ENERGY-EFFICIENT DESIGN FOR UNDERLAY COGNITIVE RADIO USING IMPROPER SIGNALING

Mohammad Soleymani*, Christian Lameiro*, Peter J. Schreier*, and Ignacio Santamaria†

*Signal and System Theory Group, Universität Paderborn, Germany, http://sst.upb.de
†Department of Communications Engineering, University of Cantabria
Emails:{mohammad.soleymani,christian.lameiro,peter.schreier}@sst.upb.de,
i.santamaria@unican.es

ABSTRACT

Improper Gaussian signaling (IGS) has been used as an effective interference management tool in interference limited systems. Improper Gaussian signals are correlated with their complex conjugates. In this paper, we investigate the optimality of IGS from an energy efficiency (EE) perspective. First, we obtain closed form optimality conditions for IGS. We then leverage these conditions to devise a bisection method that finds the optimal transmission parameters. Our results show that IGS can improve the EE of an underlay cognitive radio system.

Index Terms— Energy efficiency, improper Gaussian signaling, underlay cognitive radio.

1. INTRODUCTION

Energy consumption is always a critical parameter of modern wireless communication systems, where the power has to be used efficiently [1]. Moreover, ever-increasing demand for data rate makes it inevitable to employ resources efficiently. A way to improve resource efficiency is to employ underlay cognitive radio (UCR) systems [2,3]. In UCR systems, which are interference-limited, there are two types of users, primary users (PUs) and secondary users (SUs). PUs are the licensed users, and SUs can transmit only if they do not disturb the PUs' communications. Thus, the interference from SUs has to be limited to meet the PUs' requirements.

It has been shown that improper Gaussian signaling (IGS) can improve the performance of different interference-limited systems when interference is treated as noise [4–16]. The real and imaginary parts of an improper signal are correlated and/or do not have equal power [17]. In [4], the authors considered IGS as an interference management tool for the first time and showed that IGS can increase the degrees of freedom in a 3-user interference channel (IC). The authors in [5] showed that IGS can increase the achievable rate of a single-input single-output (SISO) two-user IC. In [6, 7], the authors showed that IGS can enlarge the
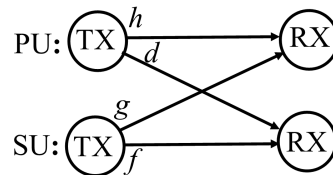

Fig. 1. UCR channel model as a SISO two-user IC.

rate region of a two-user IC and a k-user IC, respectively. In [8–10], it was shown that IGS can improve the performance of different Z-ICs. The authors in [11] showed that IGS can increase the rate of the SU in UCR, and similar results were shown in [12] for the outage probability.

IGS is usually claimed to be energy-efficient either by showing that it increases the achievable rates for given transmit power or by showing that it reduces the transmit power to achieve target data rates. However, there is a trade-off between data rate and transmit power in IGS schemes, which cannot be fully characterized when one of them is kept fixed. In order to shed light onto this trade-off, we derive in this paper a necessary and sufficient condition in closed form for IGS to be more energy efficient than PGS. We leverage this result to numerically obtain the optimal transmission parameters by the well-known bisection method. Our results show that IGS can improve the EE of the SU in UCR when interference is treated as noise.

The rest of this paper is organized as follows. In Section 2, we define the system model and formulate the EE problem. In Section 3, we present the conditions for the optimality of IGS and derive the optimal transmission parameters. In Section 4, we present some numerical results.

