Coherence

David Ramírez • Ignacio Santamaría •
Louis Scharf

Coherence

In Signal Processing and Machine Learning

Springer

David Ramírez
Universidad Carlos III de Madrid
Madrid, Spain

Ignacio Santamaría
Universidad de Cantabria
Santander, Spain

Louis Scharf
Colorado State University
Fort Collins, CO, USA

ISBN 978-3-031-13330-5 ISBN 978-3-031-13331-2 (eBook)
https://doi.org/10.1007/978-3-031-13331-2

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

To Ana Belén, Carmen, and Merche
David Ramírez

To Isabel, Cristina, Inés, and Ignacio
Ignacio Santamaría

To the next generation
Louis Scharf

Preface

This book is directed to graduate students, practitioners, and researchers in signal processing and machine learning. A basic understanding of linear algebra and probability is assumed. This background is complemented in the book with appendices on matrix algebra, (complex) multivariate normal theory, and related distributions.

The book begins in Chap. 1 with a breezy account of coherence as it has commonly appeared in science, engineering, signal processing, and machine learning. Chapter 2 is a comprehensive account of classical least squares theory, with a few original variations on cross-validation and model-order determination. Compression of ambient space dimension is analyzed by comparing multidimensional scaling and a randomized search algorithm inspired by the Johnson-Lindenstrauss lemma. But the central aim of the book is to analyze coherence in its many guises, beginning with the correlation coefficient and its multivariate extensions in Chap. 3. Chapters 4–8 contain a wealth of results on maximum likelihood theory in the complex multivariate normal model for estimating parameters and detecting signals in first- and second-order statistical models. Chapters 5 and 6 are addressed to matched and adaptive subspace detectors. Particular attention is paid to the geometries, invariances, and null distributions of these subspace detectors. Chapters 7 and 8 extend these results to detection of signals that are common to two or more channels, and to detection of spatial correlation and cyclostationarity. Coherence plays a central role in these chapters.

Chapter 9 addresses subspace averaging, an emerging topic of interest in signal processing and machine learning. The motivation is to identify subspace models (or centroids) for measurements so that images may be classified or noncoherent communication signals may be decoded. The dimension of the average or central subspace can also be estimated efficiently and applied to source enumeration in array processing. In Chap. 10, classical quadratic performance bounds on the accuracy of parameter estimators are complemented with an account of information geometry. The motivation is to harmonize performance bounds for parameter estimators with the corresponding geometry of the underlying manifold of log-likelihood random variables. Chapter 11 concludes the book with an account of other problems and methods in signal processing and machine learning where coherence is an organizing principle.

This book is more research monograph than textbook. However, many of its chapters would serve as complementary resource materials in a graduate-level course on signal processing, machine learning, or statistics. The appendices contain comprehensive accounts of matrix algebra and distribution theory, topics that join optimization theory to form the mathematical foundations of signal processing and machine learning. Chapters 2–4 would complement textbooks on multivariate analysis by covering least squares, linear minimum mean-squared error estimation, and hypothesis testing of covariance structure in the *complex* multivariate normal model. Chapters 5–8 contain an account of matched and adaptive subspace detectors that would complement a course on detection and estimation, multisensor array processing, and related topics. Chapters 9–11 would serve as resource materials in a course on advanced topics in signal processing and machine learning.

It would be futile to attempt an acknowledgment to all of our students, friends, and colleagues who have influenced our thinking and guided our educations. But several merit mention for their contribution of important ideas to this book: Yuri Abramovich, Pia Abbaddo, Javier Álvarez-Vizoso, Antonio Artés-Rodríguez, Mahmood Azimi-Sadjadi, Carlos Beltrán, Olivier Besson, Pascal Bondon, Ron Butler, Margaret Cheney, Yuejie Chi, Edwin Chong, Doug Cochran, Henry Cox, Víctor Elvira, Yossi Francos, Ben Friedlander, Antonio García Marqués, Scott Goldstein, Claude Gueguen, Alfred Hanssen, Stephen Howard, Jesús Ibáñez, Steven Kay, Michael Kirby, Nick Klausner, Shawn Kraut, Ramdas Kumaresan, Roberto López-Valcarce, Mike McCloud, Todd McWhorter, Bill Moran, Tom Mullis, Danilo Orlando, Pooria Pakrooh, Daniel P. Palomar, Jesús Pérez-Arriaga, Chris Peterson, Ali Pezeshki, Bernard Picinbono, José Príncipe, Giuseppe Ricci, Christ Richmond, Peter Schreier, Santiago Segarra, Songsri Sirianunpiboon, Steven Smith, John Thomas, Rick Vaccaro, Steven Van Vaerenbergh, Gonzalo Vázquez-Vilar, Javier Vía, Haonan Wang, and Yuan Wang. Javier Álvarez-Vizoso, Barry Van Veen, Ron Butler, and Stephen Howard were kind enough to review chapters and offer helpful suggestions.

