Performance Analysis of Transmit Antenna Selection in Broadcast MISO Channels

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***Abstract*—Time-division multiple access (TDMA) with transmit antenna selection (TAS) is a simple and feasible transmission scheme for broadcast multiple-input single-output (BC-MISO) channels. Compared to other more complex strategies, TAS exhibits two important advantages: 1) The required channel knowledge at the transmitter is relatively low and 2) it only needs a single radio-frequency chain at the transmitter. In this work we analyze the performance of TAS in BC-MISO channels. We derive simple exact closed-form expressions for the achievable sum rate of TDMA-TAS in asymmetric BC-MISO ergodic Rayleigh channels, as well as closed-form expressions for the resulting individual users' rates. Using these expressions we compare the achievable sum-rate of TAS with two more complex schemes: dirty paper coding and TDMA with transmit beamforming. The performance comparisons reveal that, in some cases, TAS could be and interesting practical alternative to other more complex schemes.**

I. Introduction

Broadcast multiple-input single-output (BC-MISO) channels, where a multi-antenna base-station (BS) transmits to a number of single-antenna receivers, are usual in practical multiuser communication systems. It is well known that dirty paper coding (DPC) [1] achieves the full capacity region of Gaussian MISO broadcast channels. But DPC is quite complex and difficult to implement in practical systems due to the high computation requirements, especially when the number of users is large. Space-division multiple access (SDMA) is a suboptimal strategy capacity that asymptotically achieves the sum-capacity of MISO-BC channels when the number of users grows [2]. Although beamforming is relatively less complex than DPC, it also requires high computation capability at transmitter and receivers, which also mades the SDMA schemes too complex for many practical systems.

Time-division multiple access (TDMA) is an orthogonal multiple access scheme much simpler than DPC and SDMA, and hence, more feasible in practical systems. In TDMA the users are scheduled according to the norms of their vector channels and the base station serves only to the selected user using conventional point-to-point transmit beamforming. Although significantly simpler than DPC and SDMA, TDMA offers lower capacity performance. Performance comparisons among DPC, SDMA and TDMA can be found in [3], [4] and [5]. As example, in a system with 4 antennas at the BS, 20 users and a an average signal-to-noise ratio of 10 dB, the sum capacity achieved by DPC is 2.4 times the sum-rate of TDMA, assuming i.i.d. Rayleigh fading. This gain reduces to 1.5 when the number of transmit antennas is 2. In practical systems, these gains could not justify the complexity of DPC compared to TDMA. As example, Qualcomm's High Data Rate (HDR) is a TDMA-based system used in the downlink of IS-856 [6], [7].

All the above transmit strategies require complete knowledge of the users' channels (CSI, channels state information) at the transmitter. Transmit antenna selection (TAS) can be an alternative to TDMA where the best combination transmit antenna-user is selected by the BS at a time. It has important advantages with respect to other schemes:

- The required amount of feedback is low. The required feedback from each user is the channel gain from the best transmit antenna. That is, a real number (channel gain) plus an integer indicating the best antenna.
- A single RF chain is used at the transmitter.

On the other hand, TAS offers lower performance than TDMA, but in some cases the performance gap does not justify the feedback load required by TDMA. In this paper we derive simple exact closed-form expressions for the achievable rate of TAS in the general case of asymmetric (i.d.d., independent and differently distributed) MISO downlink ergodic Rayleigh fading channels. In asymmetric channels the strategy to achieve the sum rate leads to different individual users' rates. We also derive simple and exact closed-form expressions for the resulting individual rates. Using the derived expressions we compare the achievable sum-rate of TAS and TDMA. The comparisons show that, in some cases, the performance penalty of TAS could not justify the amount of feedback required by TDMA.

The remainder of this paper is organized as follows. Section II describes the broadcast channel model. In section III we derive closed-form expressions for the maximum sum-rate achievable by the TAS scheme. In section IV we derive closed-form expressions for the resulting individual users' rates when the maximum sum rate is achieved. Numerical results are presented in section V comparing the sum-rate and individual rates achievable by TAS and TDMA. Finally, section VI concludes the paper.

