# SOS-BASED BLIND CHANNEL ESTIMATION IN MULTIUSER SPACE-TIME BLOCK CODED SYSTEMS

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**ABSTRACT**

*In this paper the problem of blind channel estimation in multiuser space-time block coded (STBC) systems is considered. Specifically, a new blind channel estimation technique is proposed. The method is solely based on second-order statistics (SOS) and it generalizes previously proposed techniques, which are specific to orthogonal codes, to a wide class of STBCs and multiuser settings. Additionally, the STBC identifiability conditions are analyzed, and a new transmission technique is proposed. This technique is able to resolve many of the indeterminacy problems associated to the blind channel estimation process, and it is based on the use of different codes for transmitting different data blocks (a technique that we refer to as code diversity). In the simplest case, the proposed transmission technique reduces to a set of rotations or permutations of the transmit matrices of each user (non-redundant precoding). Finally, the performance of the proposed method is demonstrated by means of some simulation examples.*

## 1. INTRODUCTION

In the last ten years, space-time block coding (STBC) has emerged as one of the most promising techniques to exploit spatial diversity in multiple-input multiple-output (MIMO) systems. This has motivated the development of different families of STBCs, including the orthogonal (OSTBCs) [1, 2], quasi-orthogonal (QSTBCs) [3], and trace-orthogonal (TOSTBCs) [4] space-time block codes. A common assumption for most of the STBCs is that perfect channel state information (CSI) is available at the receiver. Obviously, this is not true in practice, where the channel is usually estimated by means of training approaches, which implies a reduction in the bandwidth efficiency. On the other hand, the differential techniques [2, 5, 6], which do not require channel knowledge at the receiver, incur a penalty in performance of at least 3-dB as compared to the coherent maximum likelihood (ML) receiver. These shortcomings suggest the use of blind or semi-blind methods.

Unfortunately, most of the blind techniques have been proposed for the case of single-user and orthogonal STBCs (OSTBCs) [7, 8]. In the case of multiuser [9] and general STBCs [10], the ambiguities associated to the blind channel estimation problem make necessary the use of some pilot symbols. The main contributions of this paper are twofold: Firstly, we propose a blind channel estimation technique for a wider class of STBCs and multiuser settings. The method is only based on second-order statistics (SOS), and it can be considered as a deterministic approach, i.e., in the absence of noise it is able to exactly recover the channel, up to a real scalar for each user, within a finite number of observations. Secondly, the analysis of the identifiability conditions is exploited to propose a transmission technique to avoid the indeterminacy problems associated to the blind channel estimation from SOS. The proposed technique is based on the idea of code diversity, i.e., the combination of different STBCs. However, in the simplest case it reduces to a non-redundant precoding consisting in a set of rotations or permutations of the transmit antennas, which comes at virtually no cost at the transmitter. Finally, in the particular case of single-user systems, the proposed techniques are equivalent to those of [11]. Here, the performance in multiuser settings is illustrated by means of some numerical examples.

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## 2. DATA MODEL FOR SPACE-TIME BLOCK CODES

In this paper, we will use bold-faced upper case letters to denote matrices, e.g., $\mathbf{X}$, with elements $x_{i,j}$; bold-faced lower case letters for column vector, e.g., $\mathbf{x}$, and light-faced lower case letters for scalar quantities. The superscripts $(\cdot)^T$ and $(\cdot)^H$ will denote transpose and Hermitian, respectively. The real and imaginary parts will be denoted as $\Re(\cdot)$ and $\Im(\cdot)$, and superscript $\hat{(\cdot)}$ will denote estimated matrices, vectors or scalars. The trace, range (or column space) and Frobenius norm of matrix $\mathbf{A}$ will be denoted as $\text{Tr}(\mathbf{A})$, $\text{range}(\mathbf{A})$ and $\|\mathbf{A}\|$, respectively. Finally, the identity and zero matrices of the required dimensions will be denoted as $\mathbf{I}$ and $\mathbf{0}$, $\text{vec}(\mathbf{A})$ denotes the columnwise vectorized form of $\mathbf{A}$, and $E_n[\cdot]$ will denote the expectation operator with respect to $n$.

### 2.1 Single-User Space-Time Block Coding

Let us consider first the single-user scenario and assume a flat fading multiple-input multiple-output (MIMO) system with $n_T$ transmit and $n_R$ receive antennas. The $n_T \times n_R$ complex channel matrix is $\mathbf{H} = [\mathbf{h}_1 \cdots \mathbf{h}_{n_R}]$, where $\mathbf{h}_j = \left[h_{1,j}, \ldots, h_{n_T,j}\right]^T$, and $h_{i,j}$ denotes the channel response between the $i$-th transmit and the $j$-th receive antennas. The complex noise at the receive antennas is considered both spatially and temporally white with variance $\sigma^2$.

