Deterministic CCA-Based Algorithms for Blind Equalization of FIR-MIMO Channels

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***Abstract*—In this paper, a new deterministic technique for blind equalization of multiple-input multiple-output (MIMO) channels is presented. The proposed method relies on the channel order diversity of the finite impulse response-MIMO channel, and it does not make any assumption about the spectra or the finite alphabet property of the sources. Assuming that the lengths of the single-input multiple-output channels are known or estimated *a priori*, it is able to extract the sources up to a scale and rotation matrix ambiguity, which only affects to those sources associated to single-input multiple-output (SIMO) channels with equal lengths. Unlike previously proposed techniques, the method described in this paper obtains the equalizers and the best linear combination of their outputs in a single step and, thus, optimally. The reformulation of the proposed method as a set of nested canonical correlation analysis problems is exploited to obtain efficient batch and adaptive equalization algorithms. Finally, the performance of the proposed algorithms is evaluated by means of some simulation examples.**

***Index Terms*—Blind equalization, blind source separation (BSS), canonical correlation analysis (CCA), channel order diversity, convolutive mixtures, multiple-input multiple-output (MIMO).**

I. Introduction

BLIND equalization of finite impulse response (FIR) multiple-input multiple-output (MIMO) channels is a common problem encountered in wireless and mobile communications. Although a number of methods based on higher-order statistics (HOS) have been successfully applied [1]–[4], it is well known that, under mild assumptions on the source signals and on the FIR channels, second-order statistics (SOS) are sufficient for blind equalization [4], [5].

Blind equalization methods can be divided into indirect approaches [6]–[11], if they are based on the previous estimate of the MIMO channel, or direct approaches [12]–[22], which can be used in adaptive environments avoiding the process of channel inversion. Additionally, blind channel equalization methods can be considered as stochastic or deterministic techniques. The stochastic methods are based on the estimate of the correlation matrices of the observations and typically make some assumptions about the input, such as source signals with power spectra which are either known [1], [8], [11], [16]–[19], [22]–[27], or different [9], [28], [29]. This implies that the stochastic methods need a relatively large number of observations to get accurate estimates. On the other hand, the deterministic techniques are solely based on the subspace decomposition of the received data matrices and, in the absence of noise, they are able to obtain exact estimates within a finite number of observations. Unfortunately, most of the SOS-based deterministic techniques have been proposed only for SIMO channels [6], [15], [30]–[33].

In this paper, the problem of SOS-based deterministic direct blind equalization of MIMO channels is considered. The proposed criterion is based on the property of channel order diversity, which has recently been exploited to derive blind MIMO channel estimation [10], [34] and equalization techniques [35], [36], without making any assumption about the spectra of the sources. Specifically, assuming that the SIMO channel orders are known or estimated *a priori* [25], [34], the proposed criterion is able to extract the sources with the only ambiguity of a scale factor and a rotation matrix affecting those signals distorted by SIMO channels of the same order. As a consequence, the proposed technique extends the results in [35], where only the signals affected by the longest SIMO channel were extracted. If several SIMO channels of exactly the same order appear (which is a very unlikely situation in practice), the remaining ambiguity could be solved by resorting to the HOS of the sources. Finally, for those SIMO channels whose length is different from the remaining ones, the proposed criterion extracts the associated source signal with the only ambiguity of a scale factor.

The solution of the proposed criterion is given by a set of nested canonical correlation analysis (CCA) problems. CCA is a well-known technique in multivariate statistical analysis to find maximally correlated projections between two data sets, and it has been widely used in communications and statistical signal processing problems [37]–[41].

The proposed technique is equivalent to the maximum variance (MAXVAR) generalization of CCA to several data sets [42]–[44]. The CCA-MAXVAR method was proposed by Kettenring [42] and it is closely related to principal component analysis (PCA). The solution of the CCA-MAXVAR problem can be directly obtained from a generalized eigenvalue (GEV) problem [45], [46]. However, the reformulation of CCA as a set of coupled least squares (LS) regression problems has recently been exploited to derive efficient batch and adaptive CCA algorithms [45], [46]. A direct application of these CCA algorithms yields simple, and online blind MIMO equalization algorithms.

The deterministic nature of the proposed algorithms allows us to exactly recover the signals in the absence of noise within a finite number of observations. Furthermore, unlike previous techniques [12]–[15], the proposed method obtains, in a single

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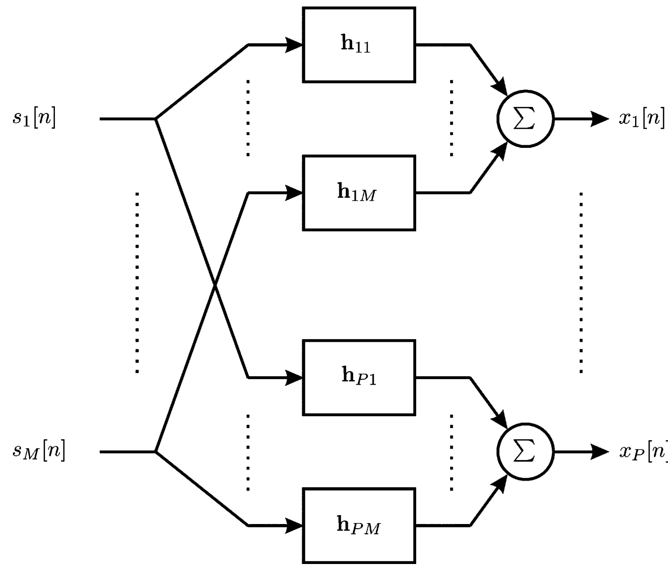

Fig. 1. MIMO system.

step, the set of equalizers and the best combination of their outputs with respect to a PCA criterion. This property is directly related with the interpretation of the obtained equalizers as estimates of the minimum mean square error (MMSE) solutions, unlike the typical zero-forcing (ZF) equalizers obtained in other SIMO [12]–[15], [22] and MIMO [17], [20] equalization techniques. Thus, the proposed method avoids the noise enhancement problem commonly associated to the ZF solutions.

The paper is structured as follows. The problem of blind equalization of MIMO systems and the main assumptions are presented in Section II. The proposed blind equalization criterion is introduced in Section III and its equivalence to a set of nested CCA problems is pointed out in Section IV. In Section V, the proposed blind MIMO equalization procedure is outlined and its relationship with previously proposed techniques is discussed. Finally, the performance of the proposed algorithms is evaluated in Section VI by means of some simulations, and the main conclusions are summarized in Section VII.

II. Blind Equalization of MIMO Channels

A. Notation and Data Model

Throughout this paper, we will use bold-faced upper case letters to denote matrices, e.g., $\mathbf{X}$, with elements x_{ji}; bold-faced lower case letters for column vector, e.g., $\mathbf{x}$, and light-faced lower case letters for scalar quantities. The superscripts $(\cdot)^T$ and $(\cdot)^H$ denote transpose and Hermitian, respectively. The superscript $\hat{(\cdot)}$ will denote estimated matrices, vectors or scalars. The Frobenius norm of a matrix $\mathbf{A}$ will be denoted as $\|\mathbf{A}\|$. $\mathbf{I}$ and $\mathbf{0}$ will denote the identity and zero matrices of the required dimensions. Finally, $*$ will denote the convolution operator.