II. Broadcast Channel Model

A narrowband BC-MISO channel with K users and M transmit antennas is considered. The transmitter is subject to an average power constraint denoted by P. We assume independent and identically distributed (i.i.d.) AWGN noise at the receivers, with single-sided power spectral density denoted by N_0. The noise power at the receivers is BN_0, where B denotes the bandwidth. The baseband-equivalent discrete channel response between the m-th transmit antenna and the k-th user is denoted by $h_{m,k}$. We assume that the channel responses $h_{m,k}$ are zero-mean circularly symmetric complex Gaussian random variables. For a fixed k, the $h_{m,k}$'s are identically distributed (i.i.d.), whereas for different k they are independent but differently distributed (i.d.d).

Let us denote $\gamma_{m,k} = |h_{m,k}|^2$. The $\gamma_{m,k}$ will be independent exponentially distributed random variables with cumulative distribution functions (c.d.f.) given by

$$F_{m,k}(x) = 1 - \exp\left(-x\, b_k\right) \ , m = 1, \ldots M, \tag{1}$$

where b_k denotes the inverse of the average power gain for the k-th user: $\bar{\gamma}_k = \mathrm{E}\left\{\gamma_k\right\} = 1/b_k$. The corresponding probability density functions (p.d.f.) will be

$$f_{m,k}(x) = b_k \exp\left(-x\, b_k\right) \ , \forall m \, . \tag{2}$$

We assume, without loss of generality, that the channel is normalized so

$$\sum_{k=1}^{K} \bar{\gamma}_k = \sum_{k=1}^{K} \frac{1}{b_k} = K. \tag{3}$$

Under the above normalization, the average SNR at the receivers will be $\rho = P/BN_0$.

We assume feedback channels that provide the transmitter with information about the highest power gains at the receivers among the transmit antennas: $\gamma_k = \max_m \left\{\gamma_{m,k}\right\}$. In addition, each user also retransmits the index of the corresponding transmit antenna. Since the $\gamma_{m,k}$ are i.i.d. for a given k, the c.d.f. of γ_k will be

$$F_k(x) = \left(F_{m,k}(x)\right)^M = \left(1 - \exp\left(-x\, b_k\right)\right)^M, \tag{4}$$

which can also be expressed as follows

$$F_k(x) = \sum_{i=0}^{M} \binom{M}{i} (-1)^i \exp(-x\, b_k\, i). \tag{5}$$

III. Maximum sum rate of TAS

Under the TAS scheme, the MISO broadcast channel is degraded, so the strategy to achieve the maximum sum rate is to transmit only to the user with the best channel from the best transmit antenna in each channel state.

Then, the sum-rate can be viewed as the capacity of an equivalent point-to-point SIMO channel with MK antennas at the receiver using selection combining (SC). Closed-form expressions for the ergodic capacity of point-to-point SIMO-SC Rayleigh channels were derived in [8], but they assume identically distributed fading at the receiver branches. Ergodic capacity expressions for dependent and differently distributed SIMO-SC channels were derived in [9] and [10], but they are restricted to dual-branch receivers. To the best of the authors' knowledge, there are not closed-form expressions for the ergodic capacity of SIMO-SC i.d.d. channels in the technical literature that could be used here.

The sum rate of TAS can also be viewed as the capacity of an equivalent point-to-point SISO (single-input single-output) channel with power gain given by $\gamma = \max_k \left\{\gamma_k\right\}$ [2].

