WIRELESS COMMUNICATIONS AND MOBILE COMPUTING
Wirel. Commun. Mob. Comput. **00**: 1–15 (2009)
Published online in Wiley InterScience
(www.interscience.wiley.com) DOI: 10.1002/wcm.0000

A New Subspace Method for Blind Estimation of Selective MIMO-STBC Channels

Javier Vía*, Ignacio Santamaría, Jesús Pérez and Luis Vielva
e-mail: {jvia,nacho,jperez,luis}@gtas.dicom.unican.es.
Telephone: +34-942201493. Fax: +34-942201488.

Department of Communications Engineering, University of Cantabria, 39005 Santander, Cantabria, Spain.

Summary

In this paper, a new technique for the blind estimation of frequency and/or time-selective multiple-input multiple-output (MIMO) channels under space-time block coding (STBC) transmissions is presented. The proposed method relies on a basis expansion model of the MIMO channel, which reduces the number of parameters to be estimated, and includes many practical STBC-based transmission scenarios, such as STBC-OFDM, space-frequency block coding (SFBC), time-reversal STBC, and time-varying STBC encoded systems. Inspired by the unconstrained blind maximum likelihood (UML) decoder, the proposed criterion is a subspace method that efficiently exploits all the information provided by the STBC structure, as well as by the reduced-rank representation of the MIMO channel. The method, which is independent of the specific signal constellation, is able to blindly recover the MIMO channel within a small number of available blocks at the receiver side. In fact, for some particular cases of interest such as orthogonal STBC-OFDM schemes, the proposed technique blindly identifies the channel using just one data block. The complexity of the proposed approach reduces to the solution of a generalized eigenvalue problem (GEV) and its computational cost is linear in the number of sub-channels. An identifiability analysis and some numerical examples illustrating the performance of the proposed algorithm are also provided.

KEY WORDS: Space-time block coding (STBC), orthogonal frequency division multiplexing (OFDM), space-frequency block coding (SFBC), time-reversal STBC, time-varying channels, blind channel estimation, second-order statistics (SOS)

1. Introduction

In the last ten years, since the well known work of Alamouti [1], and the later generalization by Tarokh et. al. [2], several families of space-time block codes (STBCs) have been proposed to exploit the spatial diversity in MIMO systems. Some examples are the orthogonal STBCs (OSTBCs) [2], quasi orthogonal STBCs (QSTBCs) [3, 4, 5], trace-orthogonal codes (TOSTBC) [6, 7], and perfect STBCs [8].

A common assumption for most of the STBCs is that perfect channel state information (CSI) is available at the receiver, which has motivated an increasing interest on blind channel estimation algorithms [9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19]. Blind techniques avoid the penalty in bandwidth efficiency or signal to noise ratio (SNR) associated, respectively, to training based approaches [20, 21, 22], or differential techniques [23, 24, 25, 26, 27, 28]. Among blind channel estimation techniques, those solely based on second-order statistics (SOS) [12, 14, 15, 16, 18, 19] are specially appealing due to their low computational complexity and their independence of the specific signal constellation.

Although the literature on blind and semiblind channel estimation under STBC transmissions is abundant, most of the research efforts have considered time-invariant flat-fading MIMO channels [10, 11, 12, 14, 15, 13, 16, 17, 18, 19]. However, the number of techniques for more general settings such as time-varying [12,29,30,31] or frequency-selective channels [9, 32, 33, 34, 35, 36, 37, 38, 39, 40] is more scarce. Specifically, the on-line algorithms in [29, 30, 31] consider OSTBC transmissions over a time-varying flat-fading channel, and they can be seen as adaptive versions of the technique proposed in [12]. On the other hand, the problem of blind estimation or equalization of frequency-selective MIMO channels has been addressed from two different points of view. Firstly, the techniques in [9, 32, 33, 34, 35] apply standard blind channel estimation or equalization techniques, which do not completely exploit the structure induced by the STBC. Moreover, they require a relatively high number of available blocks at the receiver. Secondly, in [36, 37, 38, 39, 40] the authors have proposed several subspace-based blind techniques, which require a large number of available blocks at the receiver side, and consequently long channel coherence times.

To our best knowledge, only a few techniques have considered the problem of blind decoding within a reduced number of blocks at the receiver. Specifically, for orthogonal codes the sources can be recovered by means of differential approaches [23, 24, 25, 26, 27] or the blind techniques proposed in [41, 42, 43]. However, most of these techniques introduce some constraints in the signal constellation of the sources, which might not be satisfied if the signals have been linearly precoded [7]. On the other hand, the method proposed in [43], which is independent of the specific symbol constellation, is based on a semidefinite-relaxation approach, which translates into a relatively high computational complexity.

In this paper we propose a technique for the blind estimation of frequency and/or time-selective MIMO channels, which allows us to jointly address a wide class of STBC-based systems, to name a few: orthogonal frequency division (OFDM-STBC), space-frequency block coding (SFBC), time-reversal STBC, or STBC transmissions through a time-varying channel. Firstly, the frequency and/or time-varying MIMO channel is represented by means of a basis expansion model (BEM) [44, 45], which limits the number of parameters to be estimated. Secondly, inspired by the unconstrained blind maximum-likelihood (UML) decoder, we propose a subspace-based blind channel estimation technique which reduces to the extraction of the main eigenvector of a generalized eigenvalue problem (GEV). The proposed technique is solely based on the second-order statistics (SOS) of the observations, and therefore it can be directly applied even for linearly precoded sources. Furthermore, the technique is able to recover the channels within a reduced number of available blocks at the receiver, and unlike other approaches, its computational complexity is linear in the number of MIMO sub-channels.

The structure of the paper is as follows: The channel and STBC data models are introduced in Section 2. The proposed technique for the estimation of the channel parameters is presented in Section 3. In Section 4 we prove that, under mild assumptions, the theoretical solutions of the proposed method are those of the UML decoder. Section 5 summarizes the main properties of the proposed technique. Finally, the performance of the proposed method is evaluated by means of some numerical examples in Section 6, and the concluding remarks are pointed out in Section 7.

2. Channel and Data Model

2.1. Notation

2.1.1. Vectors/Matrices

Throughout this paper we will use bold-faced upper case letters to denote matrices, e.g., $\mathbf{X}$, with elements $x_{i,j}$; bold-faced lower case letters for column vector, e.g., $\mathbf{x}$, and light-faced lower case letters for scalar quantities. Superscript $\hat{(\cdot)}$ will denote estimated matrices, vectors or scalars, the identity matrix of dimension p will be denoted as $\mathbf{I}_p$, and $\mathbf{0}$ will denote the zero matrix of the required dimensions.

2.1.2. Operators

The superscripts $(\cdot)^T$, $(\cdot)^H$ and $(\cdot)^*$ denote transpose, Hermitian and complex conjugate, respectively. The real and imaginary parts of a matrix $\mathbf{A}$ are denoted as $\Re(\mathbf{A})$ and $\Im(\mathbf{A})$. The trace, range (or column space) and Frobenius norm will be denoted as $\mathrm{Tr}(\mathbf{A})$, $\mathrm{range}(\mathbf{A})$ and $\|\mathbf{A}\|$, respectively. Finally, the column-wise vectorized version of matrix $\mathbf{A}$ will be denoted as $\mathrm{vec}(\mathbf{A})$, and $\otimes$ will denote the Kronecker product.

DOI: 10.1002/wcm

2.2. MIMO Channel Model

Let us consider a set of N_c flat fading MIMO channels. The i-th MIMO channel is represented by the $n_T \times n_R$ complex channel matrix $\mathbf{H}_i$, where the element in the k-th row and l-th column of $\mathbf{H}_i$ denotes the response of the i-th channel between the k-th transmit and the l-th receive antennas.

The correlation existing among the N_c MIMO channels is represented by means of the following BEM [44]

$$\mathbf{H}_i = \sum_{k=1}^{L_c} b_{i,k} \mathbf{\Theta}_k, \qquad i = 1, \ldots, N_c, \tag{1}$$

where $\mathbf{\Theta}_k \in \mathbb{C}^{n_T \times n_R}$ are the parameter matrices,

$$\mathbf{B} = \begin{bmatrix} b_{1,1} & \cdots & b_{1,L_c} \\ \vdots & \ddots & \vdots \\ b_{N_c,1} & \cdots & b_{N_c,L_c} \end{bmatrix}, \tag{2}$$

is some orthogonal basis,* and $L_c \leq N_c$ is the BEM order, which allows us to range from the case of perfectly correlated (i.e., identical) channels ($L_c = 1$), to the case of independent channels ($L_c = N_c$). Eq. (1) can be rewritten in matrix form as

$$\underbrace{\begin{bmatrix} \mathbf{H}_1 \\ \vdots \\ \mathbf{H}_{N_c} \end{bmatrix}}_{\mathbf{H}} = \underbrace{(\mathbf{B} \otimes \mathbf{I}_{n_T})}_{\mathcal{B}} \underbrace{\begin{bmatrix} \mathbf{\Theta}_1 \\ \vdots \\ \mathbf{\Theta}_{L_c} \end{bmatrix}}_{\mathbf{\Theta}}, \tag{3}$$

where $\mathbf{H} \in \mathbb{C}^{N_c n_T \times n_R}$, $\mathbf{\Theta} \in \mathbb{C}^{L_c n_T \times n_R}$, and $\mathcal{B} \in \mathbb{C}^{N_c n_T \times L_c n_T}$. Finally, the complex noise is considered independent for different channels, and it is assumed to be both spatially and temporally white with variance σ^2, $i = 1, \ldots, N_c$.