Suppose the noise free system shown in Fig. 1, where the signals $x_1[n], \dots, x_P[n]$, are the outputs of an unknown M-input/P-output FIR system driven by M signals $s_1[n], \dots, s_M[n]$. Assuming an FIR-MIMO channel of length L, the input-output relationship can be expressed as

$$\mathbf{x}[n] = \mathbf{H}[n] * \mathbf{s}[n] = \sum_{l=0}^{L-1} \mathbf{H}[l]\mathbf{s}[n-l]$$

where $\mathbf{x}[n] = [x_1[n], \dots, x_P[n]]^T$ is the vector of observations, $\mathbf{s}[n] = [s_1[n], \dots, s_M[n]]^T$ is the input signal and $\mathbf{H}[n] = [\mathbf{h}_1[n] \cdots \mathbf{h}_M[n]]$ is the discrete-time channel matrix defined as

$$\mathbf{H}[n] = \begin{bmatrix} h_{11}[n] & \cdots & h_{1M}[n] \\ \vdots & \ddots & \vdots \\ h_{P1}[n] & \cdots & h_{PM}[n] \end{bmatrix}$$

where $h_{ji}[n]$, $n = 0, \dots, L-1$, denotes the channel response between the ith transmit and jth receive antennas. The transfer function associated to the above model is

$$\mathbf{H}(z) = \sum_{l=0}^{L-1} \mathbf{H}[l] z^{-l}.$$

Stacking K successive observations into the vector $\tilde{\mathbf{x}}[n] = [\mathbf{x}^T[n], \dots, \mathbf{x}^T[n-K+1]]^T$, we obtain

$$\tilde{\mathbf{x}}[n] = \sum_{i=1}^{M} \mathcal{T}(\mathbf{h}_i)\tilde{\mathbf{s}}_i[n] \tag{1}$$

where $\mathbf{h}_i$ denotes the FIR-SIMO channel of length L_i associated to the ith source signal, $\tilde{\mathbf{s}}_i[n] = [s_i[n], \dots, s_i[n-K-L_i+2]]^T$, and

$$\mathcal{T}(\mathbf{h}_i) = \begin{bmatrix} \mathbf{h}_i[0] & \cdots & \mathbf{h}_i[L_i-1] & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{h}_i[0] & \cdots & \mathbf{h}_i[L_i-1] \end{bmatrix}$$

is the $PK \times (K + L_i - 1)$ filtering matrix associated to the ith FIR-SIMO channel. Equation (1) can be expressed in a more compact form as

$$\tilde{\mathbf{x}}[n] = \mathcal{T}(\mathbf{H})\tilde{\mathbf{s}}[n] \tag{2}$$

where $\tilde{\mathbf{s}}[n] = [\tilde{\mathbf{s}}_1^T[n], \dots, \tilde{\mathbf{s}}_M^T[n]]^T$, and $\mathcal{T}(\mathbf{H}) = [\mathcal{T}(\mathbf{h}_1) \cdots \mathcal{T}(\mathbf{h}_M)]$ is the MIMO channel filtering matrix.

B. Main Assumptions

The formulation given by (2), which is known as the slide-window formulation [4, Ch. 4], has typically been used to develop different blind identification/equalization algorithms. Common assumptions or constraints imposed to eliminate the inherent ambiguities of the problem in the FIR-MIMO case are the following:

- full column rank of the filtering matrix $\mathcal{T}(\mathbf{H})$: [7], [10]–[15], [18]–[22], [28], [33], [47], [48];
- spatially uncorrelated and white source signals: [1], [8], [11], [16], [17], [22]–[27];
- spatially uncorrelated source signals with known power spectra: [18], [19];
- spatially uncorrelated source signals with different power spectra: [9], [28], [29];
- previous knowledge (at least partially) of the HOS of the sources: [1]–[3], [23], [33], [48].

Unlike the previous assumptions, in this work, we resort to the channel order diversity of the MIMO channel (i.e., different SIMO channel orders). The channel order diversity has recently been exploited in [10], [11], [34], [35]. However, most of these techniques are stochastic approaches that only consider the identification problem [10], [11], [34]. Blind equalization exploiting the channel order diversity is studied in [35], although this work only considers the extraction of the signals affected by the longest SIMO channel. Moreover, none of these techniques are suited for adaptive implementations. In this paper, the channel order diversity is exploited to obtain deterministic batch and adaptive blind equalization algorithms for FIR-MIMO channels.

Specifically, the assumptions of the proposed criterion are the following.

Condition 1 (MIMO Channel): The MIMO channel filtering matrix $\mathcal{T}(\mathbf{H})$ is full column rank.

Condition 2 (Source signals): The source signals are spatially uncorrelated and, for some finite N, the matrices

$$\mathbf{S}_i^{(r)}[m]=\begin{bmatrix} s_i[m] & s_i[m-1] & \cdots & s_i[m-r+1] \\ s_i[m+1] & s_i[m] & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ s_i[m+N-1] & \cdots & \cdots & s_i[m+N-r] \end{bmatrix} \quad (3)$$

are full column rank for $i=1,\ldots,M$, $r=K+L_i$, and $m=n+1,\ldots,n+K+L-2$.

Condition 3 (SIMO Channel lengths): The SIMO channel lengths $L_1,\ldots,L_M$ are known (or estimated) *a priori*.

It can be proved [4, Ch. 4] that, in order to satisfy Condition 1, the MIMO channel $\mathbf{H}(z)$ must be *irreducible* and *column reduced*. Furthermore, it can be directly deduced that, in order to satisfy Condition 1, the $PK \times \sum_{i=1}^{M}(K+L_i-1)$ MIMO channel filtering matrix $\mathcal{T}(\mathbf{H})$ must be a tall matrix, i.e.,

$$K \geq \frac{\sum_{i=1}^{M} L_i - M}{P-M}$$

which constitutes a necessary condition. Finally, the following sufficient condition on the equalizer length is given in [49] and [50] (see also [24])

$$K \geq \sum_{i=1}^{M} L_i - M.$$

Regarding Condition 2, the full rank property of the matrices defined in (3) establishes a *persistent excitation* condition. This condition is easily satisfied, and it is assumed by most of the blind channel identification/equalization techniques. For instance, taking into account the definition of *linear complexity* given in [6], the full rank condition is satisfied if and only if the *linear complexity* of the finite sequences $s_i[m-r+1],\ldots,s_i[m+N-1]$ is greater or equal than $K+L_i$, for $i=1,\ldots,M$, and $m=n+1,\ldots,n+K+L-2$.

Two final comments regarding Condition 3 are now in order. First, this assumption makes necessary to estimate in advance the *effective* orders [51] of the SIMO channels composing the MIMO system. Analogously to the large number of contributions for estimating the order of a SIMO system [26], [31], [47], [51], the recent interest in exploiting the channel order diversity has motivated the development of some techniques for blind MIMO order estimation [25], [34] that can be used for this purpose. Second, we must point out that multiple sources affected by channels of the same length can only be estimated up to a scale and rotation (unitary) matrix. To solve this situation, we could exploit some properties of the sources, such as their belonging to a finite alphabet or their spectral properties.

III. Proposed Equalization Criterion

A. Preliminaries

Based on Condition 1, it can be easily proved that there exists a set of M matrices $\mathbf{W}_i$ of size $PK \times Q_i$, with $Q_i = K+L_i-1$, such that

$$[\mathbf{W}_1 \cdots \mathbf{W}_M]^T \mathcal{T}(\mathbf{H}) = \mathbf{W}^T \mathcal{T}(\mathbf{H}) = \mathbf{I}.$$

Denoting the kth column of $\mathbf{W}_i$ as $\mathbf{w}_{ik}$, we can write, for $i=1,\ldots,M$ and $k=1,\ldots,Q_i$

$$\mathbf{w}_{ik}^T \tilde{\mathbf{x}}[n+k-1] = s_i[n]$$

i.e., the columns of the left-inverse of $\mathcal{T}(\mathbf{H})$ provide a set of zero-forcing (ZF) equalizers with different delays for the M source signals. This implies

$$\mathbf{w}_{ik}^T \tilde{\mathbf{x}}[n+k] = \mathbf{w}_{il}^T \tilde{\mathbf{x}}[n+l], \quad k,l=1,\ldots,Q_i \quad (4)$$

which constitutes the basis of the proposed technique. However, we must note that the converse is only true in the case of SIMO channels. In general, there exist additional solutions of (4) which are not directly given by the columns of the left-inverse of $\mathcal{T}(\mathbf{H})$, as we will prove later.

The formulation in (4) has already been exploited in the derivation of direct blind ZF equalizers in the case of SIMO channels [12]–[15]. Typically, the difference among the equalizer outputs is considered as a cost function, and in order to avoid the trivial solution, a unit norm [13]–[15] or a linear constraint [12] on the equalizer coefficients is applied. The main difference among the above referred methods relies on the selection of the cost function. For instance, the difference between the outputs of consecutive equalizers ($\mathbf{w}_{ik}$ and $\mathbf{w}_{i(k+1)}$) is considered in [12] and [13], whereas the techniques proposed in [14] and [15] are based on the distance between the outputs of the equalizer with the minimum delay ($\mathbf{w}_{i1}$) and the remaining ones. Finally, a solution taking into account all possible combinations of equalizer outputs is also presented in [13]. Another interesting difference consists on the selection of the best equalizer or the best combination of equalizers that provide the estimated source signals. Typically, this problem is solved in a second step. The selection of an equalizer with a moderate delay (for instance $\mathbf{w}_{i(Q_i/2)}$) has been considered in [12]. In [14] and [15] (see also [22]) the authors propose a method to find the linear combination of the ZF equalizers providing a MMSE estimate. Finally, in [21], the authors propose to obtain the MMSE equalizers from the estimated ZF solutions in a second step.