2. SYSTEM MODEL

We consider UCR system as depicted in Fig. 1, in which the PU, unaware of the SU, employs PGS with fixed transmit power P, while the SU can employ IGS. In particular we assume that the SU transmits a zero-mean complex Gaussian random variable with variance $q = \mathbb{E}\{|x|^2\}$, and circularity coefficient $\kappa = \frac{|\mathbb{E}\{x^2\}|}{\mathbb{E}\{|x|^2\}}$, where $\kappa_x \in [0, 1]$ [17]. We call x proper if $\mathbb{E}\{x^2\} = 0$, and improper otherwise. As a result,

the rates of the PU and SU are, respectively, [6, 10]

$$R_p = \frac{1}{2} \log_2 \left(\frac{(P|h|^2 + \sigma^2 + q|g|^2)^2 - (q\kappa|g|^2)^2}{(\sigma^2 + q|g|^2)^2 - (q\kappa|g|^2)^2} \right), \quad (1)$$

$$R_s = \frac{1}{2} \log_2 \left(\frac{(\sigma^2 + q|f|^2 + P|d|^2)^2 - (q\kappa|f|^2)^2}{(\sigma^2 + P|d|^2)^2} \right), \quad (2)$$

where h, d, g, f, and σ^2 are the PU-PU, PU-SU, SU-PU, SU-SU channel coefficients, and noise variance, respectively.

The energy efficiency function for the SU is defined as the ratio of its data rate to its consumed power, i.e., $U_s = \frac{R_s}{\zeta q + q_c}$, where ζ and q_c are the power efficiency of the SU transmitter and the constant power consumed by the SU transmitter, respectively [18–21]. In this paper, we aim at maximizing the EE of the SU under the constraint that the rate of the PU is above a threshold. That is [21]

$$\underset{q,\kappa}{\text{maximize}} \quad U_s = \frac{R_s}{\zeta q + q_c} \quad (3a)$$

$$\text{s.t.} \quad R_p \geq \overline{R} = \alpha R_p^{\max}, \quad (3b)$$

$$0 \leq q \leq Q, \quad (3c)$$

$$0 \leq \kappa \leq 1, \quad (3d)$$

where $R_p^{\max} = \log_2(1 + \frac{P|h|^2}{\sigma^2})$, $\overline{R}$, and Q are the maximum rate of the PU, the rate constraint for the PU and the power budget of the SU, respectively, and $\alpha \in [0, 1]$ is the loading factor.

3. OPTIMALITY OF IMPROPER GAUSSIAN SIGNALING

In this section, we investigate the optimality of IGS as an energy efficient design for UCR. To this end, we employ the analytical results in [11], which provide the conditions for the optimality of IGS in terms of achievable rate. It is evident that IGS may improve the EE of the SU only if IGS is able to increase the rate of the SU. For convenience, we restate these conditions in the following lemma.

Lemma 1. *IGS improves the rate of the SU if and only if:*

1. $\eta = \frac{|g|^2(\sigma^2 + P|d|^2)}{|f|^2\sigma^2} + \frac{P|h|^2}{\sigma^2(2^{2\overline{R}} - 1)} > 1$, *and*

2. $Q > q_0 = \frac{1}{|g|^2} \left(\frac{P|h|^2}{2^{\overline{R}} - 1} - \sigma^2 \right)$.

Proof. Refer to [11, Theorem 1] or [22, Lemma 1]. □

As mentioned before, the conditions in Lemma 1 are necessary, but not sufficient, for the optimality of IGS in the terms of EE. Thus, in the following we assume that these conditions are satisfied when we derive the additional conditions that are required for IGS to be also optimal in terms of EE. We also relax the power constraint when we study the behavior of the EE function of the SU. However, we finally consider all conditions for deriving the optimal parameters.