2.3. STBC Data Model

Let us consider a linear space-time block code (STBC) transmitting M symbols during L uses of the i-th MIMO channel $\mathbf{H}_i \in \mathbb{C}^{n_T \times n_R}$. The transmission rate is defined as $R = M/L$, and $M' = 2M$ is the number of real symbols transmitted in each block.†

*The orthogonality condition is not restrictive and all the results in the paper can be easily generalized for any full-column rank basis $\mathbf{B}$.

†In the particular case of real STBCs we have $M' = M$ and real transmission and code matrices.

For a STBC, the n-th block of data can be expressed as

$$\mathbf{S}_i(\mathbf{s}_i[n]) = \sum_{k=1}^{M'} \mathbf{C}_{i,k} s_{i,k}[n], \qquad i = 1, \ldots, N_c, \tag{4}$$

where $\mathbf{s}_i[n] = [s_{i,1}[n], \ldots, s_{i,M'}[n]]^T$ contains the M' real information symbols transmitted through the i-th channel in the n-th STBC block, and $\mathbf{C}_{i,k} \in \mathbb{C}^{L \times n_T}$, $k = 1, \ldots, M'$, are the code matrices.‡

The complex signal at the receive antennas can be written, for $i = 1, \ldots, N_c$, as

$$\mathbf{Y}_i[n] = \mathbf{S}_i(\mathbf{s}_i[n])\mathbf{H}_i + \mathbf{N}_i[n] = \sum_{k=1}^{M'} \mathbf{W}_{i,k}(\mathbf{H}_i) s_{i,k}[n] + \mathbf{N}_i[n], \tag{5}$$

where $\mathbf{N}_i[n] \in \mathbb{C}^{L \times n_R}$ represents the white complex noise with zero mean and variance σ^2, and

$$\mathbf{W}_{i,k}(\mathbf{H}_i) = \mathbf{C}_{i,k}\mathbf{H}_i, \qquad k = 1, \ldots, M'. \tag{6}$$

Defining now $\mathbf{y}_i[n] = \text{vec}\,(\mathbf{Y}_i[n])$, $\mathbf{h}_i = \text{vec}\,(\mathbf{H}_i)$ and $\mathbf{n}_i[n] = \text{vec}\,(\mathbf{N}_i[n])$, eq. (5) can be rewritten as

$$\mathbf{y}_i[n] = \mathbf{W}_i(\mathbf{h}_i)\mathbf{s}_i[n] + \mathbf{n}_i[n], \qquad i = 1, \ldots, N_c, \tag{7}$$

where $\mathbf{W}_i(\mathbf{h}_i)$ can be seen as the i-th complex equivalent channel, whose k-th column is given by

$$\text{vec}\,(\mathbf{W}_{i,k}(\mathbf{h}_i)) = \mathbf{D}_{i,k}\mathbf{h}_i, \tag{8}$$

with $\mathbf{D}_{i,k} = \mathbf{I}_{n_R} \otimes \mathbf{C}_{i,k}$, $k = 1, \ldots, M'$.

Here, we must note that the data model in (7) can be seen as a particular case of a complex system with a non-circular (improper) source [46, 47], i.e., the real information symbols $\mathbf{s}_i[n]$ are observed through a complex equivalent channel given by $\mathbf{W}_i(\mathbf{h}_i)$. This fact has been previously exploited in [48] to equalize frequency-selective channels under STBC transmissions, and it has also been implicitly exploited by the blind OSTBC channel estimation techniques proposed in [12, 13, 15, 17, 18, 19]. In this paper we exploit the impropriety of the sources by using the following real data model

$$\tilde{\mathbf{y}}_i[n] = \tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)\mathbf{s}_i[n] + \tilde{\mathbf{n}}_i[n], \qquad i = 1, \ldots, N_c, \tag{9}$$

‡Usually, the STBC is common for all the channels, so we could drop the subindex i.

where $\tilde{\mathbf{y}}_i[n] = \left[\Re(\mathbf{y}_i^T[n]), \Im(\mathbf{y}_i^T[n])\right]^T$, $\tilde{\mathbf{n}}_i[n] = \left[\Re(\mathbf{n}_i^T[n]), \Im(\mathbf{n}_i^T[n])\right]^T$, $\tilde{\mathbf{h}}_i = \left[\Re(\mathbf{h}_i^T), \Im(\mathbf{h}_i^T)\right]^T$, and the i-th real equivalent channel is

$$\underbrace{\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)}_{2Ln_R \times M'} = \left[\Re\left(\mathbf{W}_i^T(\mathbf{h}_i)\right) \quad \Im\left(\mathbf{W}_i^T(\mathbf{h}_i)\right)\right]^T = \\ = \left[\tilde{\mathbf{D}}_{i,1}\tilde{\mathbf{h}}_i \quad \tilde{\mathbf{D}}_{i,2}\tilde{\mathbf{h}}_i \quad \cdots \quad \tilde{\mathbf{D}}_{i,M'}\tilde{\mathbf{h}}_i\right], \tag{10}$$

where

$$\tilde{\mathbf{D}}_{i,k} = \underbrace{\begin{bmatrix} \Re(\mathbf{D}_{i,k}) & -\Im(\mathbf{D}_{i,k}) \\ \Im(\mathbf{D}_{i,k}) & \Re(\mathbf{D}_{i,k}) \end{bmatrix}}_{2Ln_R \times 2n_T n_R}, \qquad k = 1, \ldots, M', \tag{11}$$

are the extended code matrices with real elements.

2.4. Linear Precoding of the Information Symbols

In general, STBCs are able to exploit the spatial diversity of the MIMO channel. However, in order to take advantage of the frequency and/or time diversity of the system, the information symbols have to be distributed among the different MIMO channel realizations. Fortunately, this can be easily done by means of linear precoding techniques [7, 45]. Thus, we can assume without loss of generality that the transmitted symbols $\mathbf{s}_i[n]$ are obtained as

$$\underbrace{\begin{bmatrix} \mathbf{s}_1[n] \\ \vdots \\ \mathbf{s}_{N_c}[n] \end{bmatrix}}_{\mathbf{s}[n]} = \left(\Re(\mathbf{G}) \otimes \mathbf{I}_{M'} + \Im(\mathbf{G}) \otimes \mathbf{J}_{M'}\right) \underbrace{\begin{bmatrix} \mathbf{d}_1[n] \\ \vdots \\ \mathbf{d}_{N_c}[n] \end{bmatrix}}_{\mathbf{d}[n]}, \tag{12}$$

where $\mathbf{d}_i[n] \in \mathbb{R}^{M' \times 1}$ is a vector containing the real and imaginary parts of the information symbols, which belong to some finite alphabet $\mathcal{S}$, $\mathbf{G} \in \mathbb{C}^{N_c \times N_c}$ is a unitary precoding matrix [7], and

$$\mathbf{J}_{M'} = \begin{bmatrix} \mathbf{0} & -\mathbf{I}_M \\ \mathbf{I}_M & \mathbf{0} \end{bmatrix}. \tag{13}$$

2.5. Some Particular Cases

The data and channel models introduced in this section are very general. Some particular cases of interest are summarized in Table I, where the matrix $\mathbf{F}_{N_c \times P}(\delta)$ is defined as

$$\mathbf{F}_{N_c \times P}(\delta) = \left[\mathbf{f}_{N_c}(\delta) \quad \mathbf{f}_{N_c}(\tfrac{1}{N_c} + \delta) \quad \cdots \quad \mathbf{f}_{N_c}(\tfrac{P-1}{N_c} + \delta)\right], \tag{14}$$

$\mathbf{f}_{N_c}(f)$ is the Fourier vector of length N_c at normalized frequency f, and δ is a frequency offset in the FFT grid. Let us illustrate these equivalences in more detail:

- **STBC-OFDM**: In this case, the sub-channels $\mathbf{H}_i$, $i = 1, \ldots, N_c$, represent the frequency response of the MIMO channel in the i-th subcarrier. The orthogonal basis $\mathbf{B}$ is given by the first L_c columns of the FFT matrix, and the parameters $\mathbf{\Theta}_k$ $(k = 1, \ldots, L_c)$ represent the finite impulse response of the MIMO channel. The STBC-OFDM scheme assumes that the frequency-selective MIMO channel remains constant during at least L (the channel uses per STBC block) OFDM symbols, and it uses STBC transmission in each subcarrier.
- **SFBC**: Space-Frequency Block Coding (SFBC) can be seen as an alternative to STBC-OFDM systems based on only one OFDM symbol [49, 50]. In this case, the temporal coherence requirement in STBC-OFDM systems is replaced by a constraint in the spectral coherence. In particular, the OFDM symbol is divided into groups of L adjacent subcarriers, which see the same flat fading MIMO channel $\mathbf{H}_i$ and are used to transmit one STBC data block.
- **Time-Reversal STBC**: Time-reversal orthogonal STBC was proposed in [51] (see also [52, 53]) as a transmission technique to exploit the multipath diversity in MIMO systems with inter-symbol interference, and it was later generalized to non-orthogonal codes [54, 55]. Interestingly, these schemes can be viewed as a particular case of a STBC-OFDM system with basis and precoding matrices

$$\mathbf{B} = \mathbf{F}_{N_c \times L_c}\left(\frac{1}{2N_c}\right) \qquad \mathbf{G} = \mathbf{F}_{N_c \times N_c}\left(\frac{1}{2N_c}\right), \tag{15}$$

which satisfy the *time-reversal* property [52, 53]

$$\mathbf{F}_{N_c \times N_c}^T\left(\frac{1}{2N_c}\right)\mathbf{F}_{N_c \times N_c}\left(\frac{1}{2N_c}\right) = \overleftarrow{\mathbf{I}}_{N_c}, \tag{16}$$

where $\overleftarrow{\mathbf{I}}_{N_c}$ is obtained from $\mathbf{I}_{N_c}$ with its columns (or rows) in reverse order.
- **Time-Varying Channels**: Let us consider a STBC transmission through a time-varying flat fading MIMO channel, which is considered static during the L channel uses of a STBC

