Effective Channel Order Estimation Based on Combined Identification/Equalization

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***Abstract*—Channel order estimation is a critical step in most blind single-input multiple-output (SIMO) channel identification/equalization algorithms. Several methods for estimating either the true channel order or its most significant part (the so-called effective channel order) have been recently proposed, but a solution able to work in practical scenarios (low or moderate signal-to-noise ratios (SNRs) and channels with small leading and/or trailing coefficients) has not been found yet. In this paper, a new criterion for effective channel order detection of SIMO channels is presented. The method is based on the fact that the cost function typically used in blind identification algorithms decreases monotonically with the estimated channel order, whereas for blind equalization algorithms, the cost function increases monotonically. In this paper, it is shown that a straightforward combination of both cost functions attains its minimum at the correct channel order even for moderate SNRs. The proposed method is able to work with small data sets, colored signals, and channels with small head and tail taps, which is a common problem in communication applications. The improvement of the proposed criterion over a number of existing algorithms is demonstrated through simulations.**

***Index Terms*—Blind identification/equalization, canonical correlation analysis (CCA), effective channel order, single-input multiple-output (SIMO).**

I. INTRODUCTION

FOLLOWING the well-known work of Tong *et al.* [1], many methods have been proposed for blind single-input multiple-output (SIMO) channel identification and equalization. Among them, the deterministic approaches [2]–[5] (see [6] for a complete review), which do not assume any specific stochastic model for the input sequence and exploit only the special structure of the multichannel matrix, are capable of recovering colored source signals and have a faster convergence than the stochastic techniques (e.g., subspace methods or linear prediction algorithms). However, these methods, as well as many other stochastic approaches, require a previous estimate of the SIMO channel order. When the effective channel order is either overestimated or underestimated, their performance degrades drastically [7], [8].

Addressing this issue, several methods based on information theoretic criteria, such as the minimum description length (MDL) or the Akaike information theoretic criterion (AIC), have been applied to this problem. In [9] and [10], the authors have found that information theoretic criteria, which assume independent identically distributed (i.i.d.) Gaussian data vectors, are not robust or accurate enough for realistic applications due to the overestimation of the effective channel order for high signal-to-noise ratios (SNRs) or for channels with small head and tail taps. Although the method proposed in [9] outperforms these criteria for high SNR, it suffers from poor performance at low SNR. Furthermore, the criterion proposed in [9], as well as other information theoretic criteria, is based on the stochastic properties of the source signals; then they need a relative large sample size for accurate channel order estimation. This is also the main drawback of the linear prediction methods for blind estimation and equalization [11]–[14] of SIMO channels.

In [15], the authors present a deterministic algorithm for joint order detection and channel estimation, which provides perfect channel order estimates in the absence of noise. However, the algorithm requires three sequential singular value decompositions, which results in poor performance for low SNRs and for channels with small impulse response terms.

In this paper, we propose a new technique for channel order estimation which consists on minimizing a combination of a blind identification cost function (which decreases with the estimated channel order) and a blind equalization cost function (which increases with the estimated channel order). Specifically, we use a least squares (LS) blind identification cost function [2] combined with a LS blind equalization cost function [16]. Under mild assumptions and in the absence of noise, it is shown in the paper that the identification term becomes zero when the estimated channel order is greater than or equal to the true one, whereas the equalization term is zero when the channel order is either exact or underestimated. By exploiting this fact, we propose a straightforward combination of both terms which is zero only for the true channel order. The proposed method is deterministic and therefore is able to work with correlated sources. Furthermore, the method provides not only an estimate of the channel order but also an estimate of the channel impulse response and, simultaneously, the best equalizers according to the criterion used in the cost function. Finally, for each channel order estimate, the evaluation of the new cost function requires solving two generalized eigenvalue problems (GEVs); therefore it is less sensitive to noise than the technique proposed in [15].

This paper is organized as follows. Sections II and III present, respectively, the LS methods for blind identification and blind equalization of SIMO channels. In Section IV, the influence of the channel order estimate on the identification and equalization terms of the cost function is analyzed. In Section V, the

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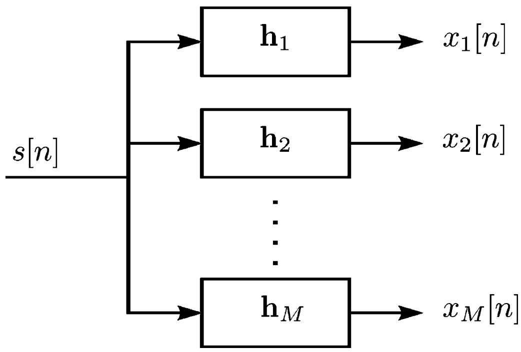

Fig. 1. Single-input multiple-output system.