In order to simplify (3), we can rewrite (3b) in a more convenient way, i.e., by writing κ as a function of q, as [11]

$$\kappa^2(q) = \begin{cases} 0 & \text{if} \quad q < q_0 \\ \frac{2^{2\overline{R}}(q|g|^2 + \sigma^2)^2 - (q|g|^2 + P|h|^2 + \sigma^2)^2}{(1 - 2^{2\overline{R}})q^2|g|^4} & \text{if} \quad q_0 \leq q \leq q_1, \end{cases} \quad (4)$$

where $q_1 = \frac{1}{2|g|^2} \left(\frac{P|h|^2}{2^{2\overline{R} - R_p^{\max}} - 1} - \sigma^2 \right)$, and $R_p^{\max}$ is defined as in (3). Note that the constraint (3b) is active only if $q_0 \leq q \leq q_1$. If $q < q_0$, (3b) is not active and the optimal circularity coefficient is $\kappa^* = 0$. Plugging (4) into (2), the SU rate as a function of the transmit power is [11, 22]

$$R_s(q) = \begin{cases} \log_2 \left(1 + \frac{|f|^2 q}{\sigma^2 + |d|^2 P} \right) & \text{if} \quad 0 \leq q \leq q_0, \\ \frac{1}{2} \log_2 (a + bq) & \text{if} \quad q_0 \leq q \leq q_1, \end{cases} \quad (5)$$

where

$$a = 1 + \frac{|f|^4}{|g|^4 (P|d|^2 + \sigma^2)^2} \left(\frac{P^2|h|^4}{2^{2\overline{R}} - 1} - \sigma^4 \right), \quad (6a)$$

$$b = \frac{2|f|^2}{P|d|^2 + \sigma^2} + \frac{2|f|^4}{|g|^2 (P|d|^2 + \sigma^2)^2} \left(\frac{P|h|^2}{2^{2\overline{R}} - 1} - \sigma^2 \right). \quad (6b)$$

Note that $\eta > 1$ is equivalent to $b > 0$, which implies that $R_s(q)$ is increasing in q when IGS is data-rate optimal. The following lemmas characterize the SU EE function.

Lemma 2. *The EE function of the SU is differentiable in $[0, q_1]$ except at q_0, where we have $\lim_{q \to q_0^-} \frac{\partial U(q)}{\partial q} > \lim_{q \to q_0^+} \frac{\partial U(q)}{\partial q}$. Moreover, the derivative of $U(q)$ with respect to q at $q \neq q_0$ is*

$$\frac{\partial U(q)}{\partial q} = \begin{cases} \frac{\frac{b'}{1 + b'q}(q + q_c') - \ln(1 + b'q)}{(\zeta \ln 2)(q + q_c')^2} & \text{if} \quad 0 \leq q < q_0, \\ \frac{\frac{b}{a + bq}(q + q_c') - \ln(a + bq)}{(\zeta 2 \ln 2)(q + q_c')^2} & \text{if} \quad q_0 < q \leq q_1, \end{cases} \quad (7)$$

where $b' = \frac{|f|^2}{\sigma^2 + |d|^2 P}$ and $q_c' = q_c/\zeta$.

Proof. Refer to Appendix A. □

Lemma 3. *The EE function of the SU is maximized at a unique power q^*, where we have $U(q) < U(q^*)$ for $q \neq q^*$.*

Proof. Refer to Appendix B. □

The following theorem presents the optimality conditions of IGS.

Theorem 1. *IGS improves the EE of the SU if and only if $\eta > 1$, $Q > q_0$ and*

$$b(q_0 + q_c') > (2 \ln 2) R_s(q_0)(2^{2R_s(q_0)}). \quad (8)$$

Proof. Since $U(q)$ is strictly increasing before the optimal point and strictly decreasing after the optimal point according to Lemma 3, it is sufficient to consider the behavior of $U(q)$ as $q \to q_0^+$ to verify optimality of IGS. $U(q)$ is strictly increasing as $q \to q_0^+$ if and only if (8) holds. Moreover, the power budget has to be sufficiently large, i.e., $Q > q_0$. □

Note that if $b < 0$, which is equivalent to $\eta < 1$, (8) does not hold. Thus, as indicated before, the optimality conditions for EE are stricter than those for data rate in Lemma 1.