DOI: 10.1002/wcm

block. Obviously, this assumption implies that the MIMO channel changes slowly, and therefore, it can be well approximated by a BEM [44]. For instance, the relationship among subsequent channel realizations could be modeled through the Fourier transform of the bandlimited time-varying channel response. Thus, N_c consecutive realizations of the channel $\mathbf{H}_1, \ldots, \mathbf{H}_{N_c}$ can be represented by eq. (3), where the orthogonal basis is

$$\mathbf{B} = \mathbf{F}_{N_c \times L_c}\left(-\frac{L_c - 1}{2N_c}\right). \quad (17)$$

There exist other alternative basis for modeling the temporal variation of the channel, such as the discrete prolate spheroidal sequences (or finite Slepian sequences) [56], which avoid the spectral leakage problem associated to the Fourier basis. However, regardless of the particular basis selection, the number L_c of parameters is directly related with the maximum Doppler frequency f_D by means of

$$\frac{f_D}{f_s} = \frac{f_c}{f_s}\frac{v_{\max}}{v_{\text{light}}} = \frac{L_c - 1}{2LN_c}, \quad (18)$$

where f_c and f_s are the carrier and symbol frequencies, respectively, $v_{\max}$ is the maximum relative speed between the transmitter and the receiver, and v_{light} is the speed of light.

- **Doubly-Selective Channels**: The above data model can be easily extended to the case of doubly-selective MIMO channels [44, 45], for which the basis $\mathbf{B}$ can be interpreted as a Kronecker product between the time and frequency bases, whereas the parameter L_c indicates the total number of degrees of freedom in the system, i.e., the product of the time and frequency diversities.

3. Blind Estimation of Selective MIMO Channels

In this section we propose a general blind channel estimation technique inspired by the blind ML receiver. Unlike other approaches, the proposed scheme is able to recover the channel up to a real scalar from a reduced number of observations (STBC-OFDM or SFBC blocks). Let us start by introducing the joint ML estimator of the channel and information symbols.

3.1. Unconstrained Blind ML Receiver

In general, the blind maximum likelihood (ML) estimation of the channel and sources is a very difficult problem, which is due to the coupling among the channels $\mathbf{H}_i$, which depend on the parameters $\boldsymbol{\Theta}$, the coupling among the sources $\mathbf{s}_i[n]$, which depend on $\mathbf{d}[n]$, and the finite alphabet properties of the information symbols $\mathbf{d}[n]$. A direct simplification is obtained by relaxing the finite alphabet constraint, which decouples the signal estimates for different channels. Thus, assuming that for each channel $\mathbf{H}_i$, a set of N STBC blocks is available at the receiver side, the unconstrained blind maximum likelihood decoder (UML) reduces to

$$\left\{\hat{\boldsymbol{\Theta}}^{\text{UML}}, \hat{\mathbf{s}}_i^{\text{UML}}[n]\right\} = \underset{\boldsymbol{\Theta}, \mathbf{s}_i[n]}{\text{argmin}} \sum_{i=1}^{N_c}\sum_{n=0}^{N-1}\left\|\tilde{\mathbf{y}}_i[n] - \tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)\mathbf{s}_i[n]\right\|^2, \quad (19)$$

and solving for $\mathbf{s}_i[n]$ we obtain[§]

$$\hat{\mathbf{s}}_i^{\text{UML}}[n] = \left(\tilde{\mathbf{W}}_i^T(\tilde{\mathbf{h}}_i)\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)\right)^{-1}\tilde{\mathbf{W}}_i^T(\tilde{\mathbf{h}}_i)\tilde{\mathbf{y}}_i[n] = \\ = \tilde{\mathbf{V}}_i(\tilde{\mathbf{h}}_i)\tilde{\boldsymbol{\Sigma}}_i^{-1}(\tilde{\mathbf{h}}_i)\tilde{\mathbf{U}}_i^T(\tilde{\mathbf{h}}_i)\tilde{\mathbf{y}}_i[n], \quad (20)$$

where $\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i) = \tilde{\mathbf{U}}_i(\tilde{\mathbf{h}}_i)\tilde{\boldsymbol{\Sigma}}_i(\tilde{\mathbf{h}}_i)\tilde{\mathbf{V}}_i^T(\tilde{\mathbf{h}}_i)$ denotes the singular value decomposition (SVD) of $\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)$. Therefore, combining (19) and (20) the UML criterion can be rewritten as

$$\hat{\boldsymbol{\Theta}}^{\text{UML}} = \underset{\boldsymbol{\Theta}}{\text{argmax}} \sum_{i=1}^{N_c}\sum_{n=0}^{N-1}\tilde{\mathbf{y}}_i^T[n]\tilde{\mathbf{U}}_i(\tilde{\mathbf{h}}_i)\tilde{\mathbf{U}}_i^T(\tilde{\mathbf{h}}_i)\tilde{\mathbf{y}}_i[n], \quad (21)$$

or equivalently

$$\hat{\boldsymbol{\Theta}}^{\text{UML}} = \underset{\boldsymbol{\Theta}}{\text{argmax}} \sum_{i=1}^{N_c}\text{Tr}\left(\tilde{\mathbf{U}}_i^T(\tilde{\mathbf{h}}_i)\mathbf{R}_{\tilde{\mathbf{y}}_i}\tilde{\mathbf{U}}_i(\tilde{\mathbf{h}}_i)\right), \quad (22)$$

where

$$\mathbf{R}_{\tilde{\mathbf{y}}_i} = \frac{1}{N}\sum_{n=0}^{N-1}\tilde{\mathbf{y}}_i[n]\tilde{\mathbf{y}}_i^T[n], \quad (23)$$

is the sample mean estimate of the correlation matrix for the observations of the i-th channel.

3.2. Proposed Blind Channel Estimation Method

Although the relaxation of the finite alphabet constraint in the information symbols simplifies

[§]We are assuming that the equivalent channels $\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)$ are full-column rank, which is a common assumption for all the STBCs.

DOI: 10.1002/wcm

the blind channel estimation criterion, the channel estimates are still coupled through the basis expansion parameters $\boldsymbol{\Theta}$. On the one hand, this reduced-rank model allows us to take into account the correlation among consecutive channels. On the other hand, the coupling in the channel estimates and the non trivial dependency of $\tilde{\mathbf{U}}_i(\tilde{\mathbf{h}}_i)$ w.r.t. the parameters $\boldsymbol{\Theta}$ preclude a direct solution of the criterion in (22). In this subsection, we present a subspace-based blind channel estimation method which provides closed-form channel estimates.

The UML estimator in (21) and (22) can be easily interpreted as a subspace technique, whose goal is to maximize the energy of the projections of the *observed signal subspaces*, obtained from $\tilde{\mathbf{y}}_i[n]$, onto the *parameter-dependent signal subspaces*, which are defined by the equivalent channel matrices $\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)$ (or $\tilde{\mathbf{U}}_i(\tilde{\mathbf{h}}_i)$). Here, we propose an alternative criterion which consists in the maximization of the following weighted sum of energies

$$\hat{\boldsymbol{\Theta}} = \underset{\boldsymbol{\Theta}}{\text{argmax}} \sum_{i=1}^{N_c} E_i \text{Tr}\left(\tilde{\mathbf{W}}_i^T(\tilde{\mathbf{h}}_i)\boldsymbol{\Phi}_{\tilde{\mathbf{y}}_i}\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)\right), \tag{24}$$

where[¶]

$$\boldsymbol{\Phi}_{\tilde{\mathbf{y}}_i} = \tilde{\mathbf{U}}_{\tilde{\mathbf{y}}_i}\tilde{\mathbf{U}}_{\tilde{\mathbf{y}}_i}^T, \qquad i = 1, \ldots, N_c, \tag{25}$$

are the projection matrices onto the *observed signal subspaces*, $\tilde{\mathbf{U}}_{\tilde{\mathbf{y}}_i} \in \mathbb{R}^{2Ln_R \times r}$ is a matrix containing the $r = \min(N, M')$ principal eigenvectors of $\mathbf{R}_{\tilde{\mathbf{y}}_i}$, and E_i denotes the signal energy in the i-th channel, which is obtained as the sum of the r largest eigenvalues of $\mathbf{R}_{\tilde{\mathbf{y}}_i}$.

The criterion in (24) can be interpreted as follows: Instead of maximizing the projections of $\tilde{\mathbf{y}}_i[n]$ onto the *parameter-dependent signal subspaces*, we maximize the projections of the equivalent channels $\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)$ onto the *observed signal subspaces*. This alternative criterion will allow us to obtain closed-form channel estimates. However, unlike (22), the energy of the channels $\tilde{\mathbf{h}}_i$ (or equivalent channels $\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)$) in (24) must be constrained to avoid trivial solutions. Although this could seem a minor problem, the selection of the constraint constitutes a key point in the derivation of the blind channel estimation criterion. Specifically, we propose the following constraint in the channel energies

$$\sum_{i=1}^{N_c} E_i \|\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)\|^2 = 1. \tag{26}$$

As will be shown later, this constraint not only avoids trivial solutions, but also ensures that, under mild assumptions, the theoretical solutions of the overall channel estimation technique are those of the UML decoder.

