profiles and cell coverage ranges has lead to a significant research interest in heterogeneous networks (HetNets) [1]. This variety of network topologies and scenarios plays a paramount role in the 5-th generation of wireless communications (5G). One of the most promising access techniques in the context of HetNets is flexible duplexing, also called reverse-time division duplex (reverse TDD) [2] or dynamic-TDD [3] in the literature. The main idea consists in allowing the coexistence of uplink and downlink cells in the same time and frequency slot. Each uplink/downlink (UL/DL) combination generates different interference levels at the receivers, having an impact on the quality of service (QoS).

Several studies on the benefits of flexible duplexing have been recently carried out, relying on different figures of merit. Most of the existing works assume that a fixed UL/DL combination is given, and present an analysis in terms of multiplexing gain [4–6], or in terms of throughput [7–11]. A frame structure for interference mitigation can be found in [12], and flexible duplexing is further proposed as a candidate for evaluation in real deployments in [13]. On the other hand, the number of works addressing the task of finding the best UL/DL combination is still scarce. Nevertheless, a joint user scheduling, precoding and UL/DL selection framework is presented in [9].

Regarding power efficiency, most of the works are still focused on orthogonal TDD access techniques [14–17]. However, a power optimization method with flexible UL/DL sets is provided in [18], and afterwards, authors in [19] analyze the coexistence of UL and DL cells in terms of downlink transmit power. A remarkable study is performed in [20], where a variation of flexible duplexing called α-duplex is introduced. Authors in [20] optimize power and spectrum overlap for a single-tier network.

Our main contribution is an algorithm, denoted by `MinPower-MaxSINR`, that addresses power minimization so that a QoS requirement is fulfilled in terms of signal-to-interference-plus-noise ratio (SINR). The scheme consists in performing an iterative procedure that minimizes transmit power for fixed receiving filters, then sets the transmitters to the solution obtained in the previous step and maximizes the SINR at the receivers. Once the `Max-SINR` filters are calculated, the sequence is repeated for the original SINR constraints with the new filters. We also propose a hierarchical switching (HS) scheme that reduces the computational cost of UL/DL selection significantly [10]. By means of Monte Carlo simulations, we show that the proposed technique outperforms other existing strategies in a 2-tier HetNet.

Notation: Uppercase (lowercase) boldface letters will be used for matrices (column vectors), and $(\cdot)^H$ for conjugate transpose (Hermitian). Additionally, we use $\bar{\mathcal{L}}$ for the logical negation of a set of Boolean variables $\mathcal{L}$, and we define the operator

$$\operatorname*{cat}_{s}(a_s)$$

as the horizontal concatenation of indexed elements a_s.

2. SYSTEM MODEL AND PROBLEM STATEMENT

The considered HetNet consists of 2 tiers, with G_m macrocells and G_s small cells for a total of $G = G_m + G_s$ cells. Each cell g has an N_g-antenna base station (BS) in the macro-tier, or access point (AP) in the small tier, whereas the user equipments (UE) have a single antenna. For each cell, a single data stream per user is transmitted, and the intra-cell interference is assumed to be handled internally. Therefore, the scenario under analysis, with a single active user per cell, can be viewed as a MIMO interference channel (IC) where the inter-cell interference is associated to the interfering links. The signal at the input of the receiver in cell g is given by

$$
\begin{aligned}
y_g = \sigma_g^2 + i_g &\left[d_{gg}^{-\alpha/2}\mathbf{h}_{gg}\mathbf{w}_g + \sum_{j=1, j\neq g}^{G} \left(i_j d_{gj}^{-\alpha/2}\mathbf{h}_{gj}\mathbf{w}_j \right. \right. \\
&\left. + (1-i_j) d_{gj}^{-\alpha/2} h_{gj} P_{UE_j}^{1/2} \right) \bigg] + (1-i_g)\left[\mathbf{u}_g^H d_{gg}^{-\alpha/2}\mathbf{h}_{gg} P_{UE_g}^{1/2} \right. \\
&\left. + \sum_{j=1, j\neq g}^{G} \left(i_j \mathbf{u}_g^H d_{gj}^{-\alpha/2}\mathbf{H}_{gj}\mathbf{w}_j + (1-i_j)\mathbf{u}_g^H d_{gj}^{-\alpha/2}\mathbf{h}_{gj} P_{UE_j}^{1/2} \right) \right]
\end{aligned}
\tag{1}
$$