We now obtain the optimal transmission parameters of the SU, which are the solution of (3). According to Theorem 1, IGS is optimal if and only if $Q > q_0$, and (8) holds. In
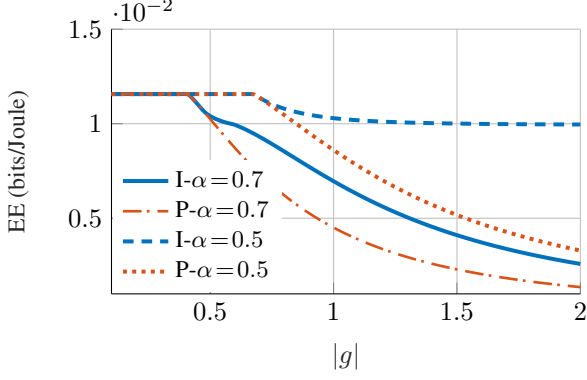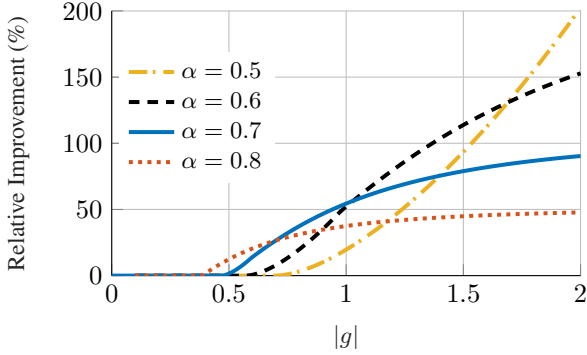
Fig. 2. EE function for different values of α.

Fig. 3. Percentage of improvement of EE by employing IGS as a function of $|g|$.

Fig. 4. Percentage of improvement of EE by employing IGS as a function of $|g|$.

Fig. 5. Percentage of improvement of EE by employing IGS as a function of α.

this case, the optimal solution is $q^* = \min(Q, q', q_1)$, where q' is the solution of $\frac{\partial U(q)}{\partial q} = 0$ for $q > q_0$, or equivalently

$$\delta(q) = b(q + q'_c) - (a + bq)\ln(a + bq) = 0. \quad (9)$$

Since there is no closed form solution for (9), we solve it numerically. As shown in the proof of Lemma 3, $\delta(q)$ is decreasing and crosses zero at only one point. As a result, if $\delta(q_1) \geq 0$, the optimal power is $q^* = \min(Q, q_1)$. Otherwise, the optimal power can be found using bisection in the interval $[q_0, q_1]$. Finally, the optimal circularity coefficient is obtained by (4) as $\kappa^* = \kappa(q^*)$.

If the conditions in Theorem 1 are not satisfied, PGS is optimal. In this case, the optimal solution is $\kappa^* = 0$ and $q^* = \min(Q, q'', q_0)$, where q'' is the solution of $\frac{\partial U(q)}{\partial q} = 0$ for $q < q_0$, or equivalently

$$\tilde{\delta}(q) = b'(q + q'_c) - (1 + b'q)\ln(1 + b'q) = 0. \quad (10)$$

Since $\tilde{\delta}(q)$ exhibits the same properties as $\delta(q)$, we can again make use of a bisection method to find q''.