Now, the dependency of $\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)$ with $\tilde{\mathbf{h}}_i$, given by eq. (10), allows us to write

$$\text{Tr}\left(\tilde{\mathbf{W}}_i^T(\tilde{\mathbf{h}}_i)\boldsymbol{\Phi}_{\tilde{\mathbf{y}}_i}\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)\right) = \sum_{k=1}^{M'} \tilde{\mathbf{h}}_i^T \tilde{\mathbf{D}}_{i,k}^T \boldsymbol{\Phi}_{\tilde{\mathbf{y}}_i} \tilde{\mathbf{D}}_{i,k} \tilde{\mathbf{h}}_i, \tag{27}$$

and

$$\|\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)\|^2 = \sum_{k=1}^{M'} \tilde{\mathbf{h}}_i^T \tilde{\mathbf{D}}_{i,k}^T \tilde{\mathbf{D}}_{i,k} \tilde{\mathbf{h}}_i. \tag{28}$$

Thus, the optimization problem given by (24) and (26) can be reformulated as

$$\hat{\boldsymbol{\Theta}} = \underset{\boldsymbol{\Theta}}{\text{argmax}} \sum_{i=1}^{N_c} \tilde{\mathbf{h}}_i^T \boldsymbol{\Xi}_i \tilde{\mathbf{h}}_i, \qquad \text{s.t.} \quad \sum_{i=1}^{N_c} \tilde{\mathbf{h}}_i^T \boldsymbol{\Psi}_i \tilde{\mathbf{h}}_i = 1, \tag{29}$$

where

$$\boldsymbol{\Xi}_i = E_i \sum_{k=1}^{M'} \tilde{\mathbf{D}}_{i,k}^T \boldsymbol{\Phi}_{\tilde{\mathbf{y}}_i} \tilde{\mathbf{D}}_{i,k}, \tag{30}$$

and

$$\boldsymbol{\Psi}_i = E_i \sum_{k=1}^{M'} \tilde{\mathbf{D}}_{i,k}^T \tilde{\mathbf{D}}_{i,k}. \tag{31}$$

The criterion proposed so far only exploits the structure imposed by the STBC. The additional structure provided by the time-frequency-selective behavior can be incorporated to the criterion through the reduced-rank BEM. In particular, defining the vectors $\boldsymbol{\theta}_k = \text{vec}(\boldsymbol{\Theta}_k)$, the channel model in (1) can be rewritten as

$$\mathbf{h}_i = \sum_{k=1}^{L_c} b_{i,k} \boldsymbol{\theta}_k, \qquad i = 1, \ldots, N_c, \tag{32}$$

or equivalently, for $i = 1, \ldots, N_c$,

$$\tilde{\mathbf{h}}_i = \sum_{k=1}^{L_c} \left(\begin{bmatrix} \Re(b_{i,k}) & -\Im(b_{i,k}) \\ \Im(b_{i,k}) & \Re(b_{i,k}) \end{bmatrix} \otimes \mathbf{I}_{n_T n_R} \right) \tilde{\boldsymbol{\theta}}_k, \tag{33}$$

[¶]The prewhitening is nos necessary in the OSTBC case. In other words, due to the orthogonality of $\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)$, $\boldsymbol{\Phi}_{\tilde{\mathbf{y}}_i}$ can be replaced by $\frac{r}{E_i}\mathbf{R}_{\tilde{\mathbf{y}}_i}$.

where $\tilde{\boldsymbol{\theta}}_k = \left[\Re(\boldsymbol{\theta}_k^T), \Im(\boldsymbol{\theta}_k^T)\right]^T$. Therefore, the real vectorized channels are given by $\tilde{\mathbf{h}}_i = \boldsymbol{\Omega}_i\tilde{\boldsymbol{\theta}}$, where $\tilde{\boldsymbol{\theta}} = \left[\tilde{\boldsymbol{\theta}}_1^T, \ldots, \tilde{\boldsymbol{\theta}}_{L_c}^T\right]^T$,

$$\boldsymbol{\Omega}_i = \Re(\mathbf{b}_i^T) \otimes \mathbf{I}_{2n_T n_R} + \Im(\mathbf{b}_i^T) \otimes \begin{bmatrix} \mathbf{0} & -\mathbf{I}_{n_T n_R} \\ \mathbf{I}_{n_T n_R} & \mathbf{0} \end{bmatrix}, \tag{34}$$

and $\mathbf{b}_i^T$ is the i-th row of the orthogonal basis $\mathbf{B}$. Thus, the combination of (29) and $\tilde{\mathbf{h}}_i = \boldsymbol{\Omega}_i\tilde{\boldsymbol{\theta}}$ allows us to rewrite the proposed blind channel estimation criterion as a function of the channel expansion coefficients $\tilde{\boldsymbol{\theta}}$

$$\hat{\tilde{\boldsymbol{\theta}}} = \underset{\tilde{\boldsymbol{\theta}}}{\operatorname{argmax}}\, \tilde{\boldsymbol{\theta}}^T \boldsymbol{\Xi}\tilde{\boldsymbol{\theta}}, \qquad \text{s.t.} \qquad \tilde{\boldsymbol{\theta}}^T \boldsymbol{\Psi}\tilde{\boldsymbol{\theta}}, \tag{35}$$

where

$$\boldsymbol{\Xi} = \sum_{i=1}^{N_c} \boldsymbol{\Omega}_i^T \boldsymbol{\Xi}_i \boldsymbol{\Omega}_i, \tag{36}$$

and

$$\boldsymbol{\Psi} = \sum_{i=1}^{N_c} \boldsymbol{\Omega}_i^T \boldsymbol{\Psi}_i \boldsymbol{\Omega}_i. \tag{37}$$

The solution of (35) is obtained as the eigenvector $\hat{\tilde{\boldsymbol{\theta}}}$ associated to the largest eigenvalue β of the following generalized eigenvalue problem (GEV)

$$\boldsymbol{\Xi}\hat{\tilde{\boldsymbol{\theta}}} = \beta\boldsymbol{\Psi}\hat{\tilde{\boldsymbol{\theta}}}. \tag{38}$$

Finally, the overall blind channel estimation algorithm is summarized in Algorithm 1.

4. Identifiability Analysis

Although some intuitive necessary conditions can be easily obtained, the analysis of the blind channel identifiability from SOS under STBC transmissions is a difficult problem yet to be solved. In particular, several efforts have been made in the case of flat fading and time-invariant STBC systems [14, 16, 19], but the identifiability properties are only partially clear in the OSTBC case [57, 58].

Here we show that, under mild assumptions, the theoretical solutions of the proposed criterion are those associated to the UML decoder. In other words, the channel estimates provided by the proposed technique are congruent with the data model.

Let us consider a noise-free scenario[‖]. From (21) and (22), it is easy to prove that the solutions $\hat{\tilde{\mathbf{h}}}_i$ of the UML criterion fulfill

$$\text{range}\left(\tilde{\mathbf{U}}_{\tilde{\mathbf{y}}_i}\right) \subseteq \text{range}\left(\tilde{\mathbf{W}}_i(\hat{\tilde{\mathbf{h}}}_i)\right), \qquad i = 1, \ldots, N_c, \tag{39}$$

where the equality is satisfied iff the signal subspace is completely determined by the observations ($N \geq M'$). To continue the analysis we must distinguish two different cases.

4.1. Case of $N \geq M'$

In this case, $\boldsymbol{\Phi}_{\tilde{\mathbf{y}}_i}$ ($i = 1, \ldots, N_c$) are the true projection matrices onto the signal subspaces. Therefore, the energy of the projections in (24) is bounded by

$$\text{Tr}\left(\tilde{\mathbf{W}}_i^T(\tilde{\mathbf{h}}_i)\boldsymbol{\Phi}_{\tilde{\mathbf{y}}_i}\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)\right) \leq \left\|\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)\right\|^2, \tag{40}$$

where the equality is satisfied iff $\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)$ spans the true signal subspace. Finally, taking into account the energy constraint in (26) it is clear that

$$\sum_{i=1}^{N_c} E_i \text{Tr}\left(\tilde{\mathbf{W}}_i^T(\tilde{\mathbf{h}}_i)\boldsymbol{\Phi}_{\tilde{\mathbf{y}}_i}\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)\right) \leq 1, \tag{41}$$

and the equality is attained by the solutions of the UML decoder, which obviously include the true MIMO channel.