Since the γ_k are i.d.d., the c.d.f. of γ will be

$$F(x) = \prod_{k=1}^{K} \sum_{i=0}^{M} \binom{M}{i} (-1)^i \exp(-x\, b_k\, i), \tag{6}$$

which can be expressed as follows

$$F(x) = \sum_{\mathbf{i} \in S} c_{\mathbf{i},M} \exp\left(-x\, \mathbf{b} \cdot \mathbf{i}\right) \tag{7}$$

where $\mathbf{b} = [b_1\, b_2 \ldots b_K]^T$, S is the set of all K-tuples of integers between 0 and M (K-dimesional integer vectors with entries between 0 and M) and the coeficients $c_{\mathbf{i},M}$ are given by

$$c_{\mathbf{i},M} = \prod_{k=1}^{K} \binom{M}{i_k} (-1)^{i_k}. \tag{8}$$

The p.d.f. of γ will be

$$f(x) = \sum_{\mathbf{i} \in S} c_{\mathbf{i},M} \left(-\, \mathbf{b} \cdot \mathbf{i}\right) \exp\left(-x\, \mathbf{b} \cdot \mathbf{i}\right). \tag{9}$$

Note that the term $\mathbf{i} = \mathbf{0}$ always equals one in (7), whereas it always equals zero in (9).

Assuming that the channel is ergodic, the sum rate is given by [11]

$$\begin{aligned} R_{sum} &= \mathrm{E}\left[\log_2\left(1 + \gamma\, \rho\right)\right] = \\ &= \int_0^\infty \log_2\left(1 + x\, \rho\right) f(x)\, dx. \end{aligned} \tag{10}$$

Considering (9), the sum rate of the TAS broadcast channel can be finally expressed as follows

$$R_{sum} = -\sum_{\mathbf{i} \in S,\, \mathbf{i} \neq \mathbf{0}} c_{\mathbf{i},M} \frac{\exp\left(\mathbf{b} \cdot \mathbf{i}/\rho\right)}{\ln 2}\, E_1\left(\mathbf{b} \cdot \mathbf{i}/\rho\right), \tag{11}$$

where $E_1(\cdot)$ is the exponential integral. When the entries of $\mathbf{b}$ are identical (i.i.d. channel), (11) coincides with the expression derived in [8] for the ergodic capacity of a single-user i.i.d. SIMO-SC Rayleigh channel, assuming constant transmit power.

The SISO BC channel can be viewed as a particular case of MISO BC channel with $M = 1$. In this case TAS and TDMA coincide and achieve the sum-capacity [12] [2] that will be given by (11) with $M = 1$. Now, the set S reduces to the 2^K binary words (entries taking values 0 or 1) of length K.

IV. Individual Users' Rates

Since the channel is asymmetric, the users' rates will be different. To derive the expression for the individual rates, let define the following effective channel gain for the s-th user

$$\gamma_s^* = \begin{cases} 0\,, & \gamma_s < \gamma_{-s} \\ \gamma_s\,, & \gamma_s > \gamma_{-s} \end{cases} \tag{12}$$

where $\gamma_{-s} = \max_{k \neq s} \{\gamma_k\}$. The p.d.f. of γ_s^* can be expressed as follows

$$f_s^*(x) = \text{Prob}\{\gamma_s < \gamma_{-s}\}\ \delta(x) + f_s(x)\, F_{-s}(x), \tag{13}$$

where $\delta(x)$ is the Dirac delta function, $f_s(x)$ is given by (2) and $F_{-s}(x)$ is the c.d.f. of γ_{-s}, which can be expressed as follows

$$\begin{aligned} F_{-s}(x) &= \prod_{k \neq s}^{K} F_k(x) = \\ &= \sum_{\mathbf{i} \in S} c_{\mathbf{i},M}\,(1 - i_s)\,\exp(-x\,\mathbf{b}\cdot\mathbf{i}) \end{aligned} \tag{14}$$

where i_s denotes the s-th component of $\mathbf{i}$. Considering (14) and (2),

$$f_s(x)\,F_{-s}(x) = -\sum_{\mathbf{i} \in S} c_{\mathbf{i},M}\,(b_s\, i_s)\,\exp(-x\,\mathbf{b}\cdot\mathbf{i})\,. \tag{15}$$

The rate for the s-th user will be the capacity of the effective point-to-point channel with power gain γ_s^*. Then,

$$\begin{aligned} R_s &= \int_0^\infty \log_2(1 + \rho\,\gamma)\, f_s^*(x)\, dx \\ &= \int_0^\infty \log_2(1 + \rho\, x)\, f_s(x)\, F_{-s}(x)\, dx. \end{aligned} \tag{16}$$