B. Proposed Criterion

The main contributions of the proposed method are the following.

- The equalizers and the best linear combination of their outputs, with respect to a PCA criterion, are obtained in a single step and, thus, optimally.
- Unlike the techniques in [12]–[15], the differences among the outputs of all the equalizers are considered.

- The proposed method is derived from a deterministic framework. In the absence of noise, it is able to exactly recover the sources within a finite number of observations.
- The constraint on the equalizer coefficients is replaced by a constraint on the energy of the equalized signals. This constraint reduces the noise enhancement problem, especially in the case of colored signals or a small number of observations [46].
- It can be proved that the obtained equalizers can be interpreted as an estimate of the MMSE equalizers, unlike the ZF solutions obtained in [12]–[15] and [22].

In order to introduce the proposed criterion, let us start by considering, without loss of generality, $L = L_1 \geq, \dots, \geq L_M$. Remember that Q_i is the number of equalizers that allows us to extract delayed versions of the ith source, which is distorted by a SIMO channel of length L_i (i.e., $Q_i = K + L_i - 1$). Then, considering a block of $Q_1 + N - 1$ observation vectors $\tilde{\mathbf{x}}[n], \dots, \tilde{\mathbf{x}}[n+Q_1+N-2]$, we can define, for $i = 1, \dots, M$, $j = 1, \dots, i$ and $k = 1, \dots, Q_1$, the following matrices:

$$\mathbf{X}_k[n] = \underbrace{[\tilde{\mathbf{x}}[n+k-1]\ \ \tilde{\mathbf{x}}[n+k]\ \ \cdots\ \ \tilde{\mathbf{x}}[n+k+N-2]]^T}_{N \times PK}$$

$$\mathbf{S}_i^j[n] = \underbrace{[\mathbf{s}_j[n]\ \ \mathbf{s}_j[n-1]\ \ \cdots\ \ \mathbf{s}_j[n-L_j+L_i]]}_{N \times (L_j - L_i + 1)}$$

$$\mathbf{s}_i[n] = \underbrace{[s_i[n], s_i[n+1], \dots, s_i[n+N-1]]^T}_{N \times 1}.$$

Based on these definitions, (4) can be rewritten as

$$\mathbf{X}_k[n]\mathbf{w}_{ik} = \mathbf{X}_l[n]\mathbf{w}_{il}, \quad k, l = 1, \dots, Q_i. \tag{5}$$

Let us now introduce the following theorem.

Theorem 1: If $L_i > L_{i+1}$ (or $i = M$), and the matrices

$$\mathcal{S}[n+k] = \left[\mathbf{S}_1^{(Q_1+1)}[n+k] \cdots \mathbf{S}_M^{(Q_M+1)}[n+k]\right]$$

are full column rank for $k = 1, \dots, Q_1 - 1$, and some N, then the set of equalizers $\mathbf{w}_{ik}$ $(k = 1, \dots, Q_i)$ is a solution of (5) iff

$$\mathbf{X}_k[n]\mathbf{w}_{ik} = \sum_{j=1}^{i} \mathbf{S}_i^j[n]\mathbf{a}_i^j$$

where $\mathbf{a}_i^j$ is an arbitrary vector of size $L_j - L_i + 1$, for $j = 1, \dots, i$.

Proof: See Appendix I. ■

Remark 1: Under Condition 2, the matrices $\mathbf{S}_i^{(Q_i+1)}[n+k]$ defined in (3) are full column rank. Furthermore, the condition of spatially uncorrelated sources implies that, for some finite number of observations N, the matrices $\mathcal{S}[n+k]$ are full column rank with probability one, and then Theorem 1 applies.

Considering that there exists a subset of M_i FIR-SIMO channels with length L_i, the implications of Theorem 1 are the following.

- The solutions $\mathbf{w}_i = [\mathbf{w}_{i1}^T, \dots, \mathbf{w}_{iQ_i}^T]^T$ of (5) are given by a subspace of dimension $M_i + M_i^{\perp}$, where
$$M_i^{\perp} = \sum_{j=1}^{i-M_i} (L_j - L_i + 1).$$
- The effect of the signals distorted by SIMO channels with length $L_j < L_i$ is cancelled.
- The interference among the M_i input signals distorted by SIMO channels of length L_i is reduced to M_i instantaneous mixtures, i.e., the ISI is eliminated.
- The interference of signals affected by longer SIMO channels $(L_j > L_i)$ is still present, but the interfering signals are distorted by equivalent FIR-SIMO channels of reduced length: $L_j - L_i$.

Several alternatives to cancel the previously extracted signals can be applied. Here, we will use a deflation procedure based on the assumption of spatially uncorrelated sources, which implies the asymptotic orthogonality $(N \to \infty)$ among the matrices $\mathbf{S}_i^j[n]$, $j = 1, \dots, i$, i.e.,

$$\lim_{N\to\infty} \frac{1}{N} \mathbf{S}_i^{jH}[n]\mathbf{S}_i^m[n] = \mathbf{0}, \quad j \neq m$$

and then the proposed technique is implicitly assuming that $\mathbf{S}_i^{jH}[n]\mathbf{S}_i^m[n] = \mathbf{0}$.

Finally, denoting the desired equalizers as $\mathbf{v}_{ik}$, for $i = 1, \dots, M$, $k = 1, \dots, Q_i$, the problem that we solve in this paper can be stated as follows.

> For $i = 1, \dots, M$, find the equalizers $\mathbf{v}_{ik}$ $(k = 1, \dots, Q_i)$ producing the unit-energy equalized signals $(\mathbf{y}_{ik}[n] = \mathbf{X}_k[n]\mathbf{v}_{ik})$ which admit the best 1-D PCA representation $\hat{\mathbf{s}}_i[n]$ uncorrelated with the previous estimates $\hat{\mathbf{s}}_j[n]$, $j = 1, \dots, i-1$, and their delayed versions.

In other words, we propose to measure the difference among the equalizer outputs by means of a PCA criterion, i.e., the difference among the equalized signals is given by the residual error of a unidimensional PCA representation. From a geometrical point of view, we are looking for the projectors $\mathbf{v}_{ik}$ providing a set of most similar unit-norm (or unit-variance) projections $\mathbf{y}_{ik}[n]$. To quantify precisely this similarity among the projections, we require that they can be projected onto a new common vector $\hat{\mathbf{s}}_i[n]$ retaining as much energy as possible. Furthermore, the vector $\hat{\mathbf{s}}_i[n]$ must be orthogonal to previously obtained vectors $\hat{\mathbf{s}}_p[n]$, $p < i$. In the next section, it is proved that the solution of this problem can be obtained by means of a set of nested CCA problems.

IV. Equivalence to Canonical Correlation Analysis

CCA is a well-known technique in multivariate statistical analysis to find maximally correlated projections between two data sets. CCA was developed by Hotelling [52] and it has been widely used in economics, meteorology and in many modern information processing fields, such as communication problems [53]–[55], statistical signal processing [37]–[41], [56], independent component analysis [57], and blind source separation (BSS) [58].

Although CCA was originally proposed for two data sets, it has been generalized to the case of several data sets. Specifically, the MAXVAR generalization proposed by Kettenring [42] solves the problem of finding the projections that can be best approximated by a 1-D PCA model. This generalization preserves the property of invariance under uncoupled nonsingular transformations [42], and for Gaussian data gives a measure of the mutual information among the data sets [57, App. A].

In this section, we prove that the solution associated to the proposed blind equalization criterion is given by a set of nested CCA-MAXVAR problems, and we propose batch and adaptive algorithms based on the reformulation of CCA as a set of coupled LS regression problems.