David Ramírez wishes to acknowledge the generous support received from the Agencia Española de investigación (AEI), the Deutsche Forschungsgemeinschaft (DFG), the Comunidad de Madrid (CAM), and the Office of Naval Research (ONR) Global.

Ignacio Santamaría gratefully acknowledges the various agencies that have funded his research over the years, in particular, the Agencia Española de investigación (AEI) and the funding received in different projects of the Plan Nacional de I+D+I of the Spanish government.

Louis Scharf acknowledges, with gratitude, generous research support from the US National Science Foundation (NSF), Office of Naval Research (ONR), Air Force Office of Scientific Research (AFOSR), and Defense Advanced Research Projects Agency (DARPA).

Madrid, Spain — David Ramírez
Santander, Spain — Ignacio Santamaría
Fort Collins, CO, USA — Louis Scharf
December 2021

Contents

1 Introduction 1
- 1.1 The Coherer of Hertz, Branly, and Lodge 2
- 1.2 Interference, Coherence, and the Van Cittert-Zernike Story 2
- 1.3 Hanbury Brown-Twiss Effect 4
- 1.4 Tone Wobble and Coherence for Tuning 6
- 1.5 Beampatterns and Diffraction of Electromagnetic Radiation by a Slit 7
- 1.6 LIGO and the Detection of Einstein's Gravitational Waves 9
- 1.7 Coherence and the Heisenberg Uncertainty Relations 10
- 1.8 Coherence, Ambiguity, and the Moyal Identities 13
- 1.9 Coherence, Correlation, and Matched Filtering 13
- 1.10 Coherence and Matched Subspace Detectors 18
- 1.11 What Qualifies as a Coherence? 21
- 1.12 Why Complex? 21
- 1.13 What is the Role of Geometry? 24
- 1.14 Motivating Problems 24
- 1.15 A Preview of the Book 26
- 1.16 Chapter Notes 30

2 Least Squares and Related 33
- 2.1 The Linear Model 35
- 2.2 Over-Determined Least Squares and Related 37
  - 2.2.1 Linear Prediction 40
  - 2.2.2 Order Determination 41
  - 2.2.3 Cross-Validation 43
  - 2.2.4 Weighted Least Squares 44
  - 2.2.5 Constrained Least Squares 45
  - 2.2.6 Oblique Least Squares 46
  - 2.2.7 The BLUE (or MVUB or MVDR) Estimator 48
  - 2.2.8 Sequential Least Squares 50
  - 2.2.9 Total Least Squares 51
  - 2.2.10 Least Squares and Procrustes Problems for Channel Identification 54
  - 2.2.11 Least Squares Modal Analysis 55

2.3 Under-determined Least Squares and Related 56
2.3.1 Minimum-Norm Solution 56
2.3.2 Sparse Solutions 57
2.3.3 Maximum Entropy Solution..................... 61
2.3.4 Minimum Mean-Squared Error Solution 63
2.4 Multidimensional Scaling 63
2.5 The Johnson-Lindenstrauss Lemma 68
2.6 Chapter Notes 75
Appendices 76
2.A Completing the Square in Hermitian Quadratic Forms 76
2.A.1 Generalizing to Multiple Measurements and Other Cost Functions 76
2.A.2 LMMSE Estimation 77

3 Coherence, Classical Correlations, and their Invariances 79
3.1 Coherence Between a Random Variable and a Random Vector ... 80
3.2 Coherence Between Two Random Vectors 86
3.2.1 Relationship with Canonical Correlations..................... 87
3.2.2 The Circulant Case 88
3.2.3 Relationship with Principal Angles 88
3.2.4 Distribution of Estimated Signal-to-Noise Ratio in Adaptive Matched Filtering 90
3.3 Coherence Between Two Time Series 91
3.4 Multi-Channel Coherence 94
3.5 Principal Component Analysis 95
3.6 Two-Channel Correlation..................... 98
3.7 Multistage LMMSE Filter 106
3.8 Application to Beamforming and Spectrum Analysis..................... 110
3.8.1 The Generalized Sidelobe Canceller 111
3.8.2 Composite Covariance Matrix 111
3.8.3 Distributions of the Conventional and Capon Beamformers..................... 113
3.9 Canonical correlation analysis 115
3.9.1 Canonical Coordinates 116
3.9.2 Dimension Reduction Based on Canonical and Half-Canonical Coordinates..................... 117
3.10 Partial Correlation 118
3.10.1 Regressing Two Random Vectors onto One 119
3.10.2 Regressing One Random Vector onto Two 121
3.11 Chapter Notes 123