Assuming a linear space-time block code (STBC) transmitting $M$ symbols during $L$ time slots and using $n_T$ antennas at the transmitter side, the transmission rate is defined as $R = M/L$, and the number of real symbols $M'$ transmitted in each block is

$$M' = \begin{cases} M & \text{for real constellations,} \\ 2M & \text{for complex constellations.} \end{cases}$$

For a STBC, the $n$-th block of data can be expressed as

$$\mathscr{S}(\mathbf{s}[n]) = \sum_{k=1}^{M'} \mathbf{C}_k s_k[n],$$

where $\mathbf{s}[n] = [s_1[n], \ldots, s_{M'}[n]]^T$ contains the $M'$ real information symbols transmitted in the $n$-th block, and $\mathbf{C}_k \in \mathbb{C}^{L\times n_T}$, $k = 1, \ldots, M'$, are the STBC code matrices. In the case of real STBCs, the transmitted matrix $\mathscr{S}(\mathbf{s}[n])$ and the code matrices $\mathbf{C}_k$ are real.

The combined effect of the STBC and the $j$-th channel can be represented by the $L \times 1$ complex vectors

$$\mathbf{w}_k(\mathbf{h}_j) = \mathbf{C}_k \mathbf{h}_j, \qquad k = 1, \ldots, M',$$

and defining $\tilde{\mathbf{w}}_k(\mathbf{h}_j) = \left[\Re(\mathbf{w}_k(\mathbf{h}_j))^T, \Im(\mathbf{w}_k(\mathbf{h}_j))^T\right]^T$ we can write $\tilde{\mathbf{w}}_k(\mathbf{h}_j) = \tilde{\mathbf{C}}_k\tilde{\mathbf{h}}_j$, where $\tilde{\mathbf{h}}_j = \left[\Re(\mathbf{h}_j)^T, \Im(\mathbf{h}_j)^T\right]^T$, and

$$\tilde{\mathbf{C}}_k = \begin{bmatrix} \Re(\mathbf{C}_k) & -\Im(\mathbf{C}_k) \\ \Im(\mathbf{C}_k) & \Re(\mathbf{C}_k) \end{bmatrix}.$$

The signal at the $j$-th receive antenna is

$$\mathbf{y}_j[n] = \mathscr{S}(\mathbf{s}[n])\mathbf{h}_j + \mathbf{n}_j[n] = \sum_{k=1}^{M'} \mathbf{w}_k(\mathbf{h}_j)s_k[n] + \mathbf{n}_j[n],$$

where $\mathbf{n}_j[n]$ is the white complex noise with variance $\sigma^2$.

Defining now the vectors $\tilde{\mathbf{y}}_j[n] = [\Re(\mathbf{y}_j[n])^T, \Im(\mathbf{y}_j[n])^T]^T$ and $\tilde{\mathbf{n}}_j[n] = [\Re(\mathbf{n}_j[n])^T, \Im(\mathbf{n}_j[n])^T]^T$, the above equation can be rewritten as

$$\tilde{\mathbf{y}}_j[n] = \sum_{k=1}^{M'} \tilde{\mathbf{w}}_k(\mathbf{h}_j)s_k[n] + \tilde{\mathbf{n}}_j[n] = \tilde{\mathbf{W}}(\mathbf{h}_j)\mathbf{s}[n] + \tilde{\mathbf{n}}_j[n],$$

where $\tilde{\mathbf{W}}(\mathbf{h}_j) = \left[\tilde{\mathbf{w}}_1(\mathbf{h}_j) \cdots \tilde{\mathbf{w}}_{M'}(\mathbf{h}_j)\right]$. Finally, stacking all the received signals into $\tilde{\mathbf{y}}[n] = \left[\tilde{\mathbf{y}}_1^T[n], \ldots, \tilde{\mathbf{y}}_{n_R}^T[n]\right]^T$, we can write

$$\tilde{\mathbf{y}}[n] = \tilde{\mathbf{W}}(\mathbf{H})\mathbf{s}[n] + \tilde{\mathbf{n}}[n],$$

where $\tilde{\mathbf{W}}(\mathbf{H}) = \left[\tilde{\mathbf{W}}^T(\mathbf{h}_1) \cdots \tilde{\mathbf{W}}^T(\mathbf{h}_{n_R})\right]^T$, and $\tilde{\mathbf{n}}[n]$ is defined analogously to $\tilde{\mathbf{y}}[n]$.

If $\mathbf{H}$ is known at the receiver, and assuming a Gaussian noise distribution, the information symbols can be optimally recovered by means of the maximum likelihood (ML) decoder, whose computational complexity depends on the specific STBC and constellation properties. In general, an alternative solution with a reduced computational complexity is given by the direct application of the linear minimum mean square error (LMMSE) criterion.

### 2.2 Multiuser Space-Time Block Coding

Let us consider the synchronous uplink channel, where $U$ different users transmit to a base station equipped with $n_R$ receive antennas. Specifically, we assume that the $u$-th user ($u = 1, \ldots, U$) employs a STBC $\mathscr{C}_u$ with rate $R(\mathscr{C}_u)$ for transmitting $M(\mathscr{C}_u)$ information symbols over $L$ time slots using $n_T(\mathscr{C}_u)$ transmit antennas. Here, we must note that the assumption of a common block length is not restrictive, because we can consider successive STBC blocks as a composite block and define a common block length $L$ as the least common multiple of the different lengths $L(\mathscr{C}_u)$.