4. NUMERICAL RESULTS

In this section we provide some numerical examples to illustrate our findings. We consider $\sigma^2 = 1$, $P = 20$, $Q = 10$, and the power efficiency $\zeta = 2.86$ as in [19, 20]. In order to illustrate the impact of the interference, we fix $|h|^2 = |d|^2 = |f|^2 = 1$ and vary $|g|^2$.
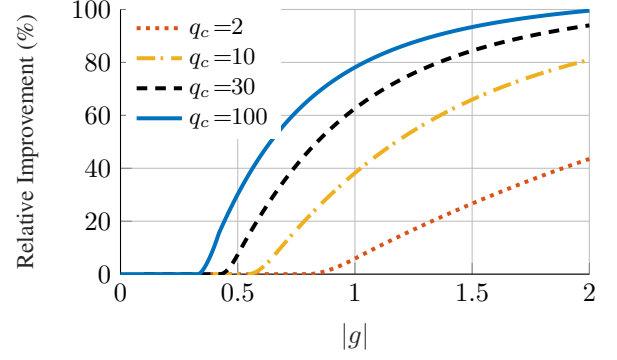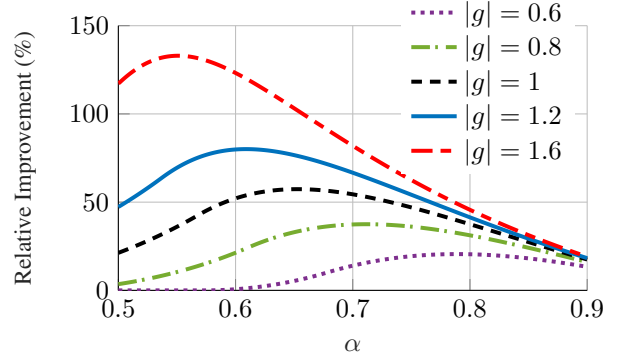
In Fig. 2, we show the EE of the SU for $q_c = 20$, and two values of α, namely, $\alpha = 0.5$ and $\alpha = 0.7$. As can be observed, IGS (indicated as "I" in the legend) can significantly improve the EE of the SU when the gain of the cross link exceeds a certain value. Interestingly, the EE is almost flat for IGS and $\alpha = 0.5$, showing a large improvement over PGS. This is due to the fact that, for $\alpha \leq 0.5$, the SU can transmit a maximally improper signal with an arbitrary large transmit power, hence this scenario is no longer being interference-limited.

In Fig. 3, we show the relative improvement of EE function by employing IGS for different α. As can be observed, the improvement is more substantial for lower values of α. However, IGS is beneficial at a lower interference level when the loading factor, α, increases.

In Fig. 4, we consider the effect of constant power consumption for $\alpha = 0.7$. As can be observed, higher constant power results in a larger improvement by employing IGS. Indeed, as q_c grows, the EE function becomes dominated by the achievable rate. Since the optimality conditions for the achievable rate are less stringent than those for EE, the behavior observed in Fig. 4 follows.

Finally, Fig. 5 shows the relative improvement of EE function by employing IGS as a function of α for $Q = 20$, $q_c = 20$, and different $|g|$. As can be observed, there is a more substantial improvement at higher $|g|$ for every value of α. Moreover, the improvement is maximized at a fixed value α, which decreases with $|g|$.

5. CONCLUSION

This work addressed the optimization of the EE function of the SU in UCR networks. We derived necessary and sufficient optimality conditions for IGS in closed form, and proposed a bisection method to find the optimal transmission parameters of PGS and IGS. Our results showed that IGS also pays off in terms of EE, although stricter conditions than those for rate optimization have to be fulfilled.

Appendix A.
PROOF OF LEMMA 2

$U(q)$ is continuous in q since $R_s(q)$ is continuous in q [11, 22]. It is also evident that $U(q)$ is differentiable for $q \neq q_0$, and its derivative is (7). In order to prove the lemma, we have to show that

$$
\frac{\frac{b'}{1+b'q_0}(q_0 + q_c') - \ln(1 + b'q_0)}{(\ln 2)(q_0 + q_c')^2} >
$$
$$
\frac{\frac{b}{a+bq_0}(q_0 + q_c') - \ln(a + bq_0)}{(2\ln 2)(q_0 + q_c')^2}. \quad (11)
$$

We know that $\log_2(a + bq_0) = 2\log_2(1 + b'q_0)$, or equivalently $a + bq_0 = (1 + b'q_0)^2$, since $R_s(q)$ is continuous. Taking this into account, we can simplify (11) as

$$
2b'(1 + b'q_0) > b, \quad (12)
$$

where $b' = \frac{|f|^2}{\sigma^2 + |d|^2 P}$, and b is defined in (6). It is easy to verify that (12) holds by replacing the corresponding parameters into it, which concludes the proof.