Substituting (15) in (16), the rate for the s-th user can be expressed as follows

$$R_s = -\sum_{\mathbf{i} \in S,\, \mathbf{i} \neq \mathbf{0}} c_{\mathbf{i},M} \frac{(b_s\, i_s)\,\exp(\mathbf{b}\cdot\mathbf{i}/\rho)}{(\mathbf{b}\cdot\mathbf{i})\,\ln 2}\, E_1(\mathbf{b}\cdot\mathbf{i}/\rho). \tag{17}$$

Note that the sum of the users' rates coincides with the sum-rate given by (11).

In the particular case of $M = 1$, the configuration reduces to a SISO BC channel where TAS and TDMA coincide. In this case, the individual rates will be given by (17) with $M = 1$, and the set S reduces to the 2^K binary words (entries taking values 0 or 1) of length K. Since TDMA achieves the sum capacity of SISO BC channels [12] [2], the expression (11), for $M = 1$, provides the coordinates of the point of the capacity region that achieves the sum capacity in asymmetric BC Rayleigh channels.

V. Numerical Results

In this section we compare the sum-rate achieved by TAS, TDMA and the sum-capacity (achieved by DPC) for a variety of MISO BC Rayleigh fading channels. The sum-rate and the users' rates achieved by TAS are computed using the closed-form expressions (11) and (17) derived in sections III and IV, respectively. The sum-rate and the users' rates achieved by TDMA and the sum capacity achieved by DPC were obtained by Monte Carlo simulations.

Figure 1 shows the relationship between the sum-rate achieved by TDMA and TAS with respect the sum-capacity in a i.i.d. Rayleigh fading channel with $K = 10$ users and $M = 2$ and 4 antennas at the BS.

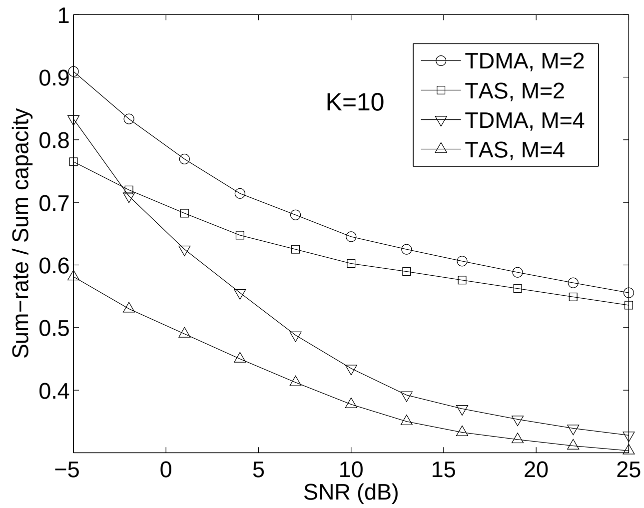

Fig. 1. Relationship between the sum-rate and sum-capacity of TDMA and TAS, as a function of the average SNR (ρ), for $K = 10$ users and different number of antennas at the BS. The channel is i.i.d. Rayleigh distributed

One can observe that the sum-rate penalty of TDMA is very low in the low-SNR regime and increases with the average SNR. Only in the high-SNR regime and for $M = 4$ transmit antennas, the TDMA rate is below half of the sum-capacity. The gap between TAS and TDMA decreases with SNR, but it is not important except in the very low-SNR regime. In any case the sum-rate penalty of both TDMA and TAS with respect the sum capacity can be acceptable for many practical systems.

Figure 2 shows again the relationship between the sum-rate achieved by TDMA and TAS with respect the sum-capacity for an i.i.d. Rayleigh channel, but now as a function of the number of users (K). Now, $M = 4$ antennas at the BS are considered. One can observe that the rate penalty of TAS and TDMA remains nearly constant when the number of users is higher than $K = 10$, regardless the SNR.