With the above formulation, and defining the vector $\mathbf{s}[n] = \left[\mathbf{s}_1^T[n], \ldots, \mathbf{s}_U^T[n]\right]^T$, where $\mathbf{s}_u[n]$ contains the $M'(\mathscr{C}_u)$ real information symbols of the $u$-th user, the set of $U$ transmission matrices

$$\mathscr{S}(\mathbf{s}[n]) = \left[\mathscr{S}(\mathbf{s}_1[n], \mathscr{C}_1) \cdots \mathscr{S}(\mathbf{s}_U[n], \mathscr{C}_U)\right],$$

can be considered as an extended STBC with $n_T = \sum_{u=1}^{U} n_T(\mathscr{C}_u)$ transmit antennas and transmitting $M = \sum_{u=1}^{U} M(\mathscr{C}_u)$ information symbols in $L$ uses of the channel. Therefore, the transmission rate of the overall STBC is $R = M/L = \sum_{u=1}^{U} R(\mathscr{C}_u)$, and the mathematical model for the received signals is $\mathbf{y}[n] = \mathscr{S}(\mathbf{s}[n])\mathbf{H}$, or

$$\tilde{\mathbf{y}}[n] = \sum_{u=1}^{U} \tilde{\mathbf{W}}(\mathbf{H}_u, \mathscr{C}_u)\mathbf{s}_u[n] + \tilde{\mathbf{n}}[n] = \tilde{\mathbf{W}}(\mathbf{H})\mathbf{s}[n] + \tilde{\mathbf{n}}[n],$$

where $\mathbf{H}_u$ is the MIMO channel for the $u$-th user, $\mathbf{H} = [\mathbf{H}_1 \cdots \mathbf{H}_U]$ is the overall MIMO channel, $\tilde{\mathbf{W}}(\mathbf{H}_u, \mathscr{C}_u)$ is the equivalent channel for the $u$-th user, and the overall equivalent channel is defined as

$$\tilde{\mathbf{W}}(\mathbf{H}) = \left[\tilde{\mathbf{W}}(\mathbf{H}_1, \mathscr{C}_1) \cdots \tilde{\mathbf{W}}(\mathbf{H}_U, \mathscr{C}_U)\right].$$

## 3. PROPOSED BLIND CHANNEL ESTIMATION TECHNIQUE

In this section a new blind channel estimation technique is proposed. The method is based on the relaxed blind ML receiver and it reduces to the extraction of the main eigenvectors of several modified correlation matrices. Moreover, although derived from a stochastic framework, it can be seen as a deterministic technique, i.e., in the absence of noise it is able to exactly recover, up to a real scalar for each user, the MIMO channel.

### 3.1 Main Assumptions

The main assumptions of the proposed technique are the following:

**Condition 1 (Number of available blocks)** *The MIMO channel is flat fading and constant during a period of $N \geq M'$ transmission blocks.*

**Condition 2 (Input signals)** *The correlation matrix of the information symbols $\mathbf{R}_{\mathbf{s}} = E_n\left[\mathbf{s}[n]\mathbf{s}^T[n]\right]$ is full rank.*

**Condition 3 (Code properties)** *All the STBC code matrices $\mathbf{C}_k(\mathscr{C}_u)$ satisfy $\sum_{k=1}^{M'(\mathscr{C}_u)} \mathbf{C}_k^H(\mathscr{C}_u)\mathbf{C}_k(\mathscr{C}_u) = c_u^2\mathbf{I}$, for some constant $c_u$, which constitutes the necessary and sufficient condition for*

$$\|\tilde{\mathbf{W}}(\mathbf{H}_u, \mathscr{C}_u)\| = c_u\|\mathbf{H}_u\|, \qquad \forall \mathbf{H}_u, \qquad u = 1, \ldots, U,$$

*i.e., the energy of the equivalent channel $\tilde{\mathbf{W}}(\mathbf{H}_u, \mathscr{C}_u)$ is proportional to that of the original MIMO channel $\mathbf{H}_u$.*

**Condition 4 (Rate and number of receive antennas)** *The number of receive antennas satisfy*

$$n_R > \begin{cases} R & \textit{for complex codes}, \\ R/2 & \textit{for real codes}, \end{cases}$$

*and the equivalent channel matrix $\tilde{\mathbf{W}}(\mathbf{H})$ is full column rank.*

Conditions 1 and 2 establish mild assumptions on the coherence time of the channel and the correlation properties of the inputs. The energy constraint in Condition 3 is directly related with the aim of reducing the effect of the channel fading, and therefore, it is satisfied by most of the practical STBCs. Finally, we must note that if $\tilde{\mathbf{W}}(\mathbf{H})$ is not full column rank, any information vector $\mathbf{s}[n] + \mathbf{z}[n]$, with $\mathbf{z}[n]$ belonging to the null subspace of $\tilde{\mathbf{W}}(\mathbf{H})$, will provide the same observations $\tilde{\mathbf{y}}[n]$ as $\mathbf{s}[n]$. Therefore, the full column rank property in Condition 4 is a desired property satisfied by most of the practical STBCs.

### 3.2 Proposed Criterion

Let us now introduce the blind maximum likelihood (ML) receiver, which is based on the minimization of the following cost function

$$\mathscr{L}(\hat{\mathbf{H}}, \hat{\mathbf{s}}[n]) = E_n\left[\left\|\tilde{\mathbf{y}}[n] - \tilde{\mathbf{W}}(\hat{\mathbf{H}})\hat{\mathbf{s}}[n]\right\|^2\right], \tag{1}$$

subject to the constraint that the estimated symbols $\hat{\mathbf{s}}[n]$ belong to some finite alphabet. Unfortunately, this is a fairly difficult problem, which is due to the fact that all the possible information symbol sequences have to be considered.