where i_g is a Boolean indicating whether each cell g is in downlink ($i_g = 1$) or in uplink ($i_g = 0$). Also, α represents the path loss exponent and σ_g^2 is the additive white Gaussian noise (AWGN) variance at the input of receiver g. The distance and the channel between the transmitter in cell j and the receiver in cell g are given by d_{gj} and $\mathbf{H}_{gj}$, respectively[1]. $\mathbf{w}_g \in \mathbb{C}^{N_g \times 1}$ is the beamformer applied at the g-th BS/AP in downlink, $\mathbf{u}_g \in \mathbb{C}^{N_g \times 1}$ is the unit-norm filter for the gth BS/AP when it is in uplink mode, and the scalar values P_{UE_g} are the transmit power at the single-antenna users.

2.1. Power Minimization in Flexible Duplexing networks

Our goal in the described scenario is to minimize the transmit power over 2 consecutive slots while satisfying an SINR requirement. We assume that all cells switch from DL to UL or viceversa every slot:

$$
\begin{aligned}
\mathcal{P}_1 : \underset{\substack{\{\mathbf{w}_g\}, \{\mathbf{u}_g\}, \\ \{P_{UE_g}\}, \mathcal{I}}}{\text{minimize}} \quad & \sum_{g=1}^{G} \left(a_g \|\mathbf{w}_g\|^2 + (1-a_g) P_{UE_g} \right) \\
\text{s.t.} \quad & \text{SINR}_g\left(\{\mathbf{w}_j\}, \{P_{UE_j}\}, \mathbf{u}_g, \mathcal{I}\right) \geq \gamma_g \quad \forall g, j \\
& \text{SINR}_g\left(\{\mathbf{w}_j\}, \{P_{UE_j}\}, \mathbf{u}_g, \bar{\mathcal{I}}\right) \geq \gamma_g \quad \forall g, j,
\end{aligned}
$$

where γ_g represents the target SINR for cell g, and $a_g \in [0,1]$ represents the predefined priority ratio be-

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[1]The notation for channel matrices varies between lowercase, uppercase and boldface letters due to the different SISO, MISO/SIMO and MIMO channels within the considered network.

Algorithm 1: Hierarchical Switching.

Input: $\{\mathbf{H}_{gj}\}$, $\{d_{gj}\}$, $\{N_g\}$, $\mathcal{I}$
Output: Sets $\{\mathbf{w}_g\}$, $\{\mathbf{u}_g\}$ and $\{P_{UE_g}\}$ and $\mathcal{I}$

```
/* Initialization                         */
𝓘 = zeros(1, G);                 // cat(i_g)
MinTotalPower = MinPower-MaxSINR(𝓘)
for g = 1 to G do
    /* Cell g switches                     */
    𝓘(g) = 𝓘̄(g)
    /* Evaluate the new setting            */
    TotalPower = MinPower-MaxSINR(𝓘)
    if TotalPower < MinTotalPower then
        MinTotalPower = TotalPower
    else
        𝓘(g) = 𝓘̄(g)
/* {w_g},{u_g},{P_UE_g} associated to 𝓘   */
return
```

tween minimizing uplink and downlink power levels. Furthermore, $\mathcal{I} \overset{\text{def}}{=} \underset{g}{\text{cat}}(i_g)$ is the set of Boolean variables i_g. Note that the SINR_g at cell g depends on the UL/DL configuration, the filter at the g-th BS/AP in uplink, and the beamformers and power levels at all the transmitters in the network. As mentioned before, all cells switch their transmit direction every slot, thus we also consider the SINR constraints for the complementary UL/DL set $\bar{\mathcal{I}}$.