4.2. Case of $N < M'$

This is a more complicated situation in which the channels are not persistently excited by the sources, i.e., the observations do not completely characterize the signal subspace, and $\boldsymbol{\Phi}_{\tilde{\mathbf{y}}_i}$ is only a projection matrix onto a rank N subspace belonging to the whole rank M' signal subspace. Thus, we must distinguish between two different cases:

4.2.1. Orthogonal STBCs (OSTBC)

In this case the channel can be unambiguously recovered by means of the proposed technique. In particular, the orthogonality property of OSTBCs is [57]

$$\tilde{\mathbf{W}}_i^T(\tilde{\mathbf{h}}_i)\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i) = \left\|\tilde{\mathbf{h}}_i\right\|^2 \mathbf{I}_{M'}, \qquad \forall\tilde{\mathbf{h}}_i, \tag{42}$$

which ensures that, in the absence of noise

$$\text{Tr}\left(\tilde{\mathbf{W}}_i^T(\tilde{\mathbf{h}}_i)\boldsymbol{\Phi}_{\tilde{\mathbf{y}}_i}\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)\right) \leq N\left\|\tilde{\mathbf{h}}_i\right\|^2, \tag{43}$$

‖The same conclusions can be obtained by assuming perfect estimates ($N \to \infty$) of the correlation matrices $\mathbf{R}_{\tilde{\mathbf{y}}_i}$.

where the equality is satisfied iff the observations $\tilde{\mathbf{y}}_i[n]$ belong to the subspace spanned by $\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)$, or equivalently

$$\text{range}\left(\tilde{\mathbf{U}}_{\tilde{\mathbf{y}}_i}\right) \subset \text{range}\left(\tilde{\mathbf{W}}_i(\hat{\tilde{\mathbf{h}}}_i)\right), \qquad i = 1, \ldots, N_c, \tag{44}$$

i.e., if the estimates are congruent with the data model. Now, taking into account the orthogonality of the equivalent channels $\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)$, the energy constraint (26) can be rewritten as

$$\sum_{i=1}^{N_c} E_i \left\|\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)\right\|^2 = M' \sum_{i=1}^{N_c} E_i \left\|\tilde{\mathbf{h}}_i\right\|^2 = 1, \tag{45}$$

which finally yields

$$\sum_{i=1}^{N_c} E_i \text{Tr}\left(\tilde{\mathbf{W}}_i^T(\tilde{\mathbf{h}}_i)\mathbf{\Phi}_{\tilde{\mathbf{y}}_i}\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)\right) \le \frac{N}{M'}, \tag{46}$$

where the equality is attained by the solutions of the UML decoder.

4.2.2. Non-Orthogonal STBCs

In this case the energies $\text{Tr}\left(\tilde{\mathbf{W}}_i^T(\tilde{\mathbf{h}}_i)\mathbf{\Phi}_{\tilde{\mathbf{y}}_i}\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)\right)$ are not necessarily maximized by the actual MIMO channels and then the channel can not be exactly recovered by means of the proposed technique. In other words, since the signal subspaces are not completely characterized by the projection matrices $\mathbf{\Phi}_{\tilde{\mathbf{y}}_i}$, the proposed technique might find spurious MIMO channels concentrating all the energy of $\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)$ in the directions defined by $\mathbf{\Phi}_{\tilde{\mathbf{y}}_i}$. However, we must note that the maximization has to be made simultaneously for the N_c sub-channels, whereas the number of effective independent channels is given by L_c. Thus, when $N_c >> L_c$ there are not enough degrees of freedom to find spurious solutions, and the proposed technique will provide very accurate estimates.

5. Computational Cost and Comparison with Previous Works

The proposed blind technique has to solve two main steps. Firstly, the N_c projection matrices $\mathbf{\Phi}_{\tilde{\mathbf{y}}_i}$ are obtained, which comes at a computational cost of order $\mathcal{O}(N_c L^3 n_R^3)$; and secondly, the channel parameters are recovered from the GEV in (38), whose computational cost is $\mathcal{O}(n_T^3 n_R^3 L_c^3)$. Therefore, the computational complexity of the proposed blind channel estimation technique is $\mathcal{O}\left(n_R^3(L^3 N_c + n_T^3 L_c^3)\right)$, i.e., it is linear in the number N_c of channels, which corresponds with the number of subcarriers in multicarrier systems, or the number of channel realizations in time-varying scenarios. This contrasts with previous applications of standard subspace techniques [36, 37] (see also [38, 39, 40]), which not only require a large number $N > M'N_c$ of STBC-OFDM blocks at the receiver, but also incur in a computational cost of $\mathcal{O}((Ln_R N_c)^3)$.

When compared with previous works, the proposed method solves the blind channel estimation problem for general STBC transmissions in a unified manner, which includes the case of time-varying channels, as well as the common STBC-OFDM, SFBC and time-reversal systems. Moreover, the proposed method avoids the finite alphabet requirement associated to the semiblind algorithm in [41, 42], or the constant energy associated to differential approaches [23,24,25, 26, 27]. Therefore, it can be directly applied when the information symbols have been linearly precoded in order to exploit the multipath or temporal diversity of the channel. Finally, as we have shown in the previous section, in the absence of noise the channel parameters can be exactly recovered within only $N = M'$ (or $N = 1$ in the OSTBC case) blocks at the receiver side.

6. Simulation Results

In this section, the performance of the proposed technique is illustrated by means of some simulation examples. All the results have been obtained by averaging 1000 independent experiments. The MIMO channels $\mathbf{H}_i$ have been generated as a Rayleigh channel with unit-variance elements. The i.i.d information symbols, which have not been linearly precoded ($\mathbf{G} = \mathbf{I}_{N_c}$), belong to a quadrature phase shift keying (QPSK) constellation. We have used MMSE receivers followed by a hard decision decoder, which in the case of OSTBC transmissions is equivalent to the ML receiver. The transmission schemes are based on two different STBCs for $n_T = 4$ transmit antennas, namely, the OSTBC presented in Eq. (7.4.10) of [52], whose parameters are $M = 3$ and $L = 4$ ($R = 3/4$), and the quasi-orthogonal (QSTBC) proposed in [3] ($M = L = 4$, $R = 1$).

The proposed method has been compared with the MMSE receiver with perfect CSI, which we refer to as clairvoyant MMSE, and with a training based approach. In the particular case of OFDM-based transmissions, the training method is based on the use of L_c equally spaced pilot subcarriers, and the channel

DOI: 10.1002/wcm

estimate is obtained by means of the well-known least squares (LS) method. Finally, in order to avoid the ambiguities associated to the QSTBC blind channel estimation problem [16, 57], we have applied the non-redundant precoding technique proposed in [19], i.e., for each subchannel $\mathbf{H}_i$ we have used a rotated version of the QSTBC in [3].

6.1. STBC-OFDM Systems

In this subsection, the proposed technique is evaluated in a STBC-OFDM MIMO system with $L_c = 4$ taps and different number N_c of subcarriers.** In the first experiment we consider the $R = 3/4$ OSTBC system with $n_R = 2$ receive antennas and $N_c = 64$ subcarriers. Fig. 1 shows the mean square error (MSE) of the channel estimate for different numbers N of available blocks at the receiver, whereas Fig. 2 shows the bit error rate (BER) after decoding. As can be seen, the proposed method outperforms the training approach based on L_c pilot carriers and, as it was expected, its accuracy increases with the number of available OSTBC-OFDM blocks. This point is also illustrated in Fig. 3, which shows the evolution of the BER with the number N of OSTBC-OFDM blocks. As can be seen, as N increases, the proposed method achieves similar results to that of the receiver with perfect channel knowledge.

In the second set of examples, the performance of the proposed method for $N = 1$ and several numbers of subcarriers N_c is evaluated. The results for the $R = 3/4$ OSTBC with $n_R = 2$ receive antennas are shown in Figs. 4 and 5. As can be seen, for a fixed number (L_c) of parameters, the performance not only improves with the number of available OSTBC-OFDM blocks, but also with the number of subcarriers N_c. This can be seen as a direct consequence of the rank-reduced channel model, which is able to properly exploit the structure introduced by the channel. In other words, while the number of unknown parameters ($L_c n_T n_R$) remains constant, the available data to estimate the channel increases with N_c, which necessarily translates into better channel estimates.

The previous experiments have been repeated for the rate-one QSTBC with $n_T = n_R = M = L = 4$. Firstly, the results for $N_c = 64$ subcarriers and different numbers of available QSTBC-OFDM blocks are shown in Figs. 6, 7 and 8. As can be seen, the proposed technique is able to exactly recover the channel, in the absence of noise, when the number of available blocks is $N \geq M'$, i.e., when the signal subspaces are completely determined by the observations. As pointed out in Subsection 4.2.2, when this condition is not satisfied ($N < M'$), the proposed technique is not able to exactly recover the channel, which explains the noise-floor effect in Figs. 6 and 7. However, as we can see in Fig. 8, as N increases, the results provided by the proposed blind channel estimation method become closer to those of the receiver with perfect channel knowledge.

Finally, the previous example has been repeated for only $N = 1$ QSTBC-OFDM block at the receiver side, $L_c = 4$ non-zero taps, and different numbers N_c of subcarriers. The results are shown in Figs. 9 and 10, where we can see that the noise floor in the channel estimate rapidly decreases with the number of subcarriers. Furthermore, we must point out that for practical SNR (or BER) values, the results provided by the proposed technique are not very far from those of the receiver with perfect channel knowledge. Actually, they are accurate enough to switch to a decision directed scheme or to provide a good starting point for an iterative implementation of the UML decoder.

6.2. Flat-Fading Time-Varying Channels

In this subsection, the performance of the proposed technique in time-varying MIMO channels is evaluated. Specifically, the time-varying channels are modeled through a Fourier†† BEM with $L_c = 5$ parameters, which correspond to a maximum Doppler frequency of

$$f_D = \frac{1}{2N_c}. \tag{47}$$

Here, we must note that in the case of time-varying channels we always consider $N = 1$, i.e., only one STBC block is transmitted over each MIMO channel $\mathbf{H}_i$. Obviously, for lower Doppler frequencies the channel could be considered constant during the transmission of N STBC blocks, and more accurate channel estimates would be obtained.

As an example, Fig. 11 shows the evolution of four MIMO channel coefficients, for $N_c = 128$ ($f_D \simeq 3.9 \cdot 10^{-3}$), during 100 symbol periods, which corresponds to the transmission of $100/L = 25$ STBC blocks. As can be seen, the channel can be considered approximately constant during the transmission of one

**Similar results have been obtained in the cases of SFBC or time-reversal STBC systems, but due to the lack of space we only present the STBC-OFDM case.