A direct simplification of the above problem is given by the relaxation of the finite alphabet constraint. Here, we must note that this relaxation introduces a real scalar ambiguity in the channel $\hat{\mathbf{H}}_u$ and signal $\hat{\mathbf{s}}_u$ estimates, which is a common ambiguity for all the SOS-based blind techniques. Therefore, from now on we will consider $\|\hat{\mathbf{H}}_u\| = \|\mathbf{H}_u\| = 1$.

Considering the signal estimates $\hat{\mathbf{s}}[n]$ minimizing (1), the cost function can be rewritten as

$$\mathscr{L}(\hat{\mathbf{H}}) = E_n\left[\|\tilde{\mathbf{y}}[n]\|^2\right] - E_n\left[\tilde{\mathbf{y}}^T[n]\tilde{\mathbf{U}}(\hat{\mathbf{H}})\tilde{\mathbf{U}}^T(\hat{\mathbf{H}})\tilde{\mathbf{y}}[n]\right],$$

where $\tilde{\mathbf{U}}(\hat{\mathbf{H}}) \in \mathbb{R}^{2Ln_R \times M'}$ is an orthogonal basis for the subspace spanned by the columns of $\tilde{\mathbf{W}}(\hat{\mathbf{H}})$. Now, taking into account the property $\mathrm{Tr}\left(\mathbf{A}^T\mathbf{B}\right) = \mathrm{Tr}\left(\mathbf{A}\mathbf{B}^T\right)$, with $\mathbf{A},\mathbf{B} \in \mathbb{R}^{p\times q}$, the relaxed blind ML decoder is reduced to the following maximization problem

$$\operatorname*{argmax}_{\hat{\mathbf{H}}} \mathrm{Tr}\left(\tilde{\mathbf{U}}^T(\hat{\mathbf{H}})\mathbf{R}_{\tilde{\mathbf{y}}}\tilde{\mathbf{U}}(\hat{\mathbf{H}})\right), \qquad (2)$$

where $\mathbf{R}_{\tilde{\mathbf{y}}} = E_n\left[\tilde{\mathbf{y}}[n]\tilde{\mathbf{y}}^T[n]\right] = \tilde{\mathbf{W}}(\mathbf{H})\mathbf{R}_{\mathbf{s}}\tilde{\mathbf{W}}^T(\mathbf{H}) + \frac{\sigma^2}{2}\mathbf{I}$ is the correlation matrix of the observations.

Unfortunately, the dependency of $\tilde{\mathbf{U}}(\hat{\mathbf{H}})$ with $\hat{\mathbf{H}}$ is rather complicated, which precludes the direct solution of (2). However, we must note that the maximization problem in (2) is equivalent to

$$\operatorname*{argmax}_{\hat{\mathbf{H}}} \mathrm{Tr}\left(\tilde{\mathbf{U}}^T(\hat{\mathbf{H}})\mathbf{\Phi}_{\tilde{\mathbf{y}}}\tilde{\mathbf{U}}(\hat{\mathbf{H}})\right),$$

where $\mathbf{\Phi}_{\tilde{\mathbf{y}}} = \tilde{\mathbf{U}}(\mathbf{H})\tilde{\mathbf{U}}^T(\mathbf{H})$ is the whitened and rank-reduced version (with rank $M'$) of the correlation matrix $\mathbf{R}_{\tilde{\mathbf{y}}}$. Finally, taking Condition 3 into account, it can be proven in a straightforward manner that the above problems can be rewritten as

$$\operatorname*{argmax}_{\hat{\mathbf{H}}} \mathrm{Tr}\left(\tilde{\mathbf{W}}^T(\hat{\mathbf{H}})\mathbf{\Phi}_{\tilde{\mathbf{y}}}\tilde{\mathbf{W}}(\hat{\mathbf{H}})\right), \qquad (3)$$

which constitutes our final channel estimation criterion. Therefore, taking into account that $\mathbf{\Phi}_{\tilde{\mathbf{y}}} = \tilde{\mathbf{U}}(\mathbf{H})\tilde{\mathbf{U}}^T(\mathbf{H})$ is a projection matrix, we can state that the proposed criterion amounts to finding the MIMO channel $\hat{\mathbf{H}}$ maximizing the energy of the projection of $\tilde{\mathbf{W}}(\hat{\mathbf{H}})$ onto the subspace defined by the equivalent channel $\tilde{\mathbf{W}}(\mathbf{H})$.

### 3.3 Practical Implementation

In practice, the theoretical correlation matrices $\mathbf{R}_{\tilde{\mathbf{y}}}$ and $\mathbf{\Phi}_{\tilde{\mathbf{y}}}$ are not exactly known, and they have to be replaced by their finite sample estimates $\hat{\mathbf{R}}_{\tilde{\mathbf{y}}}$ and $\hat{\mathbf{\Phi}}_{\tilde{\mathbf{y}}}$. Assuming a set of $N$ available blocks at the receiver, the finite sample estimate of $\mathbf{R}_{\tilde{\mathbf{y}}}$ is given by

$$\hat{\mathbf{R}}_{\tilde{\mathbf{y}}} = \frac{1}{N}\sum_{n=0}^{N-1}\tilde{\mathbf{y}}[n]\tilde{\mathbf{y}}^T[n],$$

and $\hat{\mathbf{\Phi}}_{\tilde{\mathbf{y}}}$ is obtained from the main $M'$ eigenvectors of $\hat{\mathbf{R}}_{\tilde{\mathbf{y}}}$.