A. Solution of the Proposed Problem

Taking into account Theorem 1, the criterion for the obtention of the $M_i + M_i^{\perp}$ linearly independent solutions of (5) can be stated as the problem of successively $(r = 1, \ldots, M_i + M_i^{\perp})$ finding a set of equalizers $\mathbf{v}_{ik}^{(r)}$ $(k = 1, \ldots, Q_i)$ and the corresponding outputs $\mathbf{y}_{ik}^{(r)}[n] = \mathbf{X}_k[n]\mathbf{v}_{ik}^{(r)}$, which admit the best possible 1-D PCA representation $\mathbf{z}_i^{(r)}[n]$ and subject to the constraints $\|\mathbf{y}_{ik}^{(r)}[n]\| = 1$ for $k = 1, \ldots, Q_i$ and $\mathbf{z}_i^{(r)H}[n]\mathbf{z}_i^{(p)}[n] = 0$ for $p = 1, \ldots, r-1$. The cost function to be minimized in terms of $\mathbf{v}_i^{(r)} = [\mathbf{v}_{i1}^{(r)T}, \ldots, \mathbf{v}_{iQ_i}^{(r)T}]^T$ is

$$J\left(\mathbf{v}_i^{(r)}\right) = \min_{\mathbf{z}_i^{(r)}[n], \mathbf{a}_i^{(r)}} \frac{1}{Q_i} \sum_{k=1}^{Q_i} \left\| \mathbf{z}_i^{(r)}[n] - a_{ik}^{(r)} \mathbf{y}_{ik}^{(r)}[n] \right\|^2 \quad (6)$$

where $\mathbf{a}_i^{(r)} = [a_{i1}^{(r)}, \ldots, a_{iQ_i}^{(r)}]^T$ is the vector containing the weights for the best combination of the outputs. In order to avoid the trivial solution ($\mathbf{a}_i^{(r)} = \mathbf{0}$, $\mathbf{z}_i^{(r)}[n] = \mathbf{0}$), the energy of $\mathbf{z}_i^{(r)}[n]$ or $\mathbf{a}_i^{(r)}$ has to be constrained to some value, for instance $\|\mathbf{a}_i^{(r)}\|^2 = Q_i$ [45], [46].

Taking the derivative of (6) with respect to $\mathbf{z}_i^{(r)}[n]$ and equating to zero, we get

$$\mathbf{z}_i^{(r)}[n] = \frac{1}{Q_i} \mathbf{Y}_i^{(r)}[n] \mathbf{a}_i^{(r)} \quad (7)$$

where $\mathbf{Y}_i^{(r)}[n] = [\mathbf{y}_{i1}^{(r)}[n] \cdots \mathbf{y}_{iQ_i}^{(r)}[n]]$. Now, substituting (7) into (6), the cost function becomes

$$J\left(\mathbf{v}_i^{(r)}\right) = \min_{\mathbf{a}_i^{(r)}} \left(1 - \frac{\mathbf{a}_i^{(r)H} \mathbf{Y}_i^{(r)H}[n] \mathbf{Y}_i^{(r)}[n] \mathbf{a}_i^{(r)}}{Q_i^2} \right) = 1 - \beta_i^{(r)} \quad (8)$$

where $0 \leq \beta_i^{(r)} \leq 1$ is an eigenvalue of $\mathbf{Y}_i^{(r)H}[n]\mathbf{Y}_i^{(r)}[n]/Q_i$ (which depends on $\mathbf{v}_i^{(r)}$) and $\mathbf{a}_i^{(r)}$ is the associated eigenvector scaled to $\|\mathbf{a}_i^{(r)}\|^2 = Q_i$.

Here we must point out that, in the case of the first CCA solution $(r = 1)$, (8) is the classical PCA problem and the solution is given by the eigenvector $\mathbf{a}_i^{(1)}$ associated to the largest eigenvalue $\beta_i^{(1)}$. However, in the general case $r \neq 1$, the orthogonality constraints $\mathbf{z}_i^{(r)H}[n]\mathbf{z}_i^{(p)}[n] = 0$ have the effect of modifying the PCA problem, which implies that the solution $\mathbf{a}_i^{(r)}$ is not necessarily the eigenvector associated to the largest eigenvalue $\beta_i^{(r)}$, i.e., the subsequent PCA problems are not independent.

Interestingly, the above problem is the CCA-MAXVAR generalization to several data sets $(\mathbf{X}_1[n], \ldots, \mathbf{X}_{Q_i}[n])$ proposed in [42]. In [45] and [46], we have proved that the CCA-MAXVAR solutions can be directly obtained from a GEV problem. For completeness, this result is also included here.

Theorem 2: The solutions of the CCA-MAXVAR generalization of (8) can be obtained solving the following GEV problem

$$\frac{1}{Q_i} \mathbf{R}_i \mathbf{w}_i^{(r)} = \beta_i^{(r)} \mathbf{D}_i \mathbf{w}_i^{(r)} \quad (9)$$

where $\mathbf{w}_i^{(r)} = [\mathbf{w}_{i1}^{(r)T}, \ldots, \mathbf{w}_{iQ_i}^{(r)T}]^T$, $\mathbf{w}_{ik}^{(r)} = \mathbf{v}_{ik}^{(r)} a_{ik}^{(r)}$ are the canonical vectors, the matrices $\mathbf{R}_i$ and $\mathbf{D}_i$ are defined as

$$\mathbf{R}_i = \begin{bmatrix} \mathbf{R}_{11} & \cdots & \mathbf{R}_{1Q_i} \\ \vdots & \ddots & \vdots \\ \mathbf{R}_{Q_i 1} & \cdots & \mathbf{R}_{Q_i Q_i} \end{bmatrix}, \ \mathbf{D}_i = \begin{bmatrix} \mathbf{R}_{11} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{R}_{Q_i Q_i} \end{bmatrix} \quad (10)$$

and $\mathbf{R}_{kl} = \mathbf{X}_k^H[n]\mathbf{X}_l[n]$ are the estimates of the crosscorrelation matrices. The canonical variates are defined as $\mathbf{z}_{ik}^{(r)}[n] = \mathbf{X}_k[n]\mathbf{w}_{ik}^{(r)} = a_{ik}^{(r)}\mathbf{y}_{ik}^{(r)}[n]$, and $\mathbf{z}_i^{(r)}[n] = 1/Q_i \sum_{k=1}^{Q_i} \mathbf{z}_{ik}^{(r)}[n]$ is the best 1-D PCA approximation. Finally, the generalized canonical correlations $-1/(Q_i - 1) \leq \rho_i^{(r)} \leq 1$ are defined as

$$\rho_i^{(r)} = \frac{\beta_i^{(r)} Q_i - 1}{Q_i - 1}.$$

Proof: See Appendix II. ■

B. CCA Algorithms

In [45] and [46], the CCA-MAXVAR problem has been formulated in a more intuitive manner. Specifically, if we define the successive canonical vectors (equalizers) and variates (equalized signals) as $\mathbf{w}_{ik}^{(r)}$ and $\mathbf{z}_{ik}^{(r)}[n] = \mathbf{X}_k[n]\mathbf{w}_{ik}^{(r)}$, respectively, the MAXVAR generalization can be defined as the problem of sequentially maximizing the generalized canonical correlation

$$\rho_i^{(r)} = \frac{1}{Q_i(Q_i - 1)} \sum_{\substack{k,l=1 \\ k \neq l}}^{Q_i} \mathbf{w}_{ik}^{(r)H} \mathbf{R}_{kl} \mathbf{w}_{il}^{(r)}$$

subject to the following energy and orthogonality constraints to avoid the trivial solutions:

$$\frac{1}{Q_i} \sum_{k=1}^{Q_i} \mathbf{w}_{ik}^{(r)H} \mathbf{R}_{kk} \mathbf{w}_{ik}^{(r)} = 1$$

$$\mathbf{z}_i^{(r)H}[n] \mathbf{z}_i^{(p)}[n] = 0, \quad r \neq p$$

where $\mathbf{z}_i^{(r)}[n] = (1/Q_i) \sum_{k=1}^{Q_i} \mathbf{z}_{ik}^{(r)}[n]$.

Furthermore, we have shown that the GEV problem (9) can be rewritten as a set of coupled LS regression problems, whose solutions are obtained by means of the pseudoinverses $\mathbf{X}_k^{+}[n] = (\mathbf{X}_k^H[n]\mathbf{X}_k[n])^{-1}\mathbf{X}_k^H[n]$ of the data matrices $\mathbf{X}_k[n]$ as

$$\beta_i^{(r)} \mathbf{w}_{ik}^{(r)} = \mathbf{X}_k^{+}[n] \mathbf{z}_i^{(r)}[n], \quad k = 1, \ldots, Q_i. \quad (11)$$

This LS regression framework resembles the idea of mutually referenced equalizers, which has been exploited in [12] to derive a blind equalization algorithm for FIR-SIMO channels.