4 Coherence and Classical Tests in the Multivariate Normal Model 125
4.1 How Limiting Is the Multivariate Normal Model?..................... 125
4.2 Likelihood in the MVN Model..................... 126
4.2.1 Sufficiency 127
4.2.2 Likelihood..................... 127

4.3 Hypothesis Testing 130
4.4 Invariance in Hypothesis Testing 132
4.5 Testing for Sphericity of Random Variables 134
4.5.1 Sphericity Test: Its Invariances and Null Distribution ... 134
4.5.2 Extensions 136
4.6 Testing for sphericity of random vectors 138
4.7 Testing for Homogeneity of Covariance Matrices 139
4.8 Testing for Independence 141
4.8.1 Testing for Independence of Random Variables 141
4.8.2 Testing for Independence of Random Vectors 143
4.9 Cross-Validation of a Covariance Model 145
4.10 Chapter Notes 147

5 Matched Subspace Detectors 149
5.1 Signal and Noise Models 150
5.2 The Detection Problem and Its Invariances 154
5.3 Detectors in a First-Order Model for a Signal in a Known Subspace 155
5.3.1 Scale-Invariant Matched Subspace Detector 156
5.3.2 Matched Subspace Detector 157
5.4 Detectors in a Second-Order Model for a Signal in a Known Subspace 158
5.4.1 Scale-Invariant Matched Subspace Detector 158
5.4.2 Matched Subspace Detector 161
5.5 Detectors in a First-Order Model for a Signal in a Subspace Known Only by its Dimension 162
5.5.1 Scale-Invariant Matched Direction Detector 162
5.5.2 Matched Direction Detector 163
5.6 Detectors in a Second-Order Model for a Signal in a Subspace Known Only by its Dimension 164
5.6.1 Scale-Invariant Matched Direction Detector 165
5.6.2 Matched Direction Detector 168
5.7 Factor Analysis 169
5.8 A MIMO Version of the Reed-Yu Detector 171
5.9 Chapter Notes 176
Appendices 177
5.A Variations on Matched Subspace Detectors in a First-Order Model for a Signal in a Known Subspace 177
5.A.1 Scale-Invariant, Geometrically Averaged, Matched Subspace Detector 177
5.A.2 Refinement: Special Signal Sequences 178
5.A.3 Rapprochement 179
5.B Derivation of the Matched Subspace Detector in a Second-Order Model for a Signal in a Known Subspace 180

5.C Variations on Matched Direction Detectors in a Second-Order Model for a Signal in a Subspace Known Only by its Dimension 182

6 Adaptive Subspace Detectors 185
6.1 Introduction 185
6.2 Adaptive Detection Problems 187
6.2.1 Signal Models 187
6.2.2 Hypothesis Tests 188
6.3 Estimate and Plug (EP) Solutions for Adaptive Subspace Detection 190
6.3.1 Detectors in a First-Order Model for a Signal in a Known Subspace 190
6.3.2 Detectors in a Second-Order Model for a Signal in a Known Subspace 192
6.3.3 Detectors in a First-Order Model for a Signal in a Subspace Known Only by its Dimension 192
6.3.4 Detectors in a Second-Order Model for a Signal in a Subspace Known Only by its Dimension 193
6.4 GLR Solutions for Adaptive Subspace Detection 194
6.4.1 The Kelly and ACE Detector Statistics 195
6.4.2 Multidimensional and Multiple Measurement GLR Extensions of the Kelly and ACE Detector Statistics 197
6.5 Chapter Notes 201