In the case more general case of asymmetric (or i.d.d.) broadcast Rayleigh channels, the fading statistics are determined by the vector $\mathbf{b}$. Hereafter, we present some results of sum rate and users' rares obtained for different specific vectors $\mathbf{b}$.

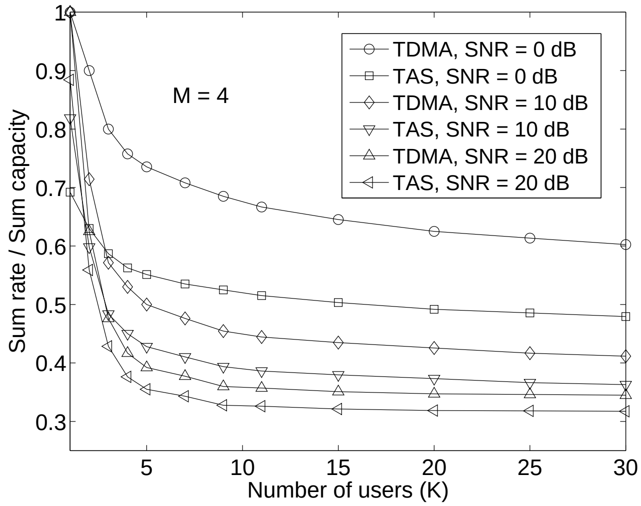

Fig. 2. Relationship between the sum-rate and sum-capacity of TDMA and TAS, as a function of the number of users (K). The channel is i.i.d. Rayleigh distributed with $M = 4$ antennas at the BS

As first example, we consider a broadcast channel where the average channel gains are linearly distributed according to:

$$\bar{\gamma}_k = a\, k \Rightarrow b_k = k^{-1}/a\,,\; k = 1, \ldots,\, K, \tag{18}$$

where a is a constant determined by the channel normalization of (3): $a = 2/(K+1)$.

Figure 3 shows the sum rate of TDMA and TAS, as a function of the number of users (K), for different values of average SNR (ρ) and $M = 4$ transmit antennas. It shows that the sum-rate gap between TDMA and TAS remains nearly constant regardless the number of users and SNR.

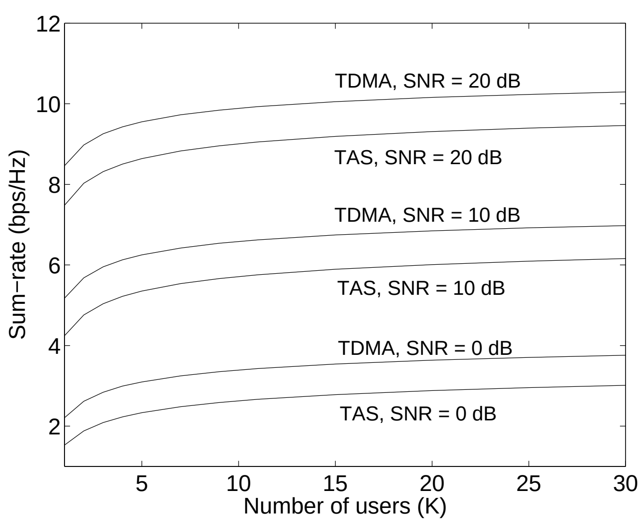

Fig. 3. Sum-rate of TDMA and TAS, as a function of the number of users (K), for different values of average SNR (ρ) and $M = 4$ transmit antennas. The users' channel gains are distributed according to (18)

Considering again the downlink channel distribution of (18), figure 4 shows the sum-rate of TDMA and TAS, as a function of the number of transmit antennas at the BS (M), for different values of average SNR (ρ) and $K = 10$ users. It shows that the sum-rate gap between TDMA and TAS is low for moderate number of transmit antennas, regardless the average SNR.