The set $\mathcal{I}$ of indication variables i_g turns $\mathcal{P}_1$ into a discrete optimization problem. For this reason, we decouple our power minimization scheme into a transmit power minimization for all (or a subset) of the possible UL/DL combinations, followed by a discrete search of the UL/DL setting that provides the optimal solution. In order to solve the problem for a given UL/DL configuration, we apply the `MinPower-MaxSINR` algorithm presented in Section 3. Finding the optimal solution for $\mathcal{P}_1$ requires evaluating all possible sets $\mathcal{I}$, i.e., a total of 2^G UL/DL combinations. Despite the networks under study being not dense in terms of number of cells, we aim to reduce the computational cost of the transmit direction selection. For this purpose, we propose a suboptimal approach, based on hierarchical switching (HS), that allows to reduce the number of evaluations to G. The main idea consists in starting in conventional TDD mode. Then, a cell in the macro-tier switches transmit direction with respect to the rest of cells, and the transmit power is minimized via `MinPower-MaxSINR`. If the required power is lower, the cell keeps the direction change, whereas if the power increases, the cell goes back to the previous state. This procedure is repeated sequentially until every macrocell has evaluated the benefit of the UL/DL switching, and then the same sequence is carried out for

$$\mathcal{P}_2: \quad \underset{\{\mathbf{w}_g\},\{\mathbf{u}_g\},\{P_{UE_g}\}}{\text{minimize}} \quad \sum_{g=1}^{G}\left(a_g i_g \|\mathbf{w}_g\|^2 + (1-a_g)(1-i_g)P_{UE_g}\right)$$

$$\text{subject to} \quad i_g \mathbf{h}_{gg}\mathbf{w}_g d_{gg}^{-\alpha}\mathbf{w}_g^H \mathbf{h}_{gg}^H + (1-i_g)\mathbf{u}_g^H \mathbf{h}_{gg}P_{UE_g}d_{gg}^{-\alpha}\mathbf{h}_{gg}^H \mathbf{u}_g$$

$$\geq \gamma_g \left(\sigma_g^2 + i_g \sum_{j=1,j\neq g}^{G} \left(\underbrace{i_j \mathbf{h}_{gj}\mathbf{w}_j d_{gj}^{-\alpha}\mathbf{w}_j^H \mathbf{h}_{gj}^H}_{\text{BS/AP to UE interference}} + \underbrace{(1-i_j)h_{gj}P_{UE_j}d_{gj}^{-\alpha}h_{gj}^H}_{\text{UE to UE interference}} \right) \right.$$

$$\left. + (1-i_g)\sum_{j=1,j\neq g}^{G} \left(\underbrace{i_j \mathbf{u}_g^H \mathbf{H}_{gj}\mathbf{w}_j d_{gj}^{-\alpha}\mathbf{w}_j^H \mathbf{H}_{gj}^H \mathbf{u}_g}_{\text{BS/AP to BS/AP interference}} + \underbrace{(1-i_j)\mathbf{u}_g^H \mathbf{h}_{gj}P_{UE_j}d_{gj}^{-\alpha}\mathbf{h}_{gj}^H \mathbf{u}_g}_{\text{UE to BS/AP interference}} \right) \right) \quad \forall g \quad (2)$$

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small cells. This hierarchical order is due to the fact that power levels are usually higher in macrocells, and hence they have a larger room for improvement. The steps to complete the HS sequence are formulated in Alg. 1.

3. THE MINPOWER-MAXSINR ALGORITHM

In order to address the power minimization for a given UL/DL combination, we rewrite $\mathcal{P}_1$ for a fixed set $\mathcal{I}$ as in (2). The filters $\mathbf{u}_g$ at the BS/AP in uplink are initialized to the maximum ratio combining (MRC) of the direct channel, turning $\mathcal{P}_2$ into a power minimization problem in an equivalent MISO interference channel with SINR constraints, which has been proven to be convex in [21]. Therefore, in order to solve $\mathcal{P}_2$ for fixed $\mathbf{u}_g$, we can rely on standard convex optimization methods [22].