7 Two-Channel Matched Subspace Detectors 203
7.1 Signal and Noise Models for Two-Channel Problems 203
7.1.1 Noise Models 205
7.1.2 Known or Unknown Subspaces 206
7.2 Detectors in a First-Order Model for a Signal in a Known Subspace 208
7.2.1 Scale-Invariant Matched Subspace Detector for Equal and Unknown Noise Variances 208
7.2.2 Matched Subspace Detector for Equal and Known Noise Variances 210
7.3 Detectors in a Second-Order Model for a Signal in a Known Subspace 210
7.3.1 Scale-Invariant Matched Subspace Detector for Equal and Unknown Noise Variances 211
7.3.2 Scale-Invariant Matched Subspace Detector for Unequal and Unknown Noise Variances 213
7.4 Detectors in a First-Order Model for a Signal in a Subspace Known Only by its Dimension 214
7.4.1 Scale-Invariant Matched Direction Detector for Equal and Unknown Noise Variances 215

7.4.2 Matched Direction Detector for Equal and Known Noise Variances ... 217
7.4.3 Scale-Invariant Matched Direction Detector in Noises of Different and Unknown Variances ... 218
7.4.4 Matched Direction Detector in Noises of Known but Different Variances ... 219
7.5 Detectors in a Second-Order Model for a Signal in a Subspace Known Only by its Dimension ... 220
7.5.1 Scale-Invariant Matched Direction Detector for Equal and Unknown Noise Variances ... 223
7.5.2 Matched Direction Detector for Equal and Known Noise Variances ... 225
7.5.3 Scale-Invariant Matched Direction Detector for Uncorrelated Noises Across Antennas (or White Noises with Different Variances) ... 226
7.5.4 Transformation-Invariant Matched Direction Detector for Noises with Arbitrary Spatial Correlation .. 230
7.6 Chapter Notes ... 233

8 Detection of Spatially Correlated Time Series ... 235
8.1 Introduction ... 236
8.2 Testing for Independence of Multiple Time Series ... 237
8.2.1 The Detection Problem and its Invariances ... 237
8.2.2 Test Statistic ... 238
8.3 Approximate GLR for Multiple WSS Time Series ... 240
8.3.1 Limiting Form of the Nonstationary GLR for WSS Time Series ... 240
8.3.2 GLR for Multiple Circulant Time Series and an Approximate GLR for Multiple WSS Time Series ... 243
8.4 Applications ... 246
8.4.1 Cognitive Radio ... 246
8.4.2 Testing for Impropriety in Time Series ... 247
8.5 Extensions ... 248
8.6 Detection of Cyclostationarity ... 249
8.6.1 Problem Formulation and Its Invariances ... 251
8.6.2 Test Statistics ... 254
8.6.3 Interpretation of the Detectors ... 256
8.7 Chapter Notes ... 257

9 Subspace Averaging ... 259
9.1 The Grassmann and Stiefel Manifolds ... 259
9.1.1 Statistics on the Grassmann and Stiefel Manifolds ... 263
9.2 Principal Angles, Coherence, and Distances Between Subspaces . 270
9.3 Subspace Averages ... 275
9.3.1 The Riemannian Mean ... 275
9.3.2 The Extrinsic or Chordal Mean ... 277

9.4 Order Estimation 278
9.5 The Average Projection Matrix 280
9.6 Application to Subspace Clustering 284
9.7 Application to Array Processing 288
9.8 Chapter Notes 296

10 Performance Bounds and Uncertainty Quantification 297
10.1 Conceptual Framework 298
10.2 Fisher Information and the Cramér-Rao Bound 299
10.2.1 Properties of Fisher Score 300
10.2.2 The Cramér-Rao Bound 301
10.2.3 Geometry 303
10.3 MVN Model 306
10.4 Accounting for Bias 309
10.5 More General Quadratic Performance Bounds 310
10.5.1 Good Scores and Bad Scores 311
10.5.2 Properties and Interpretations 311
10.6 Information Geometry 312
10.7 Chapter Notes 315

11 Variations on Coherence 317
11.1 Coherence in Compressed Sensing 318
11.2 Multiset CCA 324
11.2.1 Review of Two-Channel CCA 324
11.2.2 Multiset CCA (MCCA) 327
11.3 Coherence in Kernel Methods 331
11.3.1 Kernel Functions, Reproducing Kernel Hilbert Spaces (RKHS), and Mercer's Theorem 331
11.3.2 Kernel CCA 333
11.3.3 Coherence Criterion in KLMS 335
11.4 Mutual Information as Coherence 337
11.5 Coherence in Time-Frequency Modeling of a Nonstationary Time Series 340
11.6 Chapter Notes 343