Let us now define the real and vectorized channel $\tilde{\mathbf{h}}(\mathbf{H}_u) = \mathrm{vec}\left(\left[\Re(\mathbf{H}_u^T)|\Im(\mathbf{H}_u^T)\right]^T\right)$, and the $M'(\mathscr{C}_u)$ block diagonal matrices $\tilde{\mathbf{D}}_k(\mathscr{C}_u) \in \mathbb{R}^{2Ln_R\times 2n_T(\mathscr{C}_u)n_R}$,

$$\tilde{\mathbf{D}}_k(\mathscr{C}_u) = \begin{bmatrix} \tilde{\mathbf{C}}_k(\mathscr{C}_u) & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \tilde{\mathbf{C}}_k(\mathscr{C}_u) \end{bmatrix}, \qquad k = 1,\ldots,M'(\mathscr{C}_u),$$

where $\tilde{\mathbf{C}}_k(\mathscr{C}_u)$ are the real code matrices for the $u$-th user. Thus, it is easy to see that the $k$-th column of $\tilde{\mathbf{W}}(\mathbf{H}_u,\mathscr{C}_u)$ is given by $\tilde{\mathbf{D}}_k(\mathscr{C}_u)\tilde{\mathbf{h}}(\mathbf{H}_u)$, and (3) can be rewritten as

$$\operatorname*{argmax}_{\hat{\mathbf{H}}_u} \tilde{\mathbf{h}}^T(\hat{\mathbf{H}}_u)\hat{\mathbf{\Theta}}(\mathscr{C}_u)\tilde{\mathbf{h}}(\hat{\mathbf{H}}_u), \qquad u = 1,\ldots,U,$$

where

$$\hat{\mathbf{\Theta}}(\mathscr{C}_u) = \sum_{k=1}^{M'(\mathscr{C}_u)}\tilde{\mathbf{D}}_k^T(\mathscr{C}_u)\hat{\mathbf{\Phi}}_{\tilde{\mathbf{y}}}\tilde{\mathbf{D}}_k(\mathscr{C}_u).$$

Finally, taking into account the energy constraint $\|\tilde{\mathbf{h}}(\hat{\mathbf{H}}_u)\| = 1$, the channel estimate $\tilde{\mathbf{h}}(\hat{\mathbf{H}}_u)$ is directly given by the eigenvector associated to the largest eigenvalue of $\hat{\mathbf{\Theta}}(\mathscr{C}_u)$.

## 4. SOLUTION TO THE IDENTIFIABILITY PROBLEMS

In this section, we analyze the identifiability conditions associated to the blind channel estimation process, and propose a new transmission technique to avoid many of the indeterminacy problems. It is based on what we call code diversity, which consists in the use of different codes in consecutive data blocks, but it can be reduced to a non-redundant precoding consisting in a set of rotations or permutations of the transmit antennas.

### 4.1 Identifiability Analysis

Recently, some sufficient identifiability conditions have been obtained for the case of OSTBCs [8, 12]. However, the identifiability analysis for a wider class of codes is far from trivial. In this subsection, the ambiguities associated to the blind channel estimation process are analyzed, which will be later exploited to resolve many of the indeterminacy problems.

As previously pointed out, the blind channel estimation from SOS introduces a real scalar ambiguity in the estimate of the MIMO channels $\mathbf{H}_u$. However, in many practical cases, a more important indeterminacy problem is given by the existence of, at least, a spurious MIMO channel $\hat{\mathbf{H}}_u \neq c\mathbf{H}_u$ and signal $\hat{\mathbf{s}}_u[n] \neq c^{-1}\mathbf{s}_u[n]$, with $c$ a real scalar, satisfying

$$\tilde{\mathbf{W}}(\hat{\mathbf{H}})\hat{\mathbf{s}}[n] = \tilde{\mathbf{W}}(\mathbf{H})\mathbf{s}[n], \qquad n = 0,\ldots,N-1,$$

and it is easy to prove that, for a sufficiently large $N$, the above equality is equivalent to

$$\mathrm{range}\left(\tilde{\mathbf{W}}(\hat{\mathbf{H}})\right) = \mathrm{range}\left(\tilde{\mathbf{W}}(\mathbf{H})\right). \qquad (4)$$

Thus, taking into account the full-column rank property of $\tilde{\mathbf{W}}(\mathbf{H})$ (and $\tilde{\mathbf{W}}(\hat{\mathbf{H}})$), it is easy to see that if $\tilde{\mathbf{W}}(\mathbf{H})$ is a square matrix, then the indeterminacy condition (4) is always satisfied. Therefore, taking into account that $\tilde{\mathbf{W}}(\mathbf{H})$ is a $2Ln_R \times M'$ matrix, we obtain the following necessary identifiability condition

$$n_R > \begin{cases} R & \text{for complex codes,} \\ R/2 & \text{for real codes,} \end{cases} \qquad (5)$$

which justifies the assumption in Condition 4. Let us now introduce the following lemma:

**Lemma 1** *Assume that a full row-rank channel matrix* $\mathbf{H}$ *(*$n_R \geq n_T$*) can not be unambiguously identified, up to a set of real scalars, from SOS. Then, the set of STBCs does not allow the blind channel recovery, based solely on SOS, regardless of the number of receive antennas* $n_R$.