Let us assume that we have solved $\mathcal{P}_2$ for a set of fixed receiving filters $\mathbf{u}_g$ and a given SINR constraint, γ_g. The main intuition behind MinPower-MaxSINR is that, by fixing the transmitters to the obtained solution of $\mathcal{P}_2$ and calculating the MaxSINR filters in (3), we achieve a higher or equal SINR $\gamma_g^* \geq \gamma_g$ at the multi-antenna receivers.

$$\mathbf{u}_g^* = \text{ev}_{\max}\left(\mathbf{Q}_g^{-1/2}\mathbf{H}_{gg}P_{UE_g}d_{gg}^{-\alpha}\mathbf{H}_{gg}^H \mathbf{Q}_g^{-1/2}\right) \quad \forall g, \quad (3)$$

where $\mathbf{Q}_g$ is the interference-plus-noise covariance matrix

$$\mathbf{Q}_g = \sigma_g^2 + \sum_{j=1}^{G}\left(i_j \mathbf{H}_{gj}\mathbf{w}_j d_{gj}^{-\alpha}\mathbf{w}_j^H \mathbf{H}_{gj}^H\right.$$
$$\left. + (1-i_j)\mathbf{h}_{gj}P_{UE_j}d_{gj}^{-\alpha}\mathbf{h}_{gj}^H\right) \quad \forall g.$$

Accordingly, if we reconsider $\mathcal{P}_2$ for the initial constraints γ_g with the new filters $\mathbf{u}_g^*$, we can obtain a solution with a lower or equal power budget. We repeat this sequence until a termination criterion, Thres, is satisfied within a predefined maximum number of iterations

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Algorithm 2: MinPower-MaxSINR for a fixed UL/DL combination.

Input: $\{\mathbf{H}_{gj}\}$, $\{d_{gj}\}$, $\{N_g\}$, $\mathcal{I}$, MaxIter, Thres
Output: Sets $\{\mathbf{w}_g\}$, $\{\mathbf{u}_g\}$ and $\{P_{UE_g}\}$ and a feasibility indicator, FeasibilityFlag

/* Initialization   */
Set $\{\hat{\mathbf{u}}_g\}$ to the MRC filter
FeasibilityFlag = true, $k = 1$
while $k \leq$ *MaxIter* do
  Solve $\mathcal{P}_2$ for fixed $\{\hat{\mathbf{u}}_g\}$
  if $\{\mathbf{w}_g^*, P_{UE_g}^*\} == [\ ]$ then
    FeasibilityFlag = false
    break
  else
    $\{\mathbf{w}_g\} = \{\mathbf{w}_g^*\}$, $\{P_{UE_g}\} = \{P_{UE_g}^*\}$
    Obtain MaxSINR filters $\{\mathbf{u}_g^*\}$ as in (3)
    Power $P(k)$ = cost function in (2)
    /* Check termination condition   */
    if $(P(k-1)-P(k))/P(k-1) >$ *Thres* then
      $\{\hat{\mathbf{u}}_g\} = \{\mathbf{u}_g^*\}$, $k = k+1$
    else
      $\{\mathbf{u}_g\} = \{\mathbf{u}_g^*\}$
      return
return

MaxIter. The main steps to implement the proposed MinPower-MaxSINR scheme can be found in Alg. 2.

Notice that different UL/DL sets $\mathcal{I}$ will lead to different interference levels at the input of the receivers and hence to different solutions for the minimization problem. Therefore, a discrete search considering the different UL/DL combinations is required in order to achieve a solution for $\mathcal{P}_1$ as described in Section 2.1.

3.1. Feasibility and complexity

Throughout Section 3, we have assumed that $\mathcal{P}_2$ is feasible for the given SINR constraints. However, for some
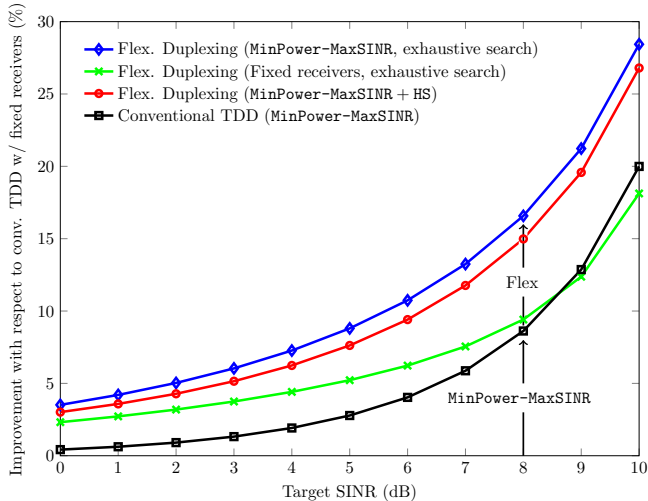
Fig. 1. Total power benefits, with respect to conventional TDD w/ fixed receivers, by `MinPower-MaxSINR`, flexible duplexing and the suboptimal `HS` scheme.