††Similar results have been obtained in the case of discrete prolate spheroidal sequences.

DOI: 10.1002/wcm

STBC block, which is a common requirement for all the STBC-based systems, but it rapidly changes during the whole transmission frame.

In the first experiment, we consider the $R = 3/4$ OSTBC with $n_R = 2$ receive antennas. The BER after decoding for different Doppler frequencies is shown in Fig. 12. As it was expected, the performance of the proposed technique improves with the temporal-coherence of the channel, which increases with N_c. Furthermore, for moderate Doppler frequencies, the performance of the blind technique is close to that of the clairvoyant receiver, and it avoids the 3-dB penalty associated to differential approaches.

Finally, the previous experiment has been repeated with the QSTBC code and $n_R = 4$ receive antennas. The results are shown in Fig. 13, where we can observe the previously commented noise floor. However, for moderate Doppler frequencies, the blind technique still provides accurate results and its performance degradation with respect to the clairvoyant receiver is lower than the minimal loss (3-dB) associated to the QSTBC differential technique proposed in [28].

7. Conclusions

In this paper a new technique for the blind estimation of frequency and/or time-selective MIMO channels, under space-time block coded (STBC) transmissions, has been presented. The proposed technique is based on a low-rank representation of the MIMO channel, which reduces the number of parameters to be estimated, and can be easily particularized to the cases of STBC-OFDM, space-frequency block coding (SFBC), time-reversal STBC, and STBC transmissions over time-varying channels. The method, which is inspired by the unconstrained blind maximum likelihood (UML) decoder, reduces to the solution of a generalized eigenvalue (GEV) problem, and its computational complexity is linear in the number of orthogonal channels (subcarriers in the particular case of STBC-OFDM systems). Furthermore, unlike other previously proposed approaches, in the absence of noise it is able to exactly recover the channels within a few data blocks at the receiver side. Finally, the proposed algorithm has been evaluated by means of numerical examples, showing that the overall system performance is close to that of the receiver with perfect channel knowledge.

Acknowledgements

This work was supported by the Spanish Government under projects TEC2007-68020-C04-02/TCM (MultiMIMO) and CONSOLIDER-INGENIO 2010 CSD2008-00010 (COMONSENS).

References

1. Alamouti S. A simple transmit diversity technique for wireless communications. *IEEE Journal on Selected Areas in Communications* 1998; **45**(9):1451–1458.
2. Tarokh V, Jafarkhani H, Calderbank AR. Space-time block codes from orthogonal designs. *IEEE Transactions on Information Theory* 1999; **45**(5):1456–1467.
3. Jafarkhani H. *Space-Time Coding: Theory and Practice*. Cambridge University Press, 2005.
4. Sharma N, Papadias CB. Full-rate full-diversity linear quasi-orthogonal space-time codes for any number of transmit antennas. *EURASIP Journal on Applied Signal Processing* Aug 2004; (9):1246–1256.
5. Sezgin A, Oechtering T. Complete characterization of the equivalent MIMO channel for quasi-orthogonal space-time codes. *IEEE Transactions on Information Theory* Jul 2008; **54**(7):3315–3327.
6. Zhang JK, Liu J, Wong KM. Trace-orthogonal full diversity cyclotomic space-time codes. *Space-Time processing for MIMO communications*, Wiley, 2005.
7. Barbarossa S. *Multiantenna Wireless Communication Systems*. Artech House Publishers, 2005.
8. Oggier F, Rekaya G, Belfiore JC, Viterbo E. Perfect Space-Time Block Codes. *IEEE Transactions on Information Theory* Sep 2006; **52**(9):3885–3902.
9. Swindlehurst L, Leus G. Blind and semi-blind equalization for generalized space-time block codes. *IEEE Transactions on Signal Processing* 2002; **50**(10):2489–2498.
10. Stoica P, Ganesan G. Space–time block codes: Trained, blind, and semi–blind detection. *Digital Signal Processing* Jan 2003; **13**:93–105.
11. Larsson E, Stoica P, Li J. Orthogonal space-time block codes: Maximum likelihood detection for unknown channels and unstructured interferences. *IEEE Transactions on Signal Processing* Feb 2003; **51**(2):362–372.
12. Shahbazpanahi S, Gershman AB, Manton JH. Closed-form blind MIMO channel estimation for orthogonal space-time block codes. *IEEE Transactions on Signal Processing* Dec 2005; **53**(12):4506–4517.
13. Ma WK, Vo BN, Davidson TN, Ching PC. Blind ML detection of orthogonal space-time block codes: Efficient high-performance implementations. *IEEE Transactions on Signal Processing* Feb 2006; **54**(2):738–751.
14. Ammar N, Ding Z. Channel identifiability under orthogonal space-time coded modulations without training. *IEEE Transactions on Wireless Communications* May 2006; **5**(5):1003–1013.
15. Shahbazpanahi S, Gershman AB, Giannakis GB. Semiblind multiuser MIMO channel estimation using Capon and MUSIC techniques. *IEEE Transactions on Signal Processing* Sep 2006; **54**(9):3581–3591.
16. Ammar N, Ding Z. Blind channel identifiability for generic linear space-time block codes. *IEEE Transactions on Signal Processing* Jan 2007; **55**(1):202–217.
17. Zhou L, Zhang JK, Wong K. A novel signaling scheme for blind unique identification of Alamouti Space-Time Block-Coded channel. *IEEE Transactions on Signal Processing* Jun 2007; **55**(6):2570–2582.

18. Vía J, Santamaría I. Correlation matching approaches for blind OSTBC channel estimation. *IEEE Transactions on Signal Processing* Dec 2008; **56**(12):5950–5961, doi:10.1109/TSP.2008.929661.
19. Vía J, Santamaría I, Prez J. Code combination for blind channel estimation in general MIMO-STBC systems. *EURASIP Journal on Advances in Signal Processing* 2009; .
20. Naguib AF, Tarokh V, Seshadri N, Calderbank AR. A space-time coding modem for high-data-rate wireless communications. *IEEE Journal on Selected Areas in Communications* Oct 1998; **16**(8):1459–1478.
21. Hassibi B, Hochwald BM. How much training is needed in multiple-antenna wireless links? *IEEE Transactions on Information Theory* Apr 2003; **49**(4):951–963.
22. Pohl V, Nguyen P, Jungnickel V, von Helmolt C. Continuous flat fading MIMO channels: Achievable rate and the optimal length of the training and data phase. *IEEE Transactions on Wireless Communications* Jul 2005; **4**(4):1889–1900.
23. Tarokh V, Jafarkhani H. A differential detection scheme for transmit diversity. *IEEE Journal on Selected Areas in Communications* Jul 2000; **18**(7):1169–1174.
24. Hughes BL. Differential space-time modulation. *IEEE Transactions on Information Theory* Nov 2000; **46**(7):2567–2578.
25. Hochwald B, Sweldens W. Differential unitary space-time modulation. *IEEE Transactions on Communications* Dec 2000; **48**(12):2041–2052.
26. Jafarkhani H, Tarokh V. Multiple transmit antenna differential detection from generalized orthogonal designs. *IEEE Transactions on Information Theory* 2001; **47**(6):2626–2631.
27. Ganesan G, Stoica P. Differential modulation using space-time block codes. *IEEE Signal Processing Letters* Feb 2002; **9**(2):57–60.
28. Zhu Y, Jafarkhani H. Differential modulation based on quasi-orthogonal codes. *IEEE Transactions on Wireless Communications* Nov 2005; **4**(6):3005–3017.
29. Vía J, Santamaría I, Pérez J, Ramírez D. Blind decoding of MISO-OSTBC systems based on principal component analysis. *IEEE International Conference on Acoustic, Speech, and Signal Processing (ICASSP 2006)*, Toulouse, France, 2006.
30. Vía J, Santamaría I, Pérez J. Blind identification of MIMO-OSTBC channels combining second and higher order statistics. *European Signal Processing Conference (EUSIPCO 2006)*, Florence, Italy, 2006.
31. Balakumar B, Shahbazpanahi S, Kirubarajan T. Joint MIMO channel tracking and symbol decoding using kalman filtering. *IEEE Transactions on Signal Processing* Dec 2007; **55**(12):5873–5879.
32. Choi J. Equalization and semi-blind channel estimation for space-time block coded signals over a frequency-selective fading channel. *IEEE Transactions on Signal Processing* Mar 2004; **52**(3):774–785.
33. Ding Z, Ward DB. Subspace approach to blind and semi-blind channel estimation for space-time block codes. *IEEE Transactions on Wireless Communications* Mar 2005; **4**(2):357–362.
34. Ding Z, Ward DB. Direct semi-blind MMSE equalization for STBC. *IEEE Signal Processing Letters* May 2005; **12**(5):380–383.
35. Ding Z, Ward D, Chin WH. A general scheme for equalization of Space-Time Block-Coded systems with unknown CSI. *IEEE Transactions on Signal Processing* Jul 2006; **54**(7):2737–2746.
36. Liu Z, Giannakis G, Muquet B, Zhou S. Space-time coding for broadband wireless communications. *Wireless Communications and Mobile Computing* 2001; **1**(1):35–53.
37. Liu Z, Giannakis G, Barbarossa S, Scaglione A. Transmit-antenna space-time block coding for generalized OFDM in the presence of unknown multipath. *IEEE Journal on Selected Areas in Communications* Jul 2001; **19**(7):1352–1364.
38. Ammar N, Ding Z. Frequency selective channel estimation in time-reversed space-time coding. *IEEE Wireless Communications and Networking Conference (WCNC)*, Atlanta, USA, 2004; 1838–1843.
39. Wu JY, Lee TS. Nonredundant precoding-assisted blind channel estimation for single-carrier space-time block-coded transmission with frequency-domain equalization. *IEEE Transactions on Signal Processing* Mar 2007; **55**(3):1062–1080.
40. Wu JY, Lee TS. Periodic-modulation-based blind channel identification for single-carrier block transmission with frequency-domain equalization. *IEEE Transactions on Signal Processing* Mar 2006; **54**(3):1114–1130.
41. Chang TH, Ma WK, Chi CY. Semiblind ML OSTBC-OFDM detection in block fading channels. *IEEE International Conference on Acoustic, Speech, and Signal Processing (ICASSP 2007)*, Honolulu, Hawaii, USA, 2007.
42. Chang TH, Ma WK, Chi CY. Maximum-likelihood detection of orthogonal space-time block coded OFDM in unknown block fading channels. *IEEE Transactions on Signal Processing* Apr 2008; **56**(4):1637–1649.
43. Sarmadi N, Gershman AB, Shahbazpanahi S. Blind channel estimation in MIMO-OFDM systems using semi-definite relaxation. *IEEE International conference on acoustic, speech and signal processing (ICASSP)*, Las Vegas, USA, 2008.
44. Giannakis G, Tepedelenlioglu C. Basis expansion models and diversity techniques for blind identification and equalization of time-varying channels. *Proceedings of the IEEE* Oct 1998; **86**(10):1969–1986.
45. Giannakis GB, Liu Z, Ma X, Zhou S. *Space Time Coding for Broadband Wireless Communications*. Wiley-Interscience, 2006.
46. Picinbono B, Chevalier P. Widely linear estimation with complex data. *IEEE Transactions on Signal Processing* Aug 1995; **43**(8):2030–2033, doi:10.1109/78.403373.
47. Schreier P, Scharf L, Mullis C. Detection and estimation of improper complex random signals. *IEEE Transactions on Information Theory* Jan 2005; **51**(1):306–312.
48. Gerstacker W, Obernosterer F, Schober R, Lampe ALA, Gunreben P. Equalization concepts for Alamouti's space-time block code. *IEEE Transactions on Communications* Jul 2004; **52**(7):1178–1190.
49. Lee K, Williams D. A space-frequency transmitter diversity technique for OFDM systems. *IEEE Global Telecommunications Conference (GLOBECOM)*, vol. 3, 2000; 1473–1477.
50. Perez J, Ibañez J, Vielva L, Santamaria I. Analytical approximations for the capacity of orthogonal SFBC. *3rd International Symposium on Wireless Communication Systems (ISWCS 06)*, 2006; 466–470.
51. Lindskog E, Paulraj A. A transmit diversity scheme for channels with intersymbol interference. *Proc. IEEE Int. Conf. Communication (ICC)*, New Orleans, USA, 2000; 307–311.
52. Larsson EG, Stoica P. *Space-Time Block Coding for Wireless Communications*. Cambridge University Press: New York, USA, 2003.
53. Zhou S, Giannakis G. Single-carrier space-time block-coded transmissions over frequency-selective fading channels. *IEEE Transactions on Information Theory* Jan 2003; **49**(1):164–179.
54. Mheidat H, Uysal M, Al-Dhahir N. Quasi-orthogonal time-reversal spacetime block coding for frequency-selective fading channels. *IEEE Transactions on Signal Processing* Feb 2007; **55**(2):772–778.
55. Fazel F, Jafarkhani H. Quasi-orthogonal space-frequency and space-time-frequency block codes for MIMO OFDM channels. *IEEE Transactions on Wireless Communications* Jan 2008; **7**(1):184–192.
56. Slepian D. Prolate spheroidal wave functions, Fourier analysis,