12 Epilogue 345

A Notation 347

B Basic Results in Matrix Algebra 353
B.1 Matrices and their Diagonalization 353
B.2 Hermitian Matrices and their Eigenvalues 356
B.2.1 Characterization of Eigenvalues of Hermitian Matrices . 357
B.2.2 Hermitian Positive Definite Matrices 359
B.3 Traces 360
B.4 Inverses 360
B.4.1 Patterned Matrices and their Inverses 361

B.4.2 Matrix Inversion Lemma or Woodbury Identity 363
B.5 Determinants 364
B.5.1 Some Useful Determinantal Identities and Inequalities .. 365
B.6 Kronecker Products 369
B.7 Projection Matrices 370
B.7.1 Gramian, Pseudo-Inverse, and Projection 372
B.8 Toeplitz, Circulant, and Hankel Matrices 373
B.9 Important Matrix Optimization Problems 377
B.9.1 Trace Optimization 377
B.9.2 Determinant Optimization 378
B.9.3 Minimize Trace or Determinant of Error Covariance in Reduced-Rank Least Squares 380
B.9.4 Maximum Likelihood Estimation in a Factor Model 381
B.10 Matrix Derivatives 382
B.10.1 Differentiation with Respect to a Real Matrix 382
B.10.2 Differentiation with Respect to a Complex Matrix 384

C The SVD 387
C.1 The Singular Value Decomposition 387
C.2 Low-Rank Matrix Approximation 390
C.3 The CS Decomposition and the GSVD 391
C.3.1 CS Decomposition 391
C.3.2 The GSVD 392

D Normal Distribution Theory 395
D.1 Introduction 395
D.2 The Normal Random Variable 397
D.3 The Multivariate Normal Random Vector 398
D.3.1 Linear Transformation of a Normal Random Vector 399
D.3.2 The Bivariate Normal Random Vector 399
D.3.3 Analysis and Synthesis 401
D.4 The Multivariate Normal Random Matrix 402
D.4.1 Analysis and Synthesis 403
D.5 The Spherically Invariant Bivariate Normal Experiment 403
D.5.1 Coordinate Transformation: The Rayleigh and Uniform Distributions 404
D.5.2 Geometry and Spherical Invariance 404
D.5.3 Chi-Squared Distribution of $\mathbf{u}^T\mathbf{u}$ 405
D.5.4 Beta Distribution of $\rho^2 = \frac{\mathbf{u}^T\mathbf{P}_1\mathbf{u}}{\mathbf{u}^T\mathbf{u}}$ 406
D.5.5 F-Distribution of $\mathtt{f} = \frac{\mathbf{u}^T(\mathbf{I}_2-\mathbf{P}_1)\mathbf{u}}{\mathbf{u}^T\mathbf{P}_1\mathbf{u}}$ 407
D.5.6 Distributions for Other Derived Random Variables 408
D.5.7 Generation of Standard Normal Random Variables 409

D.6 The Spherically Invariant Multivariate Normal Experiment 410
D.6.1 Coordinate Transformation: The Generalized Rayleigh and Uniform Distributions 410
D.6.2 Geometry and Spherical Invariance 412
D.6.3 Chi-Squared Distribution of $\mathbf{u}^T\mathbf{u}$ 412
D.6.4 Beta Distribution of $\rho_p^2 = \frac{\mathbf{u}^T\mathbf{P}_p\mathbf{u}}{\mathbf{u}^T\mathbf{u}}$ 412
D.6.5 F-Distribution of $\pounds_p = \frac{p}{L-p}\frac{\mathbf{u}^T(\mathbf{I}_L-\mathbf{P}_p)\mathbf{u}}{\mathbf{u}^T\mathbf{P}_p\mathbf{u}}$ 414
D.6.6 Distributions for Other Derived Random Variables 415
D.7 The Spherically Invariant Matrix-Valued Normal Experiment 415
D.7.1 Coordinate Transformation: Bartlett's Factorization 415
D.7.2 Geometry and Spherical Invariance 417
D.7.3 Wishart Distribution of $\mathbf{U}\mathbf{U}^T$ 417
D.7.4 The Matrix Beta Distribution 417
D.7.5 The Matrix F-Distribution 419
D.7.6 Special Cases 420
D.7.7 Summary 424
D.8 Spherical, Elliptical, and Compound Distributions 425
D.8.1 Spherical Distributions 425
D.8.2 Elliptical Distributions 426
D.8.3 Compound Distributions 428

E The complex normal distribution 433
E.1 The Complex MVN Distribution.................................. 433
E.2 The Proper Complex MVN Distribution........................... 436
E.3 An Example from Signal Theory 438
E.4 Complex Distributions... 439