channel realizations, spatial distributions ($\{d_gj\}$), and/or SINR requirements γ_g, $\mathcal{P}_2$ could be infeasible. In such cases, we are left with two straightforward approaches, namely, discard the infeasible scenario, or more practically, reduce the SINR requirement so that the problem becomes feasible. Nevertheless, a deeper analysis on feasibility is beyond the scope of this work.

Regarding the computational complexity of the algorithm for $\mathcal{P}_2$, let us recall that the proposed scheme is an iterative combination of well-known convex optimization techniques and the closed-form `MaxSINR` receivers. Details on time complexity for the former can be found in [22]. On the other hand, each calculation of a `MaxSINR` filter implies a singular value decomposition (SVD). Therefore, it has complexity $\mathcal{O}\left(\min\{mn^2, m^2n\}\right)$, being m and n the number of rows and columns of the input matrix.

As mentioned before, the task of finding the optimal set $\mathcal{I}$ (i.e., the UL/DL with minimum transmit power), requires 2^G evaluations of `MinPower-MaxSINR`. Nevertheless, we have reduced the computational cost to G evaluations with the suboptimal `HS` approach.

4. SIMULATION RESULTS

In order to verify the performance of the proposed algorithm in terms of transmit power, we consider a 2-tier HetNet comprised of $G_m = 2$ macrocells and $G_s = 2$ small cells. The BS and AP are equipped with $N_g = N = 7$ antennas $\forall g$, and the baseline SNR is set to 20 dB. Additionally, we take into account the inherent UL/DL asymmetry by setting the priority variables $a_g = 0.8$ $\forall g$, and

we assume free space propagation ($\alpha = 2$). As a termination criterion for Alg. 2, we set an improvement threshold `Thres` $= 0.01$ with respect to the previous iteration, and we consider a maximum of `MaxIter` $= 5$ iterations. In this context, we evaluate the proposed algorithm for a range of SINR constraints. As a benchmark for comparison, we also consider the method in [21], which has been conveniently adapted to work in the scenario under study. The results of 1000 independent channel realizations have been averaged.

Figure 1 shows the total transmit power benefits over 2 time slots with respect to the conventional TDD approach with fixed receiving filters [21]. From Fig. 1, we characterize two main sources of benefit.

- The black curve with square markers represents the benefit of applying `MinPower-MaxSINR` instead of the benchmark method.

- The additional improvement due to flexible duplexing is associated to the gap between the black, square marked line and the diamond marked line in blue.

Notice that, despite the benefits of flexible duplexing being significant, the power results obtained by `MinPower-MaxSINR` in conventional TDD mode are close to those attained by the method in [21] for the best UL/DL combination. Hence, we can state that, besides flexible duplexing, the proposed minimization technique applied for every UL/DL setting plays an important role.

Finally, the red curve with circle markers in Fig. 1 represents the total power benefit that can be achieved by implementing `MinPower-MaxSINR` and flexible duplexing with `HS`. Specifically, we focus on the comparison between the exhaustive search strategy and the suboptimal `HS` approach. Notice that the results provided by `HS` (G total evaluations of $\mathcal{P}_2$) are significantly close to those obtained by exhaustive search (2^G total evaluations), with the advantage of having a much lower computational cost.

5. CONCLUSION

We have presented an algorithm that is capable of minimizing the total transmit power in a given flexible duplexing setting. Additionally, we determine the best UL/DL combination, and we propose a scheme based on the hierarchy of the different nodes in the network that allows to reduce the number of flexible UL/DL evaluations, hence reducing the computational cost. Our simulation results show that the proposed scheme outperforms other techniques in the literature. Finally, we prove that significant benefits in terms of total transmit power can be achieved by implementing flexible duplexing. The impact of channel estimation errors on the proposed techniques is considered as a further line.

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