V. Proposed Blind MIMO Equalization Procedure

In this section, the CCA-MAXVAR technique is applied to the blind equalization of FIR-MIMO channels, obtaining efficient batch and adaptive algorithms based on the idea of coupled LS regression problems.

A. Outline of the CCA-Based Algorithms

Let us start by considering $L_1 \geq, \ldots, \geq L_M$, and assuming, without loss of generality, an index i such that $L_i > L_{i+1}$ (or $i = M$). Then, we can introduce the following lemma.

Lemma 1: Under the conditions in Section II-B and in the absence of noise, the largest canonical correlation of the CCA-MAXVAR problem with matrices $\mathbf{X}_1[n], \ldots, \mathbf{X}_{Q_i}[n]$ is equal to one and has multiplicity $M_i + M_i^{\perp}$, and the associated canonical variates $(\mathbf{z}_i^{(1)}[n], \ldots, \mathbf{z}_i^{(M_i+M_i^{\perp})}[n])$, or equalizer outputs, span the subspace defined by

$$\left[\mathbf{S}_i^1[n] \cdots \mathbf{S}_i^i[n]\right].$$

Proof: This is a direct consequence of Theorem 1. ■

Lemma 1 implies that, in the absence of noise, the first $M_i + M_i^{\perp}$ solutions of the CCA-MAXVAR problem with $\mathbf{X}_1[n], \ldots, \mathbf{X}_{Q_i}[n]$ form an orthogonal basis for the desired M_i signals affected by SIMO channels of length L_i, and the $i - M_i$ spurious signals (and their delayed versions) affected by SIMO channels with $L_j > L_i$. When noise is present, the orthogonal basis can be approximately obtained from the main $M_i + M_i^{\perp}$ CCA-MAXVAR solutions, and the associated canonical correlations will be $\rho_i^{(r)} < 1$.

The deflation process to avoid the interference of the spurious signals ($L_j > L_i$) is very simple: we only have to project the $M_i + M_i^{\perp}$ first solutions of our CCA problem onto the complementary subspace to that defined by the previously estimated set of matrices, i.e.,

$$\left[\hat{\mathbf{S}}_i^1[n] \cdots \hat{\mathbf{S}}_i^{i-M_i}[n]\right].$$

This deflation step can be directly incorporated in the CCA algorithms proposed in [45] and [46]. In fact, we only have to include the previously estimated interfering signals in the orthogonality constraints (as if they were the main $M_i^{\perp}$ CCA solutions), and extract the following M_i solutions applying the CCA algorithms based on coupled LS regressions. Thus, the canonical vectors will provide the equalizers for the extraction of the basis of the M_i desired signals. As pointed out in Theorem 2, these equalizers are scaled by the corresponding PCA weights, and then, the best linear combination of the equalizer outputs is directly obtained as the mean.

The batch procedure for the sequential extraction of the source signals is summarized in Algorithm 1, and a direct application of the RLS-based adaptive CCA algorithm presented in [45] and [46] is used in this paper to develop an adaptive version of the MIMO blind equalization technique.[1]

[1]Due to the lack of space, we do not provide details here.

Algorithm 1: Summary of the CCA procedure for blind equalization of FIR MIMO channels.

Initialize $i = 1$ and arrange $L_1 \geq \ldots \geq L_M$.

while $i \leq M$ **do**

Obtain the number of signals to extract M_i and update $i = i + M_i - 1$.

Obtain the number of restrictions to apply $M_i^{\perp} = \sum_{j=1}^{i-M_i}(L_j - L_i + 1)$.

Form the CCA problem with data sets $\mathbf{X}_1[n], \ldots, \mathbf{X}_{Q_i}[n]$.

Assume $[\hat{\mathbf{S}}_i^1[n] \cdots \hat{\mathbf{S}}_i^{i-M_i}[n]]$ as the $M_i^{\perp}$ main solutions $\mathbf{z}_i^{(1)}[n], \ldots, \mathbf{z}_i^{(M_i^{\perp})}[n]$.

Obtain the basis $[\mathbf{z}_i^{(M_i^{\perp}+1)}[n] \cdots \mathbf{z}_i^{(M_i^{\perp}+M_i)}[n]]$ for the next M_i signals $\hat{\mathbf{s}}_{i-M_i+1}[n], \ldots, \hat{\mathbf{s}}_i[n]$.

Update $i = i + 1$.

end while

B. Further Comments and Relationship With Other Techniques

- One of the advantages of the proposed criterion is that the obtained equalizers can be seen as estimates of the MMSE equalizers. In order to clarify this point, let us notice that (11) implies that the obtained equalizers satisfy the following LS regression problems: $\mathbf{w}_{ik} \propto \mathbf{X}_k^{+}[n]\hat{\mathbf{s}}_i[n] = \left(\mathbf{X}_k^H[n]\mathbf{X}_k[n]\right)^{-1}\mathbf{X}_k^H[n]\hat{\mathbf{s}}_i[n]$.

  Taking into account that $\hat{\mathbf{s}}_i[n]$ is an (hopefully accurate) estimate of the source $\mathbf{s}_i[n]$, the above equation can be interpreted as an estimate of the Wiener filter. This result contrasts with the techniques proposed in [12]–[15], and [17], which only consider the obtention of ZF equalizers or their best combination in terms of MMSE[2] [14], [15], [22].
- In [12], the authors have presented an adaptive algorithm for blind equalization of SIMO channels, which is based on a linear constraint on the equalizers and the inversion, by means of the RLS, of a correlation matrix of dimensions $KPQ-1$ (with $Q = K+L-1$). In the case of SIMO channels, the CCA-based technique solves Q_1 regression problems, of size KP. However, taking into account the relationship among the matrices $\mathbf{X}_1[n], \ldots, \mathbf{X}_{Q_1}[n]$, the computational cost is equivalent to a unique RLS problem of size KP. This implies that the computational complexity of the proposed technique is lower than that of [12].
- In [20], the authors have proposed a deterministic technique for blind equalization of FIR-MIMO channels. This technique can be considered as a deterministic generalization to MIMO channels of the methods in [12] and [13]. It assumes SIMO channels with equal lengths and the final

[2]See also [21] for the obtention, in a second step, of the MMSE equalizers from the estimated ZF ones.

TABLE I
EQUALIZATION TEST (EQX) POWER PROFILE

path	1	2	3	4	5	6
delay (μs)	0.0	2.2	4.4	6.6	8.8	11
attenuation (dB)	0	0	0	0	0	0

TABLE II
TYPICAL URBAN (TU) POWER PROFILE

path	1	2	3	4	5	6
delay (μs)	0	0.2	0.4	0.6	0.8	1.2
attenuation (dB)	−4	−3	0	−2	−3	−5
path	7	8	9	10	11	12
delay (μs)	1.4	1.8	2.4	3.0	3.2	5
attenuation (dB)	−7	−5	−6	−9	−11	−10

TABLE III
IMPULSE RESPONSES OF THE MIMO CHANNEL USED IN THE SIMULATION EXAMPLES

i	n	$h_{1i}[n]$	$h_{2i}[n]$	$h_{3i}[n]$	$h_{4i}[n]$
1	0	$0.0956 - 0.3002j$	$-0.0980 - 0.2201j$	$-0.2776 - 0.0478j$	$0.2144 + 0.0747j$
	1	$0.0600 + 0.0803j$	$-0.0173 + 0.1515j$	$0.1830 + 0.0034j$	$0.1270 - 0.1551j$
	2	$0.1543 - 0.4667j$	$-0.2532 - 0.2064j$	$-0.0627 + 0.2967j$	$-0.0182 + 0.2506j$
	3	$-0.0706 - 0.1395j$	$0.1338 + 0.1219j$	$-0.1152 + 0.1564j$	$-0.0394 + 0.0220j$
2	0	$0.0939 - 0.5288j$	$-0.4634 - 0.1178j$	$0.1161 + 0.1043j$	$0.3468 - 0.1314j$
	1	$-0.1292 + 0.1043j$	$-0.2997 - 0.0906j$	$0.2353 - 0.1223j$	$0.0829 - 0.3210j$

extraction of the sources is based on their belonging to a finite alphabet. Another interesting blind equalization technique for MIMO channels is proposed in [35] (see also [36]). This technique is also based on the diversity of the channel order. However, it only considers the extraction of the signals affected by the SIMO channels with the largest order.