**Proof 1** *If the channel* $\mathbf{H}$ *can not be unambiguously identified, we can find at least another channel* $\hat{\mathbf{H}}_u \neq c\mathbf{H}_u$ *and signal* $\hat{\mathbf{s}}_u[n] \neq c^{-1}\mathbf{s}_u[n]$*, such that*

$$\mathscr{S}(\mathbf{s}[n])\mathbf{H} = \mathscr{S}(\hat{\mathbf{s}}[n])\hat{\mathbf{H}}, \qquad \forall \mathbf{s}[n].$$

*Therefore, taking into account that any other channel* $\bar{\mathbf{H}} \in \mathbb{C}^{n_T\times\bar{n}_R}$ *can be written as* $\bar{\mathbf{H}} = \mathbf{H}\mathbf{B}$*, with* $\mathbf{B} \in \mathbb{C}^{n_R\times\bar{n}_R}$*, we have*

$$\mathscr{S}(\mathbf{s}[n])\bar{\mathbf{H}} = \mathscr{S}(\hat{\mathbf{s}}[n])\hat{\mathbf{H}}\mathbf{B}, \qquad \forall \mathbf{s}[n],$$

*which implies that the channel can not be unambiguously identified.*

Lemma 1 implies that, assuming that the number of receive antennas is $n_R = n_T$ and that the $n_R$ overall multiple-input single-output (MISO) channels are linearly independent, the addition of more receive antennas does not provide any additional information (from the identifiability point of view) for the blind recovery of the channel. A direct consequence of Lemma 1 and the necessary identifiability condition given in (5) is that the full-rate codes ($R = n_T$), such as trace orthogonal STBCs (TOSTBCs) [4] do not allow the blind recovery of the MIMO channel regardless of the number of receive antennas.

Finally, we must point out that the solutions to (3) belong to the subspace defined by (4), and therefore, the ambiguities are associated to the blind channel estimation problem and not to the proposed criterion. Furthermore, from a practical point of view, the existence of spurious solutions is translated into a multiplicity $P_u > 1$ of the largest eigenvalue of $\hat{\mathbf{\Theta}}(\mathscr{C}_u)$, for at least one value of $u$. Therefore, the $P_u$ principal eigenvectors of $\hat{\mathbf{\Theta}}(\mathscr{C}_u)$ constitute a basis $\mathbf{G}_u \in \mathbb{R}^{2n_T(\mathscr{C}_u)n_R \times P_u}$ for the subspace containing all the solutions $\tilde{\mathbf{h}}(\hat{\mathbf{H}}_u)$ to the proposed blind channel estimation criterion.

### 4.2 Code Diversity

From the identifiability discussion in the previous subsection we know that the true channel $\tilde{\mathbf{h}}(\mathbf{H}_u)$ belongs to the subspace defined by the matrix $\mathbf{G}_u(\mathbf{H},\mathscr{C}) \in \mathbb{R}^{2n_T(\mathscr{C}_u)n_R \times P_u}$, where we have explicitly included the dependency on the channel $\mathbf{H}$ and the codes $\mathscr{C} \triangleq \{\mathscr{C}_1,\ldots,\mathscr{C}_U\}$, i.e.,

$$\tilde{\mathbf{h}}(\mathbf{H}_u) \in \mathrm{range}\left(\mathbf{G}_u(\mathbf{H},\mathscr{C})\right).$$

Let us now consider $K$ different sets of codes $\mathscr{C}^k \triangleq \{\mathscr{C}_1^k,\ldots,\mathscr{C}_U^k\}$, $k = 1,\ldots,K$. Then, it is obvious that

$$\tilde{\mathbf{h}}(\mathbf{H}_u) \in \left\{\mathrm{range}\left(\mathbf{G}_u(\mathbf{H},\mathscr{C}^1)\right) \cap \cdots \cap \mathrm{range}\left(\mathbf{G}_u(\mathbf{H},\mathscr{C}^K)\right)\right\},$$

i.e., the true channel belongs to the intersection of the $K$ different subspaces, of size $P_u(\mathscr{C}^k)$, defined by the matrices $\mathbf{G}_u(\mathbf{H},\mathscr{C}^k)$. However, in a general case, there is no reason to think that the rank of such intersection will be larger than 1, i.e., the spurious solutions to the blind channel estimation problem for codes $\mathscr{C}^k$ do not necessarily maximize the criterion (2) when a different code $\mathscr{C}^l$ ($l \neq k$) is used.