F Quadratic Forms, Cochran's Theorem, and Related 441
F.1 Quadratic Forms and Cochran's Theorem 441
F.2 Decomposing a Measurement into Signal and Orthogonal Subspaces 442
F.3 Distribution of Squared Coherence 443
F.4 Cochran's Theorem in the Proper Complex Case 444

G The Wishart distribution, the Bartlett factorization, and related 447
G.1 Bartlett's Factorization 447
G.2 Real Wishart Distribution and Related............................. 449
G.3 Complex Wishart Distribution and Related......................... 453
G.4 Distribution of Sample Mean and Sample Covariance............. 454

H Null Distribution of Coherence Statistics 457
H.1 Null Distribution of the Tests for Independence..... 457
H.1.1 Testing Independence of Random Variables 458
H.1.2 Testing Independence of Random Vectors 460
H.2 Testing for Block-Diagonal Matrices of Different Block Sizes ... 462
H.3 Testing for Block-Sphericity 463

References..... 467

Alphabetical Index 483

Acronyms

Applications of signal processing and machine learning are so wide ranging that acronyms, descriptive of methodology or application, continue to proliferate. The following is an exhausting, but not exhaustive, list of acronyms that are germane to this book.

ACE	Adaptive coherence estimator
ASD	Adaptive subspace detector
BIC	Bayesian information criterion
BLUE	Best linear unbiased estimator
CBF	Conventional (or Capon) beamformer
CCA	Canonical correlation analysis
cdf	Cumulative density function
CFAR	Constant false alarm rate
CG	Conjugate gradient
chf	Characteristic function
CRB	Cramér-Rao bound
CS	Cyclostationary
DFT	Discrete Fourier transform
DOA	Direction of arrival
EP	Estimate and plug
ETF	Equi-angular tight frame
EVD	Eigenvalue decomposition
FA	Factor analysis
FIM	Fisher information matrix
GLR	Generalized likelihood ratio
GLRT	Generalized likelihood ratio test
GSC	Generalized sidelobe canceler
HBT	Hanbury-Brown-Twiss
i.i.d.	Independent and identically distributed
ISI	Intersymbol interference
JL	Johnson-Lindenstrauss (lemma)
KAF	Kernel adaptive filtering
KCCA	Kernel canonical correlation analysis

KL	Kullback-Leibler (divergence)
KLMS	Kernel least mean square
LASSO	Least absolute shrinkage and selection operator
LDU	Lower-diagonal-upper (decomposition)
LHS	Left hand side
LIGO	Laser Interferometer Gravitational-Wave Observatory
LMMSE	Linear minimum mean square error
LMPIT	Locally most powerful invariant test
LMS	Least mean square
LS	Least squares
LTI	Linear time-invariant
MAXVAR	Maximum variance
MCCA	Multiset canonical correlation analysis
MDD	Matched direction detector
MDL	Minimum description length
MDS	Multidimensional scaling
mgf	Moment generating function
MIMO	Multiple-input multiple-output
ML	Maximum likelihood
MMSE	Minimum mean square error
MP	Matching pursuit
MSC	Magnitude squared coherence
MSD	Matched subspace detector
MSE	Mean square error
MSWF	Multistage Wiener filter
MVDR	Minimum variance distortionless response
MVN	Multivariate normal
MVUB	Minimum variance unbiased (estimator)
OBLS	Oblique least squares
OMP	Orthogonal matching pursuit
PAM	Pulse amplitude modulation
PCA	Principal component analysis
pdf	Probability density function
PDR	Pulse Doppler radar
PMT	Photomultiplier tube
PSD	Power spectral density
PSF	Point spread function
RHS	Right hand side
RIP	Restricted isometry property
RKHS	Reproducing kernel Hilbert space
RP	Random projection
rv	Random variable
s.t.	Subject to
SAR	Synthetic aperture radar
SAS	Synthetic aperture sonar

SIMO	Single-input multiple-output
SNR	Signal-to-noise ratio
SUMCOR	Sum-of-correlations
SVD	Singular value decomposition
SVM	Support vector machine
TLS	Total least squares
ULA	Uniform linear array
UMP	Uniformly most powerful
UMPI	Uniformly most powerful invariant
wlog	Without loss of generality
WSS	Wide-sense stationary

NB: We have adhered to the convention in the statistical sciences that cdf, chf, i.i.d., mgf, pdf, and rv are lowercase acronyms.