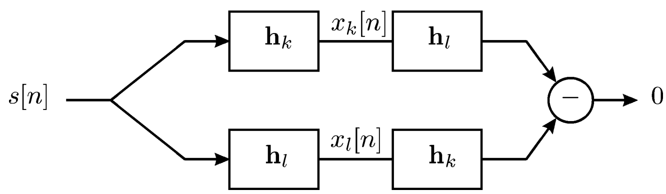

Fig. 2. LS blind identification of a noiseless SIMO system.

algorithm for blind channel order estimation is described, and in Section VI, its performance is compared through some simulation examples with information theoretic criteria as well as with the techniques proposed in [9] and [15]. The main conclusions are summarized in Section VI.

II. Blind Identification of SIMO Channels

A. Problem Formulation

Suppose we have the SIMO system shown in Fig. 1, where $s[n]$ is a source signal that is sent through M different finite impulse response (FIR) channels of order L. Let us define the following data matrices:

$$\mathbf{X}_k(\hat{L})=\begin{bmatrix} x_k[\hat{L}] & \cdots & x_k[0] \\ \vdots & \ddots & \vdots \\ x_k[N-1] & \cdots & x_k[N-\hat{L}-1] \end{bmatrix},\ k=1,\dots,M$$

where N is the number of data samples (snapshots), $\hat{L}$ is the estimated channel order, and $x_k[n]=s[n]*h_k[n]$ denotes the output signal of the kth channel. Denoting the impulse response vectors as

$$\mathbf{h}_k(\hat{L})=\left[h_k[0],\dots,h_k[\hat{L}]\right]^T,\quad k=1,\dots,M$$

it can be easily proved (see Fig. 2) that, in a noiseless situation, and for $\hat{L}\geq L$

$$\mathbf{X}_k(\hat{L})\mathbf{h}_l(\hat{L})=\mathbf{X}_l(\hat{L})\mathbf{h}_k(\hat{L}),\quad k,l=1,\dots,M. \quad (1)$$

B. Least Squares Solution

When the channel noise is taken into account, an approximate solution to (1) can be found using a least squares approach. Denoting the channel estimate of order $\hat{L}$ as $\hat{\mathbf{h}}_k(\hat{L})$, the well-known LS method proposed in [2] minimizes the following cost function:

$$J_{id}(\hat{L})=\frac{1}{2}\sum_{k,l=1}^{M}\left\|\mathbf{X}_k(\hat{L})\hat{\mathbf{h}}_l(\hat{L})-\mathbf{X}_l(\hat{L})\hat{\mathbf{h}}_k(\hat{L})\right\|^2 \quad (2)$$

subject to some constraint to avoid the trivial solution. Typically, a unit-norm constraint on the channel coefficients is applied. With this restriction and if the channel order is known, the channel estimate is collinear with the null space of a special data matrix constructed from (2). The LS estimator is closely related to other subspace-based techniques [18]. In fact, for $M=2$, both methods are identical [19].

In this paper, we consider an alternative constraint on the energy of the output signals (see Fig. 2). In particular, we use

$$\sum_{\substack{k,l=1\\k\neq l}}^{M}\left\|\mathbf{X}_k(\hat{L})\hat{\mathbf{h}}_l(\hat{L})\right\|^2=1. \quad (3)$$

Expanding (2) and using the constraint in (3), it is easy to show that the LS cost function can be rewritten as

$$J_{id}(\hat{L})=1-\rho_{id}(\hat{L}) \quad (4)$$

where

$$\rho_{id}(\hat{L})=\sum_{\substack{k,l=1\\k\neq l}}^{M}\hat{\mathbf{h}}_l^H(\hat{L})\mathbf{R}_{kl}(\hat{L})\hat{\mathbf{h}}_k(\hat{L}) \quad (5)$$

and $\mathbf{R}_{kl}(\hat{L})=\mathbf{X}_k^H(\hat{L})\mathbf{X}_l(\hat{L})$ denotes the cross-correlation matrices for each pair of channel outputs.

From (4) we can see that, under the proposed constraint (3), minimizing the LS error between each pair of outputs (2) is equivalent to maximizing their correlation (5). For the purpose of this paper (i.e., channel order estimation), an advantage of using (3) is that the resulting cost function is bounded between zero and one: $0\leq 1-\rho_{id}(\hat{L})\leq 1$. As we will see later, the equalization cost function is similar to (4) and is also bounded between zero and one; therefore, both cost functions can be combined without weighting parameters. In other words, with the chosen constraint, the cost function is invariant with respect to an arbitrary scaling in the signals, which is an unavoidable ambiguity in any blind channel identification method. Another advantage of replacing the unit norm constraint on the channels coefficients by the restriction on the energy of the output signals is the increased robustness to colored sources or small data blocks, avoiding noise enhancement problems.