- In a realistic scenario, the effective channel orders [51] must be estimated, and, hence, the performance of the proposed technique depends on the accuracy of the channel order estimation algorithms. This dependence is illustrated in the next section by means of a simulation example.

VI. SIMULATION RESULTS

In this section, the performance of the proposed blind MIMO technique is evaluated by means of some numerical examples. In all the simulations the results of 300 independent realizations are averaged. We consider source signals distorted by a SIMO or MIMO channel and corrupted by zero-mean white Gaussian noise.

A. Channel Model

The simulation examples are similar to those in [12]. We assume a digital wireless communication system at 900 MHz with $P = 2$ or $P = 4$ sensors distributed on a uniform circular array of radius $\lambda/4 \approx 8.33$ cm. The propagation model is generated based on the model of Clarke [12], [59] and the two following power profiles are used:

- Equalization Test (EQx) model for highly dispersive channels (see Table I).
- Typical urban (TU) model (see Table II).

The symbols are unit-variance 16-QAM with duration 3.7μs. Squared root raised cosine filters with roll-off 0.5 are used at the transmitter and receiver, and the continuous-time $1 \times P$ SIMO channels are sampled at the baud rate and are normalized to have gain P. Although the theoretical channel lengths (noiseless case) could be considered larger than five taps for both models, in the simulation examples we will consider the effective channel lengths (number of taps that concentrate most of the channel energy [51]) as 4 and 2 for the EQx and TU, respectively. Finally, the central taps of the sampled SIMO channels are shown in Table III.

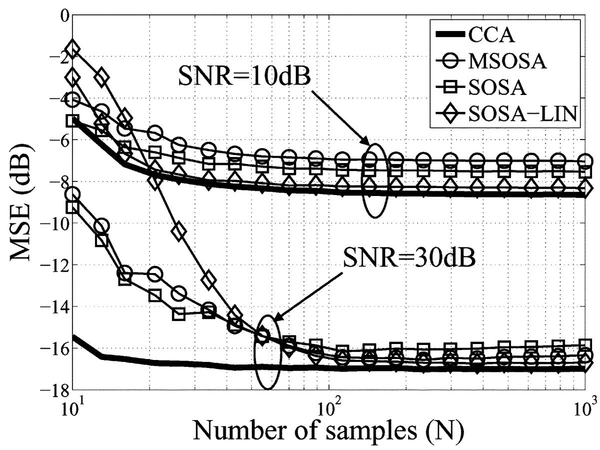

Fig. 2. Performance of the blind SIMO equalization techniques as a function of the number of observations; 1×2 SIMO channel with length $L_2 = 2$. Equalizer length $K = 2$.

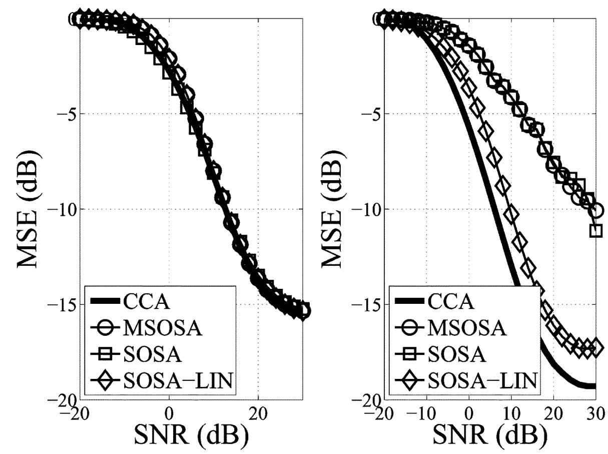

Fig. 3. Effect of increasing the number of receive antennas; (left) 1×2 and (right) 1×4 SIMO channel with length $L_2 = 2$. Number of data samples $N = 1000$. Equalizer length $K = 1$.

B. SIMO Examples

In this section, the proposed method (referred to as CCA) is compared with the second-order statistics algorithm (SOSA) and modified SOSA (MSOSA) proposed in [13], and the technique proposed in [12] (referred to as SOSA-LIN). These techniques impose either a quadratic (SOSA and MSOSA) or a linear constraint (SOSA-LIN) on the equalizer coefficients. Moreover, they do not resolve the problem of finding the best linear combination of the equalizer outputs. For these methods, the final equalized signal has been obtained as the best linear combination of the outputs [14], [15].

In the first example, a source signal is distorted by a 1×2 SIMO channel obtained from antennas 1 and 3 of the second SIMO channel ($i = 2$, $L_2 = 2$) shown in Table III. Low ($\text{SNR} = 30$ dB) and moderate ($\text{SNR} = 10$ dB) noise levels have been considered. Fig. 2 shows the MSE of the equalized signals as a function of the number of available samples N. The deterministic nature of the CCA method and the constraint on the energy of the outputs explain the good performance for high SNR and small number of data samples.

In the second example, we have considered the second SIMO channel with $P = 2$ and $P = 4$ receive antennas. The equalizer length is $K = 1$ and the number of observations is $N = 1000$. Fig. 3 shows the MSE for the equalized signals. Surprisingly, for the SOSA and MSOSA techniques, the performance is degraded when the number of receive antennas is increased. This is due to the fact that for this example there exists a set of $Q_2 = K + L_2 - 1 = 2$ equalizers in the null subspace of $\mathcal{T}(\mathbf{h}_2)$ (which for $P = 4$ and $K = 1$ is of dimension 2). This pair of equalizers cancel the source signal at their outputs. This is translated into a high noise enhancement effect, which is partially avoided by the linear constraint on the equalizers (SOSA-LIN), and completely eliminated when the constraint on the energy of the equalizer outputs is imposed (CCA).

The proposed on-line algorithm (CCA) has been compared with the RLS-based adaptive version of the SOSA-LIN [12], the constant modulus algorithm (CMA) [2], and the RLS with training symbols. Fig. 4 shows the convergence curves for the previous 1×2 SIMO channel with length $L_2 = 2$, and the 1×4 SIMO channel with length $L_1 = 4$. The forgetting factor for the RLS problems is $\lambda = 0.9$ and the signal to noise ratio is $\text{SNR} = 30$ dB. As can be seen, the performance of the SOSA-LIN technique is degraded for problems with moderate complexity ($P = 4$, $L_1 = 4$), even for high forgetting factors ($\lambda = 1$), which is due to the large RLS problem to solve and the possible ill conditioning of the correlation matrix. On the other hand, the proposed CCA technique is faster than the HOS-based CMA.

C. MIMO Examples

In this section, the performance of the CCA technique is evaluated and compared with the deterministic method proposed in [20] (denoted here as MIMO-SOSA) and the MIMO-CMA algorithm [2]. In all the examples we have considered the 2×4 MIMO channel presented in Table III, and the equalizer length parameter has been set to $K = 4$.

In the first example, the performance of the CCA and MIMO-SOSA methods is evaluated. The number of observations is $N = 1000$, and the instantaneous mixture of signals given by the MIMO-SOSA has been resolved using the knowledge of the sources. Fig. 5 shows the MSE of the equalized signals, where we can see that the proposed method outperforms the MIMO-SOSA, which fails to extract the second signal ($L_2 = 2$). This is due to the implicit assumption of SIMO channels of the same length [20]. Furthermore, we must note that the noise floor for the proposed technique is due to the effect of the heading and trailing terms of the channel and not to the deflation process (which only affects to the second signal). Thus, we can conclude that the effect of the deflation technique is irrelevant in realistic cases.

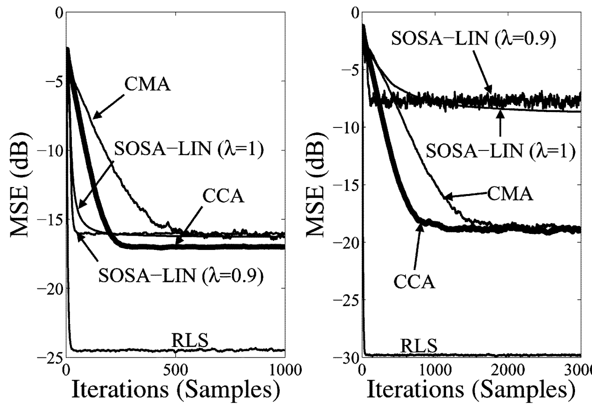

Fig. 4. Convergence of the adaptive algorithms; (left) 1×2 SIMO channel with length $L_2 = 2$ and (right) 1×4 SIMO channel with length $L_1 = 4$. SNR $= 30$ dB, $K = 2$. RLS forgetting factor $\lambda = 0.9$.