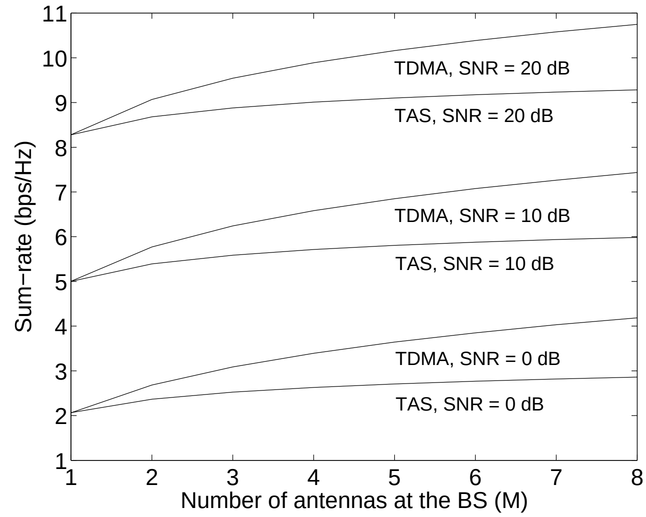

Fig. 4. Sum-rate of TDMA and TAS, as a function of the number of transmit antennas (M), for different values of average SNR (ρ) and $K = 10$ users. The users' channel gains are distributed according to (18)

For the same channel distribution, figure 5 shows the users' rates using TAS, as a function of the average SNR, for $M = 4$ transmit antennas at the BS and $K = 10$ users. The rates for the worse users remain very low at the high SNR regime, whereas, for the better channels, they increase significantly with the average SNR.

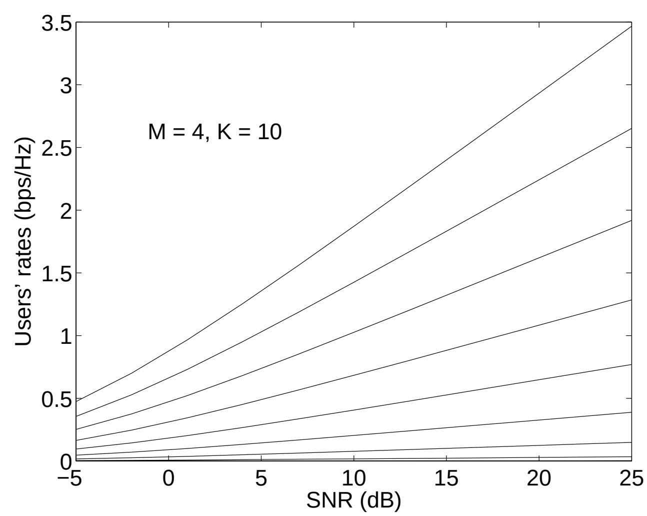

Fig. 5. Users' rates for TAS, as a function of the average SNR (ρ) for ($M = 4$) transmit antennas and $K = 10$ users. The users' channel gains are distributed according to (18)

Figure 6 refers to a downlink scenario where the users' channels are grouped in two sets of $K/2$ users each. In each set, the average power gain of the users' channels are identical.

Therefore,

$$\bar{\gamma}_k = \begin{cases} a\,,\ k = 1,\ldots,K/2 \\ a\,\Delta\,,\ k = K/2+1,\ldots,K \end{cases}$$

$$\Rightarrow b_k = \begin{cases} 1/a\,,\ k = 1,\ldots,K/2 \\ 1/(a\,\Delta)\,,\ k = K/2+1,\ldots,K \end{cases} \tag{19}$$

To fulfill the channel normalization (equation (3)), the constant $a = 2/(\Delta+1)$. All the users in a set will have the same rate. Figure 6 shows the rates for the users in each set, as a function of the parameter Δ, for $K = 10$ users, $\rho = 10$ dB and $M = 4$ antennas at the BS. Note that Δ determines the difference, on the average power gain, between the two sets of users. As it is expected, when Δ grows, the individual rates for the users of the second set (users with better channels) increase, whereas the rates for the users of the first set (users with worse channels) decrease. For the first group of users, the difference between TDMA and TAS is negligible. In fact, for high values of Δ, the rate achieved using TAS slightly overcomes the rate of TDMA.