By grouping now the subchannel estimates into the vector $\hat{\mathbf{h}}(\hat{L})=[\hat{\mathbf{h}}_1^T(\hat{L}),\dots,\hat{\mathbf{h}}_M^T(\hat{L})]^T$, it is easy to show that the solution that minimizes (4) can be obtained as the eigenvector associated to the largest eigenvalue of the following GEV:

$$\mathbf{R}(\hat{L})\hat{\mathbf{h}}(\hat{L})=\rho_{id}(\hat{L})\mathbf{D}(\hat{L})\hat{\mathbf{h}}(\hat{L}) \quad (6)$$

where

$$\mathbf{R}(\hat{L})=\begin{bmatrix} \mathbf{0} & \mathbf{R}_{21}(\hat{L}) & \cdots & \mathbf{R}_{M1}(\hat{L}) \\ \mathbf{R}_{12}(\hat{L}) & \mathbf{0} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{R}_{M(M-1)}(\hat{L}) \\ \mathbf{R}_{1M}(\hat{L}) & \cdots & \mathbf{R}_{(M-1)M}(\hat{L}) & \mathbf{0} \end{bmatrix}$$

and $\mathbf{D}(\hat{L})$ is a block diagonal matrix given by

$$\mathbf{D}(\hat{L}) = \begin{bmatrix} \sum_{k=2}^{M} \mathbf{R}_{kk}(\hat{L}) & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \sum_{k=1}^{M-1} \mathbf{R}_{kk}(\hat{L}) \end{bmatrix}.$$

III. Blind Equalization of SIMO Channels

A. Problem Formulation

Instead of identifying the channel and then inverting it to recover the source signal $s[n]$, a more direct approach is to find a set of equalizers along with the optimal coefficients to combine their outputs. From this point of view, a number of deterministic techniques have been proposed in recent years [3], [4]. In this section, we present the problem formulation in which these deterministic techniques are based.

Let us start by defining the row vectors $\tilde{\mathbf{x}}[n] = [x_1[n], \ldots, x_M[n]]$, $\tilde{\mathbf{h}}[n] = [h_1[n], \ldots, h_M[n]]$, and the matrices shown in the equation at the bottom of the page and

$$\tilde{\mathcal{T}} = \begin{bmatrix} \tilde{\mathbf{h}}[0] & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \tilde{\mathbf{h}}[0] & \ddots & \vdots \\ \tilde{\mathbf{h}}[L] & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{h}}[L] & \ddots & \tilde{\mathbf{h}}[0] \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \mathbf{0} & \tilde{\mathbf{h}}[L] \end{bmatrix}$$

where $0 \leq k \leq K + \hat{L}$, K is a parameter determining the equalizer length, $\tilde{\mathcal{T}}$ is a $(K + L + 1) \times (K + 1)M$ filtering channel matrix, and $N > L + \hat{L} + 3K + 1$ is the number of data samples.

In a noiseless case, the SIMO system output can be written as

$$\tilde{\mathbf{X}}_k(\hat{L}) = \tilde{\mathbf{S}}_k \tilde{\mathcal{T}}.$$

Therefore, if $\tilde{\mathcal{T}}$ is full row rank, there exists a matrix $\mathbf{W}(L) = [\mathbf{w}_0(L) \cdots \mathbf{w}_{K+L}(L)]$ such that

$$\tilde{\mathbf{X}}_k(\hat{L})\mathbf{W}(L) = \tilde{\mathbf{S}}_k$$

and, for $k, l = 0, \ldots, K + L$, this matrix will satisfy

$$\tilde{\mathbf{X}}_k(\hat{L})\mathbf{w}_k(L) = \tilde{\mathbf{X}}_l(\hat{L})\mathbf{w}_l(L) \tag{7}$$

which can be used as an equalization criterion [3], [17]. Finally, it can been proved [17] that in a general situation (see [21] and [22] for further details), the conditions to obtain a full row rank matrix $\tilde{\mathcal{T}}$ are the following.

1) The SIMO channel satisfies the *length-and-zero condition.*
   a) $h_k[0] \neq 0$ and $h_l[L] \neq 0$, for some $1 \leq k, l \leq M$.
   b) The M channels are coprime, i.e., they do not share any common zeros.
2) The equalizer length satisfies

$$K \geq \frac{L + 1 - M}{M - 1}.$$

B. Least Squares Solution

Similarly to the identification problem, when noise is present, (7) cannot be exactly satisfied. Again, defining $\hat{\mathbf{w}}_k(\hat{L})$ as the estimate of $\mathbf{w}_k(\hat{L})$, a simple alternative in this situation is to minimize the following LS cost function [17]:

$$J_{eq}(\hat{L}) = \frac{1}{2(K+\hat{L})} \sum_{k,l=0}^{K+\hat{L}} \left\| \tilde{\mathbf{X}}_k(\hat{L})\hat{\mathbf{w}}_k(\hat{L}) - \tilde{\mathbf{X}}_l(\hat{L})\hat{\mathbf{w}}_l(\hat{L}) \right\|^2 \tag{8}$$