The proposed technique is based on the previous idea. Assuming that the MIMO channel remains constant during a large enough interval, the first $M(\mathscr{C}^1)$ information symbols are transmitted during the first $L(\mathscr{C}^1)$ time slots using the set of codes $\mathscr{C}^1$. In the following $L(\mathscr{C}^2)$ channel uses, $M(\mathscr{C}^2)$ new information symbols are transmitted by means of the codes $\mathscr{C}^2$, and the same procedure is used with the $K$ sets of STBCs. Thus, considering the $K$ consecutive blocks transmitted by each user as a composite STBC, the proposed blind channel estimation technique can be directly applied. Furthermore, it is easy to prove that the solutions $\tilde{\mathbf{h}}(\hat{\mathbf{H}}_u)$ to the channel estimation criterion belong to the intersection of the subspaces spanned by $\mathbf{G}_u(\mathbf{H},\mathscr{C}^k)$, for $k = 1,\ldots,K$.

### 4.3 Non-Redundant Precoding

Here we propose a particularly single code combination strategy, which is based on only one STBC for each user. Specifically, each STBC is modified by means of a non-redundant precoding consisting in the rotation of the transmit antennas. Thus, considering $KU$ unitary matrices $\mathbf{Q}_u^k$, for $k = 1,\ldots,K$, $u = 1,\ldots,U$, and assuming $U$ codes $\mathscr{C}_u$, we define the following transmission matrices

$$\mathscr{S}(\mathbf{s}_u[n],\mathscr{C}_u^k) = \mathscr{S}(\mathbf{s}_u[n],\mathscr{C}_u)\mathbf{Q}_u^k, \quad k = 1,\ldots,K, \quad u = 1,\ldots,U,$$

which are associated to $K$ different sets of codes $\mathscr{C}^k$. Thus, the code diversity is obtained by rotating the transmission matrices of the original STBCs and, since the effect of the rotations can be considered as part of the channel, the code properties are preserved. Finally, we must point out that the matrices $\mathbf{Q}_u^k$ can be chosen as permutation matrices, which does not increase the complexity of the transmitter and preserves the power properties associated to each transmit antenna.

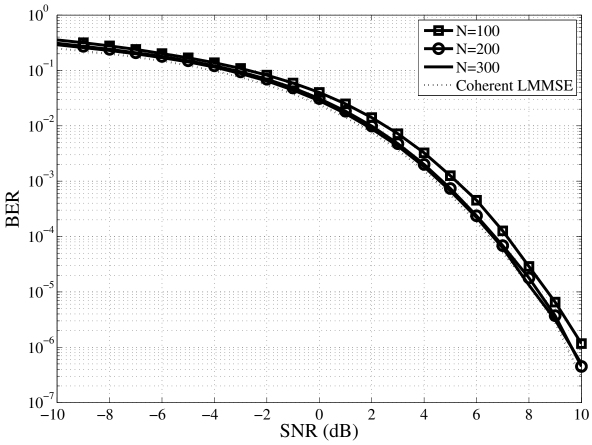


**Fig. 1**. Bit Error Rate (BER) versus SNR. Example with $U = 2$ users and $R = 3/4$ OSTBC. $n_T = L = 4$, $M = 3$, $n_R = 4$.

## 5. SIMULATION RESULTS

In this section, the performance of the proposed technique is illustrated by means of some simulation examples. All the results have been obtained by averaging 5000 independent experiments, where the MIMO channel $\mathbf{H}_u$ for each user has been generated as a Rayleigh channel with unit-variance elements. The signal to noise ratio (SNR) is the same for all the users, and the noise is temporally and spatially white and Gaussian. The i.i.d information symbols belong to a quadrature phase shift keying (QPSK) constellation and the receivers have been designed based on the LMMSE and a hard decision decoder.

In order to avoid the ambiguity problems, the non-redundant precoding technique with $K = 4$ permutations has been applied. Specifically, the permutations of the transmit antennas are based on the following matrices:

$$\mathbf{Q}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad \mathbf{Q}_2 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$
$$\mathbf{Q}_3 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \qquad \mathbf{Q}_4 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

In all the examples the users share a common STBC with $n_T = 4$ transmit antennas, which is modified by means of the non-redundant precoding technique. Specifically, the transmission matrices for the first 4 blocks of $U = 4$ different users are given by

$$\mathscr{S}_1 = \begin{bmatrix} \mathscr{S}(\mathbf{s}_1[0])\mathbf{Q}_1 \\ \mathscr{S}(\mathbf{s}_1[1])\mathbf{Q}_2 \\ \mathscr{S}(\mathbf{s}_1[2])\mathbf{Q}_3 \\ \mathscr{S}(\mathbf{s}_1[3])\mathbf{Q}_4 \end{bmatrix}, \qquad \mathscr{S}_2 = \begin{bmatrix} \mathscr{S}(\mathbf{s}_2[0])\mathbf{Q}_2 \\ \mathscr{S}(\mathbf{s}_2[1])\mathbf{Q}_3 \\ \mathscr{S}(\mathbf{s}_2[2])\mathbf{Q}_4 \\ \mathscr{S}(\mathbf{s}_2[3])\mathbf{Q}_1 \end{bmatrix},$$
$$\mathscr{S}_3 = \begin{bmatrix} \mathscr{S}(\mathbf{s}_3[0])\mathbf{Q}_3 \\ \mathscr{S}(\mathbf{s}_3[1])\mathbf{Q}_4 \\ \mathscr{S}(\mathbf{s}_3[2])\mathbf{Q}_1 \\ \mathscr{S}(\mathbf{s}_3[3])\mathbf{Q}_2 \end{bmatrix}, \qquad \mathscr{S}_4 = \begin{bmatrix} \mathscr{S}(\mathbf{s}_4[0])\mathbf{Q}_4 \\ \mathscr{S}(\mathbf{s}_4[1])\mathbf{Q}_1 \\ \mathscr{S}(\mathbf{s}_4[2])\mathbf{Q}_2 \\ \mathscr{S}(\mathbf{s}_4[3])\mathbf{Q}_3 \end{bmatrix},$$

i.e., all the users employ the same STBC and the same permutation matrices, but the permutations are shifted in time. With this scheme, each user has a virtually different STBC and the ambiguity problems are avoided.