Appendix B.
PROOF OF LEMMA 3

In order to prove the lemma, it is sufficient to show that the EE function of the SU is strictly increasing before the optimal point and strictly decreasing after the optimal point. Let us first consider the case $q_0 < q \leq q_1$. In this case, $U(q)$ is strictly increasing if and only if its derivative is non-negative, which yields

$$
\delta(q) = b(q + q_c') - (a + bq)\ln(a + bq) > 0. \quad (13)
$$

The function $\delta(q)$ decreases with q if and only if

$$
b < b\ln(a + bq) + b, \quad (14)
$$

which always holds since $a + bq > 1$. Therefore, if $\lim_{q \to q_0^+} \frac{\partial U(q)}{\partial q} < 0$, $U(q)$ is always decreasing in $q_0 < q \leq q_1$. Otherwise, there is a unique q' such that $U(q)$ is increasing in $q_0 < q \leq q'$ and decreasing in $q > q'$. However, it might happen that $q' > q_1$, in which case $U(q)$ is strictly increasing in $q_0 < q \leq q_1$.

Let us now consider the case $q < q_0$. $U(q)$ is increasing if and only if

$$
\tilde{\delta}(q) = b'(q + q_c') - (1 + b'q)\ln(1 + b'q) > 0. \quad (15)
$$

First, it is easy to verify that the above condition holds for $q = 0$. In order to find out how $\tilde{\delta}(q)$ behaves as q increases, we analyze its derivative with respect to q. Thus, we obtain that $\tilde{\delta}(q)$ is decreasing in q if and only if

$$
b' < b'\ln(1 + b'q) + b', \quad (16)
$$

which is always true since $1 + b'q > 1$. This means that there exists q'' such that $U(q)$ is increasing in $q < q''$ and decreasing in $q > q''$. Again, it might happen that $q'' > q_0$, in which case $U(q)$ is strictly increasing in the interval $0 \leq q < q_0$.

Since by Lemma 2, $\lim_{q \to q_0^-} \frac{\partial U(q)}{\partial q} > \lim_{q \to q_0^+} \frac{\partial U(q)}{\partial q}$, this behavior is maintained when regarding the whole interval $0 \leq q \leq q_1$, which concludes the proof.

6. REFERENCES

[1] S. Buzzi, I. Chih-Lin, T. E. Klein, H. V. Poor, C. Yang, and A. Zappone, "A survey of energy-efficient techniques for 5G networks and challenges ahead," *IEEE J. Sel. Areas Commun.*, vol. 34, no. 4, pp. 697–709, 2016.

[2] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 2, pp. 201–220, 2005.

[3] A. Goldsmith, S. A. Jafar, I. Maric, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: An information theoretic perspective," *proc. IEEE*, vol. 97, no. 5, pp. 894–914, 2009.

[4] V. R. Cadambe, S. A. Jafar, and C. Wang, "Interference alignment with asymmetric complex signaling–Settling the Høst-Madsen-Nosratinia conjecture," *IEEE Trans. Inf. Theory*, vol. 56, no. 9, pp. 4552–4565, 2010.

[5] Z. K. Ho and E. Jorswieck, "Improper Gaussian signaling on the two-user SISO interference channel," *IEEE Trans. Wireless Commun.*, vol. 11, no. 9, pp. 3194–3203, 2012.

[6] Y. Zeng, C. M. Yetis, E. Gunawan, Y. L. Guan, and R. Zhang, "Transmit optimization with improper Gaussian signaling for interference channels," *IEEE Trans. Signal Process.*, vol. 61, no. 11, pp. 2899–2913, 2013.