On the contrary, for the second group of users, the rate achieved by TDMA is always higher than TAS. In summary, the gap between the TDMA rate and the TAS rate is significant only for the better channels, and negligible or negative for the worse channels. In other words, full CSI feedback is useful for the best users but it is useless for the worse users.

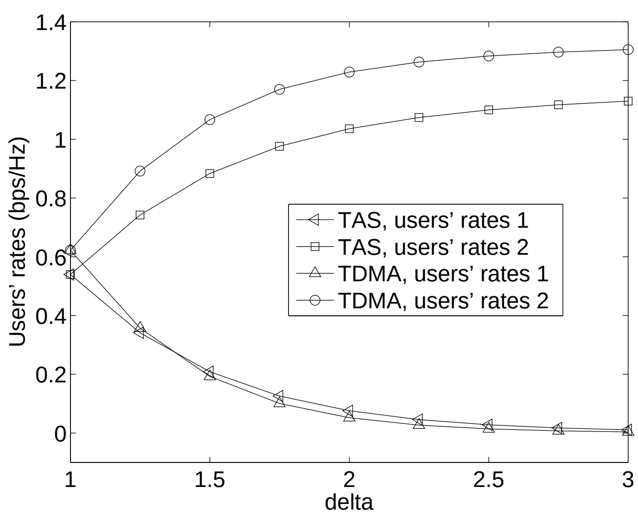

Fig. 6. Individual users' rates as a function of parameter Δ for a channel distribution given by (19), as a function of the parameter Δ. The average SNR at the receivers is $\rho = 10$ dB and the BS is equiped with $M = 4$ antennas.

Similar behavior is observed in figure 7 where the sum-rates are presented as a function of the number of users (K), for $\Delta = 1.5$. The average SNR and the number of transmit antennas are again $\rho = 10$ dB and $M = 4$, respectively. Regardless the number of users, the gap between the TDMA rate and the TAS rate is significant only for the better channels, and negligible or negative for the worse ones.

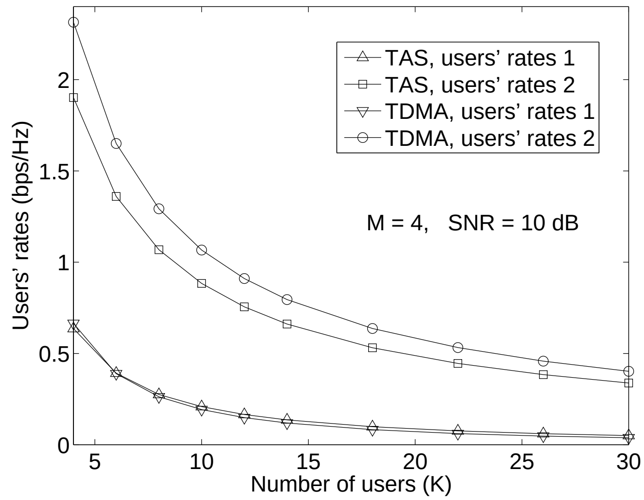

Fig. 7. Individual users' rates as a function of the number of users for a channel distribution given by (19) with $\Delta = 1.5$. The average SNR at the receivers is $\rho = 10$ dB and the BS is equipped with $M = 4$ antennas.

VI. Conclusion

In this work we have analyzed the performance, in terms of sum rate, of TDMA-TAS in BC-MISO channels. We have derived exact and simple closed-form expressions for the sum and individual users' rates in asymmetric ergodic Rayleigh channels. By using the derived expressions we have compared the performance of TAS with other more complex schemes like dirty paper coding and TDMA with transmit beamforming in different BC-MISO channels. The comparisons reveal that the performance gap between such schemes and TAS is low in the low-SNR regime and/or the number of transmit antennas is low. Taking into account its simplicity, feasibility and low requirements, we conclude that, for practical systems, TAS could be an interesting alternative to more complex schemes.

Acknowledgment

This work has been supported by the Spanish Government under the projects TEC2007- 68020-C04-02/TCM, Consolider-Ingenio2010 CSD2008-00010 (COMONSENS) and grant FPI-BES-2008-002085.

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