DOI: 10.1002/wcm

and uncertainty. V: the discrete case. *The Bell System Technical Journal* Jun 1978; **57**(5):1371–1430.
57. Vía J, Santamaría I. On the blind identifiability of orthogonal space-time block codes from second order statistics. *IEEE Transactions on Information Theory* Feb 2008; **54**(2):709–722.
58. Ma WK. Blind ML detection of orthogonal space-time block codes: Identifiability and code construction. *IEEE Transactions on Signal Processing* Jul 2007; **55**(7):3312–3324.

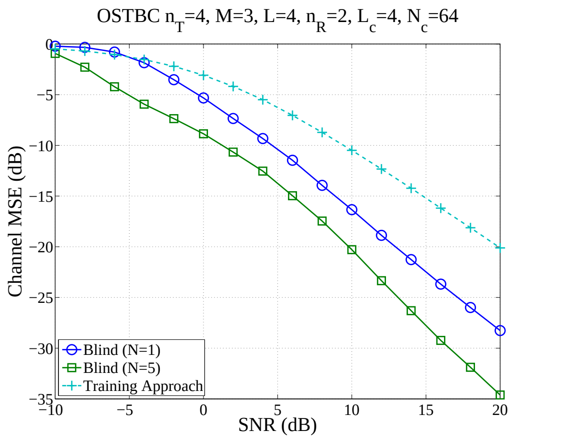

Fig. 1. MSE in the channel estimate for a $R = 3/4$ OSTBC. $N_c = 64$ and different numbers of available blocks at the receiver.

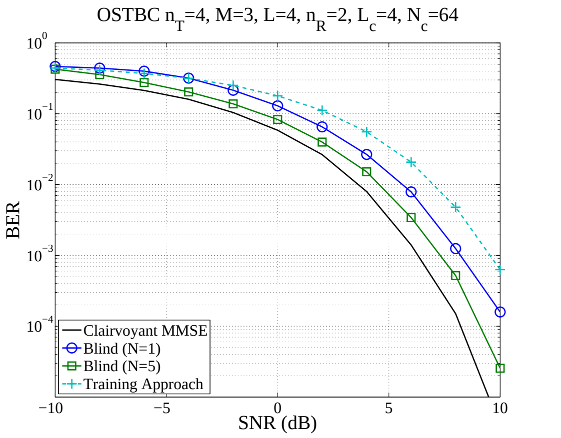

Fig. 2. BER after decoding for a $R = 3/4$ OSTBC. $N_c = 64$ and different numbers of available blocks at the receiver.

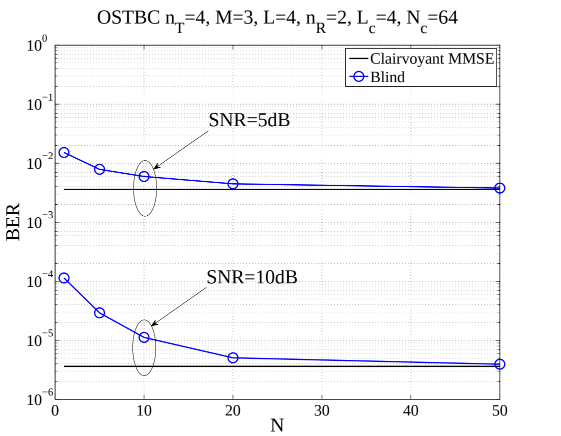

Fig. 3. BER after decoding for a $R = 3/4$ OSTBC. Effect of N on the BER for two different SNR values. $N_c = 64$.

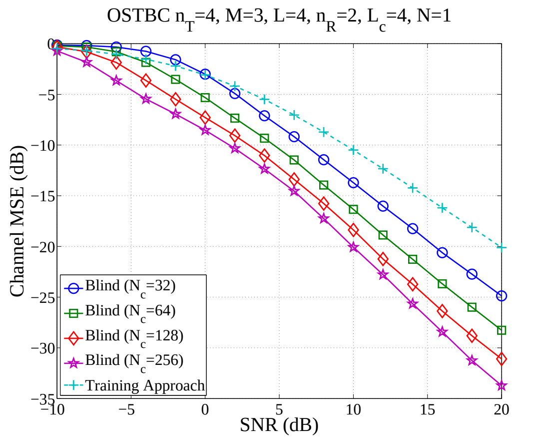

Fig. 4. MSE in the channel estimate for a $R = 3/4$ OSTBC. $N = 1$ and different numbers of subcarriers.

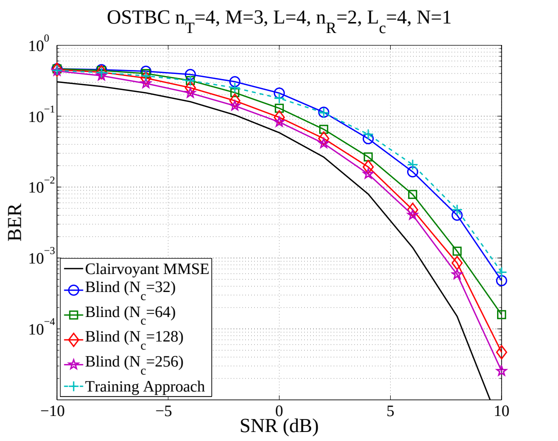

Fig. 5. BER after decoding for a $R = 3/4$ OSTBC. $N = 1$ and different numbers of subcarriers.