[7] Y. Zeng, R. Zhang, E. Gunawan, and Y. L. Guan, "Optimized transmission with improper Gaussian signaling in the K-user MISO interference channel," *IEEE Trans. Wireless Commun.*, vol. 12, no. 12, pp. 6303–6313, 2013.

[8] S. Lagen, A. Agustin, and J. Vidal, "On the superiority of improper Gaussian signaling in wireless interference MIMO scenarios," *IEEE Trans. Commun.*, vol. 64, no. 8, pp. 3350–3368, 2016.

[9] E. Kurniawan and S. Sun, "Improper Gaussian signaling scheme for the Z-interference channel," *IEEE Trans. Wireless Commun.*, vol. 14, no. 7, pp. 3912–3923, 2015.

[10] C. Lameiro, I. Santamaría, and P. J. Schreier, "Rate region boundary of the SISO Z-interference channel with improper signaling," *IEEE Trans. Commun.*, vol. 65, no. 3, pp. 1022–1034, 2017.

[11] ——, "Benefits of improper signaling for underlay cognitive radio," *IEEE Wireless Commun. Lett.*, vol. 4, no. 1, pp. 22–25, 2015.

[12] O. Amin, W. Abediseid, and M.-S. Alouini, "Underlay cognitive radio systems with improper Gaussian signaling: Outage performance analysis," *IEEE Trans. Wireless Commun.*, vol. 15, no. 7, pp. 4875–4887, 2016.

[13] H. D. Nguyen, R. Zhang, and S. Sun, "Improper signaling for symbol error rate minimization in K-user interference channel," *IEEE Trans. Commun.*, vol. 63, no. 3, pp. 857–869, 2015.

[14] L. Yang and W. Zhang, "Interference alignment with asymmetric complex signaling on MIMO X channels," *IEEE Trans. Commun.*, vol. 62, no. 10, pp. 3560–3570, 2014.

[15] H.-Y. Shin, S.-H. Park, H. Park, and I. Lee, "A new approach of interference alignment through asymmetric complex signaling and multiuser diversity," *IEEE Trans. Wireless Commun.*, vol. 11, no. 3, pp. 880–884, 2012.

[16] M. Gaafar, M. G. Khafagy, O. Amin, R. F. Schaefer, and M.-S. Alouini, "Full-duplex relaying with improper gaussian signaling over Nakagami-m fading channels," *IEEE Trans. Commun.*, vol. 66, no. 1, pp. 64–78, 2018.

[17] P. J. Schreier and L. L. Scharf, *Statistical Signal Processing of Complex-Valued Data: the Theory of Improper and Noncircular Signals*. Cambridge University Press, 2010.

[18] A. Zappone and E. Jorswieck, "Energy efficiency in wireless networks via fractional programming theory," *Found Trends® in Commun. Inf. Theory*, vol. 11, no. 3-4, pp. 185–396, 2015.

[19] C. Isheden, Z. Chong, E. Jorswieck, and G. Fettweis, "Framework for link-level energy efficiency optimization with informed transmitter," *IEEE Trans. Wireless Commun.*, vol. 11, no. 8, pp. 2946–2957, 2012.

[20] Y. Chen, S. Zhang, and S. Xu, "Impact of non-ideal efficiency on bits per Joule performance of base station transmissions," in *Proc. VTC - Spring*. IEEE, 2011, pp. 1–5.

[21] J. Mao, G. Xie, J. Gao, and Y. Liu, "Energy efficiency optimization for OFDM-based cognitive radio systems: A water-filling factor aided search method," *IEEE Trans. Wireless Commun.*, vol. 12, no. 5, pp. 2366–2375, 2013.

[22] M. Soleymani, C. Lameiro, P. J. Schreier, and I. Santamaria, "Improper signaling for OFDM underlay cognitive radio systems," in *Proc. IEEE Statistical Signal Processing Workshop (SSP)*, 2018, pp. 722–726.