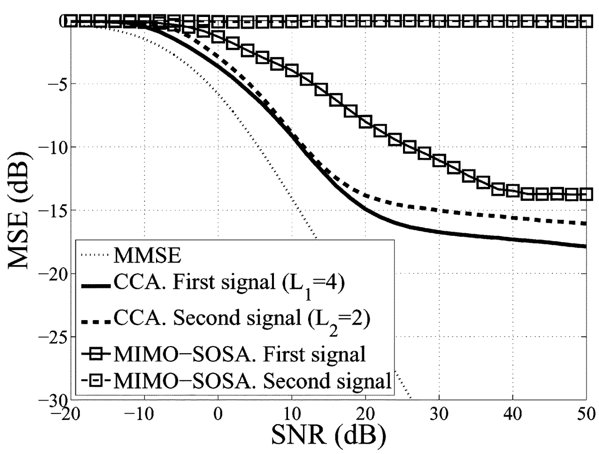

Fig. 5. Blind MIMO equalization algorithms; 2×4 MIMO channel, $N = 1000, K = 4$.

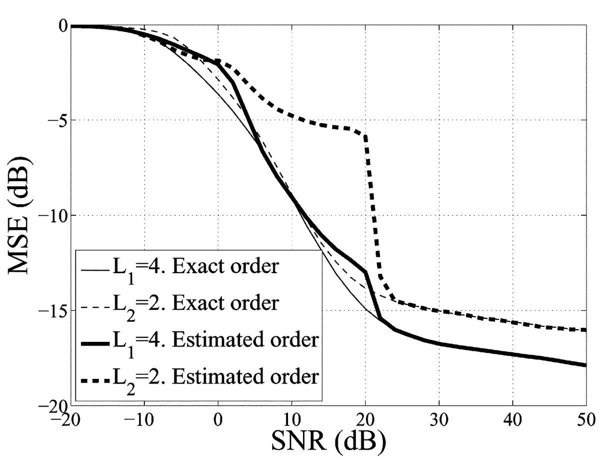

Fig. 6. Blind MIMO equalization algorithm including the channel order estimation; 2×4 MIMO channel, $N = 1000, K = 4$.

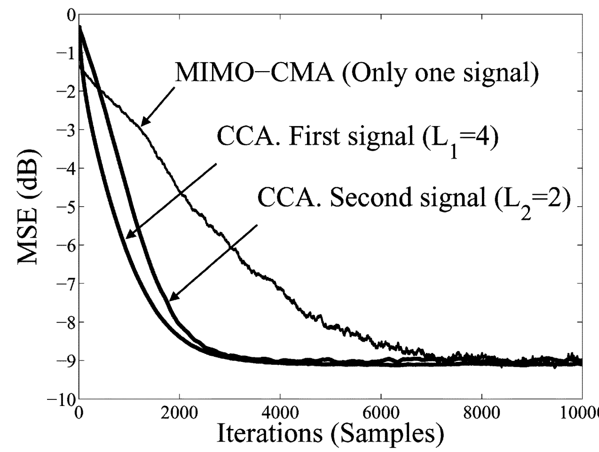

Fig. 7. Adaptive blind MIMO equalization algorithms; 2×4 MIMO channel. RLS forgetting factor $\lambda = 0.99, K = 4$, SNR $= 10$ dB.

In order to evaluate the performance of the proposed technique with estimated channel orders, the previous example has been repeated including the channel order estimation step. The technique proposed in [25] has been applied excluding the denoising step.[3] Fig. 6 shows the simulation results, where we can see that the MSE for the first signal is close to the MSE in the case of known channel order, whereas the second signal is more affected by channel order estimation errors.

Finally, the CCA adaptive algorithm has been compared with the MIMO-CMA algorithm [2]. The RLS forgetting factor has been set to $\lambda = 0.99$, and we have found that the MIMO-CMA fails to extract the second signal. Fig. 7 shows the convergence curves for the extraction of the sources. As can be seen, the proposed criterion is able to extract both sources and it outperforms the MIMO-CMA technique in terms of convergence speed.

VII. Conclusion

In this paper, a new deterministic technique for blind equalization of FIR-MIMO channels has been presented. The proposed method does not impose any constraint on the spectra of the source signals, which is a common assumption for most of the blind algorithms for MIMO channel estimation/equalization. The main assumption of the proposed technique relies on the order diversity of the MIMO channel. Exploiting this property, the proposed method is able to extract the source signals up to a scale and rotation matrix indeterminacy, which only affects to those sources associated to FIR-SIMO channels of the same length. The reformulation of the blind equalization problem as a set of nested CCA problems has been exploited to obtain, simultaneously, the set of equalizers for each source and the best combination of their outputs. Furthermore, it has been proved that the proposed solutions can be interpreted as estimates of the MMSE equalizers, unlike the classical blind zero-forcing solutions. Finally, batch and adaptive algorithms have been obtained by reformulating CCA as a set of coupled LS regression problems. The performance of the proposed algorithms has been shown by means of some simulation examples.

[3]By means of simulations, we have verified that the denoising step does not provide good results for channels with small heading and trailing terms.

Appendix I
Proof of Theorem 1

In this Appendix we show that, if $L_i > L_{i+1}$ (or $i = M$), and the matrices $\mathcal{S}[n+k] = \left[\mathbf{S}_1^{(Q_1+1)}[n+k] \cdots \mathbf{S}_M^{(Q_M+1)}[n+k]\right]$ are full column rank for $k = 1, \ldots, Q_1 - 1$, and some N, then the set of equalizers $\mathbf{w}_{ik}$ ($k = 1, \ldots, Q_i$) is a solution of (5) iff

$$\mathbf{X}_k[n]\mathbf{w}_{ik} = \sum_{j=1}^{i} \mathbf{S}_i^j[n]\mathbf{a}_i^j, \quad k = 1, \ldots, Q_i \tag{12}$$

where $\mathbf{a}_i^j$ is an arbitrary vector of size $L_j - L_i + 1$, for $j = 1, \ldots, i$.

Proof: It is obvious that (12) implies (5). In order to prove that (5) implies (12), let us start by writing

$$\mathbf{X}_k[n] = \sum_{j=1}^{M} \mathbf{S}_j^{(Q_j)}[n+k-1]\mathcal{T}^T(\mathbf{h}_j)$$

and

$$\mathbf{X}_k[n]\mathbf{w}_{ik} = \sum_{j=1}^{M} \mathbf{S}_j^{(Q_j)}[n+k-1]\mathbf{g}_{ik}^j \tag{13}$$

where $\mathbf{g}_{ik}^j = [g_{ik}^j[0], \ldots, g_{ik}^j[Q_j - 1]]^T = \mathcal{T}^T(\mathbf{h}_j)\mathbf{w}_{ik}$ is the composite channel-equalizer response. Then, (5) can be rewritten as

$$\sum_{j=1}^{M} \mathbf{S}_j^{(Q_j)}[n+k-1]\mathbf{g}_{ik}^j = \sum_{j=1}^{M} \mathbf{S}_j^{(Q_j)}[n+l-1]\mathbf{g}_{il}^j$$

for $k, l = 1, \ldots, Q_i$, or equivalently, for $k = 1, \ldots, Q_i - 1$

$$\sum_{j=1}^{M} \mathbf{S}_j^{(Q_j)}[n+k-1]\mathbf{g}_{ik}^j = \sum_{j=1}^{M} \mathbf{S}_j^{(Q_j)}[n+k]\mathbf{g}_{i(k+1)}^j. \tag{14}$$

Taking into account that the first $Q_j - 1$ columns of $\mathbf{S}_j^{(Q_j)}[n+k-1]$ and the last $Q_j - 1$ columns of $\mathbf{S}_j^{(Q_j)}[n+k]$ coincide, (14) can be rewritten as

$$\sum_{j=1}^{M} \mathbf{S}_j^{(Q_j+1)}[n+k]\mathbf{f}_{ik}^j = \mathbf{0}, \quad k = 1, \ldots, Q_i - 1$$

where $\mathbf{f}_{ik}^j = [0, \mathbf{g}_{ik}^{jT}]^T - [\mathbf{g}_{i(k+1)}^{jT}, 0]^T$. Defining now $\mathbf{f}_{ik} = [\mathbf{f}_{ik}^{1T}, \ldots, \mathbf{f}_{ik}^{MT}]^T$ the above equality yields

$$\mathcal{S}[n+k]\mathbf{f}_{ik} = \mathbf{0}, \quad k = 1, \ldots, Q_i - 1. \tag{15}$$