In the first example, $U = 2$ different users transmit with the $R = 3/4$ ($n_T = L = 4$, $M = 3$) amicable-design OSTBC given in

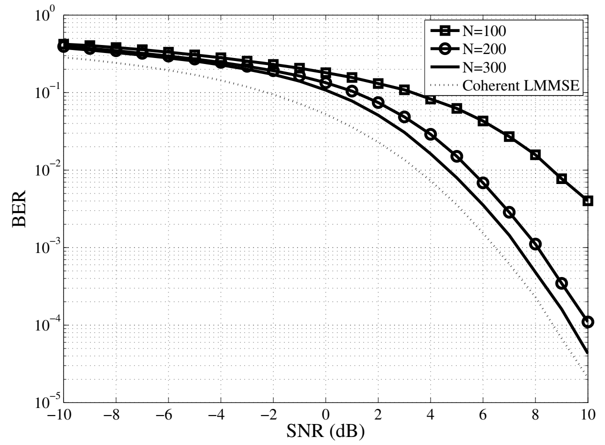


**Fig. 2**. Bit Error Rate (BER) versus SNR. Example with $U = 2$ users and $R = 1$ QSTBC. $n_T = L = M = 4$, $n_R = 4$.

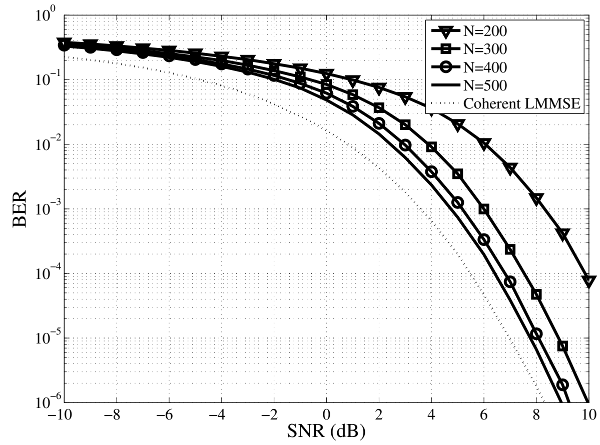


**Fig. 3**. Bit Error Rate (BER) versus SNR. Example with $U = 4$ users and $R = 1$ QSTBC. $n_T = L = M = 4$, $n_R = 8$.

[2], and the receiver is equipped with $n_R = 4$ receive antennas. We must note that this code is the same used in the simulation examples in [9], where the indeterminacy problem due to the use of a common STBC for all the users is resolved by means of some pilot symbols (semi-blind technique). Here, the non-redundant precoding solves the indeterminacy problem, and the proposed channel estimation algorithm is completely blind. Fig. 1 shows the BER as a function of the SNR for different values of available blocks at the receiver $N$. As can be seen, for $N \geq 100$, the performance loss with respect to the coherent receiver is lower than 1dB.

A similar scenario is considered in the second example, where $U = 2$ users transmit with the QSTBC design for $n_T = L = M = 4$ [3], and the number of receive antennas is $n_R = 4$. Fig. 2 shows the BER after decoding, where we can see that, in order to obtain accurate estimates, the number $N$ of blocks at the receiver must be higher than in the OSTBC case. This difference can be seen as a consequence of the higher complexity of the code (higher transmission rate and non-orthogonal signals).

In the final example, the QSTBC code with $n_T = L = M = 4$ is shared by $U = 4$ users, and the receiver is equipped with $n_R = 8$ receive antennas. Figure 3 shows the simulation results, which allow us to conclude that, for a sufficiently large number of available blocks $N$ (which depends on the problem complexity), the performance of the proposed technique is close to that of the coherent receiver.

## 6. CONCLUSIONS

In this paper, a new blind channel estimation technique for multiple-input multiple-output (MIMO) space-time block coded (STBC) multiuser systems has been proposed. The technique is solely based on second order statistics (SOS), and therefore independent of the specific signal constellation. Furthermore, it generalizes previously proposed blind channel estimation algorithms, and it does not require any assumption about the correlation matrix of the sources, which is translated into the fact that the technique can be seen as a deterministic approach, i.e., in the absence of noise it is able to exactly recover the channels, up to a real scalar for each user, within a finite number of observations. Additionally, the channel identifiability conditions have been analyzed and exploited to propose a transmission technique which avoids many of the indeterminacy problems. The main idea consists in the use of different codes for transmitting different blocks of data (code diversity), and in the simplest case, it reduces to a set of rotations or permutations of the transmit antennas (non-redundant precoding). Finally, the performance of the proposed techniques has been illustrated by means of some numerical examples.

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