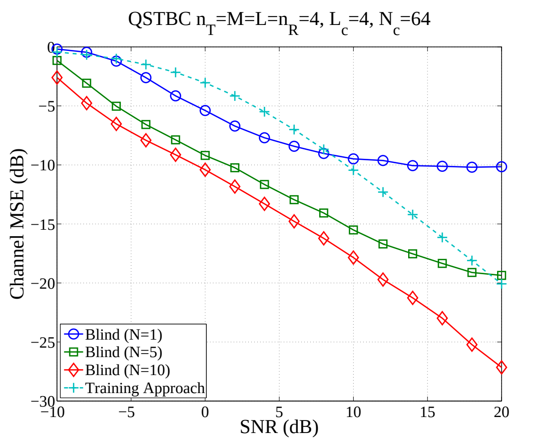

Fig. 6. MSE in the channel estimate for a $R = 1$ QSTBC. $N_c = 64$ and different numbers of available blocks at the receiver.

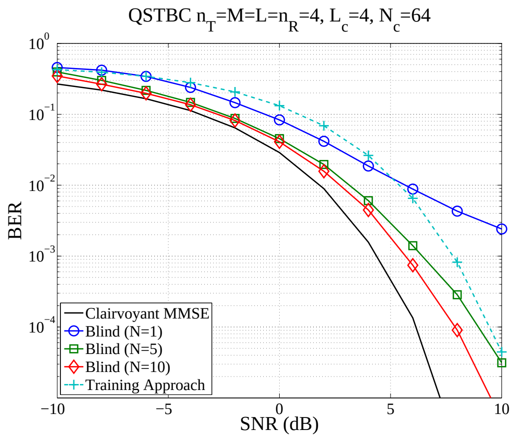

Fig. 7. BER after decoding for a $R = 1$ QSTBC. $N_c = 64$ and different numbers of available blocks at the receiver.

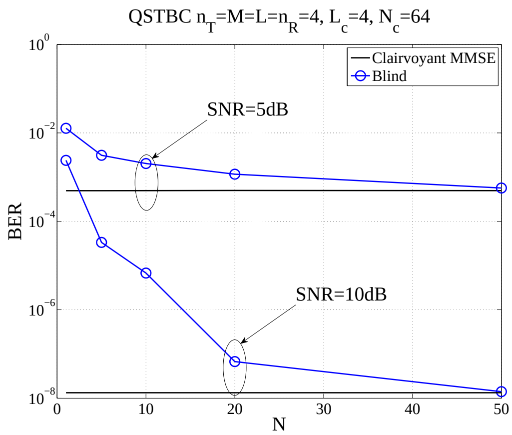

Fig. 8. BER after decoding for a $R = 1$ QSTBC. Effect of N on the BER for two different SNR values. $N_c = 64$.

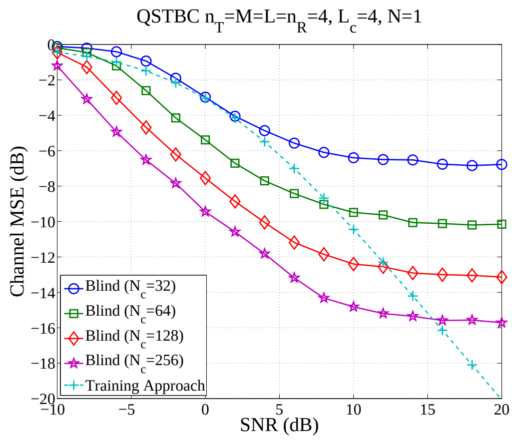

Fig. 9. MSE in the channel estimate for a $R = 1$ QSTBC. $N = 1$ and different numbers of subcarriers.

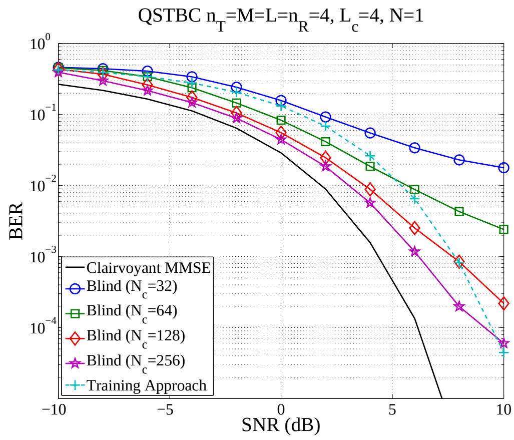

Fig. 10. BER after decoding for a $R = 1$ QSTBC. $N = 1$ and different numbers of subcarriers.

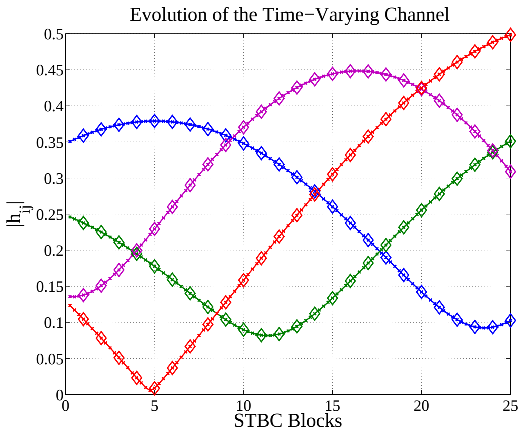

Fig. 11. Evolution of a time-varying channel during 100 symbol periods. $L_c = 5$, $N_c = 128$, $L = 4$, $f_D \simeq 3.9 \cdot 10^{-3}$.

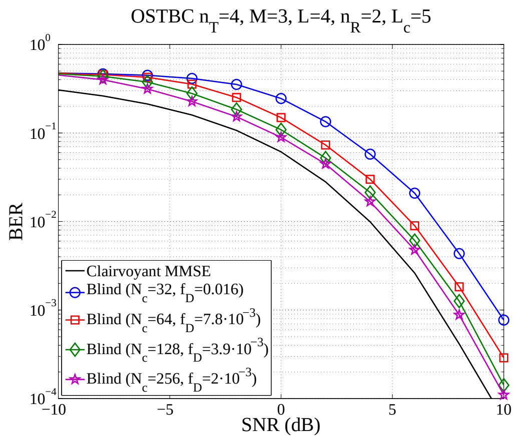

Fig. 12. BER after decoding for a $R = 3/4$ OSTBC. Time varying channels with different Doppler frequencies.

DOI: 10.1002/wcm

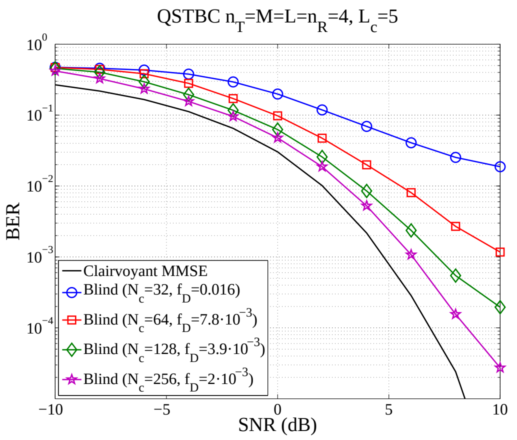

Fig. 13. BER after decoding for the $R = 1$ QSTBC. Time varying channels with different Doppler frequencies.

Table I. Correspondence between the proposed data model and several well-known STBC-based communication systems.

System	Basis $\mathbf{B}$	Precoding $\mathbf{G}$	Other Parameters
STBC-OFDM	$\mathbf{B} = \mathbf{F}_{N_c \times L_c}(0)$	Several options: • No precoding: $\mathbf{G} = \mathbf{I}_{N_c}$. • Minimum MSE [7]: e.g., $\mathbf{G} = \mathbf{F}_{N_c \times N_c}(0)$.	• L OFDM symbols. • N_c subcarriers. • L_c non-zero taps.
SFBC	$\mathbf{B} = \mathbf{F}_{N_c \times L_c}(0)$	Analogous to STBC-OFDM.	• 1 OFDM symbol. • $N_c L$ subcarriers. • L_c non-zero taps.
Time-Reversal STBC	$\mathbf{B} = \mathbf{F}_{N_c \times L_c}\left(\frac{1}{2N_c}\right)$	$\mathbf{G} = \mathbf{F}_{N_c \times N_c}\left(\frac{1}{2N_c}\right)$	• Blocks of length $N_c L$. • L_c non-zero taps.
Time-Varying	Several options: • Fourier Basis. • Discrete prolate spheroidal sequences [56].	Analogous to STBC-OFDM.	• $N_c L$ channel uses. • Maximum Doppler: $\frac{f_D}{f_s} = \frac{L_c - 1}{2LN_c}$.
Doubly-Selective	$\mathbf{B}$ is the Kronecker product of the time and frequency basis.	Several options: • No precoding. • Precoding in time and/or frequency.	This is a combination of the previous cases.

DOI: 10.1002/wcm

Collect N consecutive observation vectors $\tilde{\mathbf{y}}_i[n]$, for $i = 1, \ldots, N_c$ and $n = 0, \ldots, N-1$.
Obtain the estimates of the correlation matrices $\mathbf{R}_{\tilde{\mathbf{y}}_i}$ with (23).
Obtain the matrices $\mathbf{\Phi}_{\tilde{\mathbf{y}}_i}$ and signal energies E_i form the EV decomposition of $\mathbf{R}_{\tilde{\mathbf{y}}_i}$.
Using the code matrices $\tilde{\mathbf{D}}_{i,k}$, obtain $\mathbf{\Xi}_i$ and $\mathbf{\Psi}_i$ with (30) and (31).
Using the BEM, obtain $\mathbf{\Xi}$ and $\mathbf{\Psi}$ with (36) and (37).
Obtain the channel estimate as the principal eigenvector of the GEV in (38).

Algorithm 1: Summary of the proposed blind channel estimation algorithm.

DOI: 10.1002/wcm