Since the matrices $\mathcal{S}[n+k]$ are full column rank, from (15), we can conclude that $\mathbf{f}_{ik} = \mathbf{0}$ and then, for $j = 1, \ldots, M$, $k = 1, \ldots, Q_i - 1$, and $l = 1, \ldots, Q_j - 1$

$$g_{ik}^j[l-1] = g_{i(k+1)}^j[l]$$
$$g_{ik}^j[Q_j - 1] = g_{i(k+1)}^j[0] = 0$$

which implies

$$g_{ik}^j[l] = \begin{cases} a_i^j[l-k+1], & \text{if } 0 \le (l-k+1) \le (L_j - L_i) \\ 0, & \text{otherwise} \end{cases}$$

or in a more intuitive manner

$$\mathbf{g}_{ik}^j = \begin{cases} [\underbrace{0, \ldots, 0}_{k-1}, \underbrace{a_i^j[0], \ldots, a_i^j[L_j - L_i]}_{L_j - L_i + 1}, \underbrace{0, \ldots, 0}_{Q_i - k}]^T, & L_i \le L_j \\ \mathbf{0}, & L_i > L_j \end{cases}$$

where $a_i^j[k]$, $k = 0, \ldots, L_j - L_i$, is some constant. Finally, it is straightforward to prove that the above equation yields, for $i = 1, \ldots, M$ and $k = 1, \ldots, Q_i$

$$\mathbf{S}_j^{(Q_j)}[n+k-1]\mathbf{g}_{ik}^j = \begin{cases} \mathbf{S}_i^j[n]\mathbf{a}_i^j, & j \le i \\ \mathbf{0}, & j > i \end{cases}$$

where $\mathbf{a}_i^j = [a_i^j[0], \ldots, a_i^j[L_j - L_i]]^T$. Replacing the above equality into (13) we obtain (12), which concludes the proof. ■

Appendix II
Proof of Theorem 2

In this appendix, we prove the equivalence between the CCA-MAXVAR problem and the GEV in (9). For notational simplicity, we will omit here the temporal index $[n]$.

Proof: Let us start writing $\mathbf{y}_{ik}^{(r)} = \mathbf{X}_k\mathbf{v}_{ik}^{(r)} = \mathbf{U}_k\mathbf{f}_{ik}^{(r)}$, where $\mathbf{X}_k = \mathbf{U}_k\mathbf{\Sigma}_k\mathbf{V}_k^H$ is the singular value decomposition (SVD) of $\mathbf{X}_k$, and $\mathbf{f}_{ik}^{(r)} = \mathbf{\Sigma}_k\mathbf{V}_k^H\mathbf{v}_{ik}^{(r)}$ satisfies $\|\mathbf{f}_{ik}^{(r)}\| = 1$ as a direct consequence of the constraint $\|\mathbf{y}_{ik}^{(r)}\| = 1$. Taking (8) into account, we can write

$$\beta_i^{(r)} = \frac{\mathbf{a}_i^{(r)H}\mathbf{Y}_i^{(r)H}\mathbf{Y}_i^{(r)}\mathbf{a}_i^{(r)}}{Q_i^2} = \frac{\mathbf{b}_i^{(r)H}\mathcal{U}_i^H\mathcal{U}_i\mathbf{b}_i^{(r)}}{Q_i^2} \tag{16}$$

where $\mathcal{U}_i = [\mathbf{U}_1 \cdots \mathbf{U}_{Q_i}]$, $\mathbf{b}_i^{(r)} = \mathbf{F}_i^{(r)}\mathbf{a}_i^{(r)}$ and

$$\mathbf{F}_i^{(r)} = \begin{bmatrix} \mathbf{f}_{i1}^{(r)} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{f}_{iQ_i}^{(r)} \end{bmatrix}.$$

Since $\mathbf{F}_i^{(r)H}\mathbf{F}_i^{(r)} = \mathbf{I}$ and $\|\mathbf{a}_i^{(r)}\|^2 = Q_i$, we have $\|\mathbf{b}_i^{(r)}\|^2 = Q_i$, and the solution $\mathbf{b}_i^{(r)}$ maximizing (16) is given by the eigenvector associated to the rth eigenvalue $\beta_i^{(r)}$ of $\mathcal{U}_i^H\mathcal{U}_i/Q_i$, i.e.,

$$\frac{\mathcal{U}_i^H\mathcal{U}_i}{Q_i}\mathbf{b}_i^{(r)} = \beta_i^{(r)}\mathbf{b}_i^{(r)}. \tag{17}$$

Defining now $\mathcal{X}_i = [\mathbf{X}_1 \cdots \mathbf{X}_{\mathbf{Q}_i}] = \mathcal{U}_i\mathbf{\Lambda}_i\mathcal{V}_i^H$, where

$$\mathbf{\Lambda}_i = \begin{bmatrix} \mathbf{\Sigma}_1 & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{\Sigma}_{\mathbf{Q}_i} \end{bmatrix}, \quad \mathcal{V}_i = \begin{bmatrix} \mathbf{V}_1 & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{V}_{\mathbf{Q}_i} \end{bmatrix}$$

we can rewrite (17) as

$$\mathbf{\Lambda}_i^{-1}\mathcal{V}_i^H\frac{\mathcal{X}_i^H\mathcal{X}_i}{Q_i}\mathcal{V}_i\mathbf{\Lambda}_i^{-1}\mathbf{b}_i^{(r)} = \beta_i^{(r)}\mathbf{b}_i^{(r)}$$
$$\frac{\mathcal{X}_i^H\mathcal{X}_i}{Q_i}\mathcal{V}_i\mathbf{\Lambda}_i^{-1}\mathbf{b}_i^{(r)} = \beta_i^{(r)}\mathcal{V}_i\mathbf{\Lambda}_i\mathbf{b}_i^{(r)}$$
$$\frac{\mathcal{X}_i^H\mathcal{X}_i}{Q_i}\mathcal{V}_i\mathbf{\Lambda}_i^{-1}\mathbf{b}_i^{(r)} = \beta_i^{(r)}\mathcal{V}_i\mathbf{\Lambda}_i^2\mathcal{V}_i^H\mathcal{V}_i\mathbf{\Lambda}_i^{-1}\mathbf{b}_i^{(r)}$$

and taking into account that $\mathbf{R}_i = \mathcal{X}_i^H\mathcal{X}_i$ and $\mathbf{D}_i = \mathcal{V}_i\mathbf{\Lambda}_i^2\mathcal{V}_i^H$ are the matrices defined in (10), we obtain

$$\frac{1}{Q_i}\mathbf{R}_i\mathbf{w}_i^{(r)} = \beta_i^{(r)}\mathbf{D}_i\mathbf{w}_i^{(r)}$$

where $\mathbf{w}_i^{(r)} = [\mathbf{w}_{i1}^{(r)T}, \ldots, \mathbf{w}_{iQ_i}^{(r)T}]^T = \mathcal{V}_i \mathbf{\Lambda}_i^{-1} \mathbf{b}_i^{(r)}$. Thus, it can be proved in a straightforward manner that

$$\mathbf{w}_{ik}^{(r)} = a_{ik}^{(r)} \mathbf{V}_k \mathbf{\Sigma}_k^{-1} \mathbf{f}_{ik}^{(r)} = a_{ik}^{(r)} \mathbf{v}_{ik}^{(r)}$$

which implies $\mathbf{z}_{ik}^{(r)} = \mathbf{X}_k \mathbf{w}_{ik}^{(r)} = a_{ik}^{(r)} \mathbf{y}_{ik}^{(r)}$. Finally, the best 1-D PCA approximation of the projections $\mathbf{y}_{ik}^{(r)}$ is

$$\mathbf{z}_i^{(r)} = \frac{1}{Q_i} \sum_{k=1}^{Q_i} a_{ik}^{(r)} \mathbf{y}_{ik}^{(r)} = \frac{1}{Q_i} \sum_{k=1}^{Q_i} \mathbf{z}_{ik}^{(r)}$$

which concludes the proof. ■

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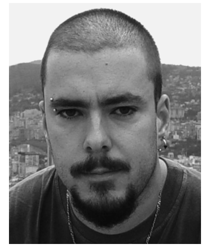

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