subject to some nontrivial constraint. In particular, if we enforce the restriction

$$\sum_{k=0}^{K+\hat{L}} \left\| \tilde{\mathbf{X}}_k(\hat{L})\hat{\mathbf{w}}_k(\hat{L}) \right\|^2 = 1 \tag{9}$$

the cost function (8) can be rewritten as

$$J_{eq}(\hat{L}) = 1 - \rho_{eq}(\hat{L}) \tag{10}$$

$$\tilde{\mathbf{X}}_k(\hat{L}) = \begin{bmatrix} \tilde{\mathbf{x}}[K + k] & \cdots & \tilde{\mathbf{x}}[k] \\ \tilde{\mathbf{x}}[K + k + 1] & \cdots & \tilde{\mathbf{x}}[k + 1] \\ \vdots & \ddots & \vdots \\ \tilde{\mathbf{x}}[N - \hat{L} - K + k - 1] & \cdots & \tilde{\mathbf{x}}[N - \hat{L} - 2K + k - 1] \end{bmatrix}$$

$$\tilde{\mathbf{S}}_k = \begin{bmatrix} s[K + k] & \cdots & s[k - L] \\ s[K + k + 1] & \cdots & s[k - L + 1] \\ \vdots & \ddots & \vdots \\ s[N - \hat{L} - K + k - 1] & \cdots & s[N - \hat{L} - 2K + k - L - 1] \end{bmatrix}$$

where

$$\rho_{eq}(\hat{L}) = \frac{1}{K+\hat{L}} \sum_{\substack{k,l=0 \\ k \neq l}}^{K+\hat{L}} \hat{\mathbf{w}}_k^H(\hat{L})\tilde{\mathbf{R}}_{kl}(\hat{L})\hat{\mathbf{w}}_l(\hat{L})$$

and $\tilde{\mathbf{R}}_{kl}(\hat{L}) = \tilde{\mathbf{X}}_k^H(\hat{L})\tilde{\mathbf{X}}_l(\hat{L})$ denote the crosscorrelation matrices between the outputs of the SIMO channel.

Like in the identification problem, (9) yields a cost function bounded between zero and one and provides better results in presence of strongly colored sources or small data blocks than any alternative restriction on the energies of the equalizers. Moreover, for blind SIMO equalization, it has been shown that this constraint transforms the original LS problem into a canonical correlation analysis (CCA) problem [16], [20]. CCA is a well-known multivariate statistical analysis technique that finds maximally correlated projections among several input data sets. It was originally proposed by Hotelling for two data sets [23] and later generalized by Kettenring to an arbitrary number of data sets [24]. Recently, adaptive algorithms for CCA have been described in [25] and [26] for the case of two data sets, and in [20] for the case of several data sets. An adaptive version of this CCA-based blind equalization algorithm has been recently proposed in [16] and [20], showing improved performance and faster convergence than the modified second-order statistics based algorithm [17].

The solution that minimizes (10) is given again by the eigenvector associated to the maximum eigenvalue of the following GEV problem:

$$\frac{1}{K+\hat{L}}\tilde{\mathbf{R}}(\hat{L})\hat{\mathbf{w}}(\hat{L}) = \rho_{eq}(\hat{L})\tilde{\mathbf{D}}(\hat{L})\hat{\mathbf{w}}(\hat{L}) \tag{11}$$

where $\hat{\mathbf{w}}(\hat{L}) = [\hat{\mathbf{w}}_0^T(\hat{L}), \ldots, \hat{\mathbf{w}}_{K+\hat{L}}^T(\hat{L})]^T$

$$\tilde{\mathbf{D}}(\hat{L}) = \begin{bmatrix} \tilde{\mathbf{R}}_{00}(\hat{L}) & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \tilde{\mathbf{R}}_{(K+\hat{L})(K+\hat{L})}(\hat{L}) \end{bmatrix}$$

and $\tilde{\mathbf{R}}(\hat{L})$ is defined as shown in the equation at the bottom of the page.

Finally, the equalized signal is constructed as

$$\hat{\mathbf{s}}(\hat{L}) = \frac{1}{K+\hat{L}+1} \sum_{k=0}^{K+\hat{L}} \tilde{\mathbf{X}}_k(\hat{L})\hat{\mathbf{w}}_k(\hat{L}) \tag{12}$$

where $\hat{\mathbf{s}}(\hat{L}) = [\hat{s}[K], \ldots, \hat{s}[N-\hat{L}-K-1]]^T$. Here, it is interesting to point out that, according with the maximum variance generalization of CCA [20], [24], the estimate $\hat{\mathbf{s}}(\hat{L})$ can be interpreted as the one-dimensional principal component analysis (PCA) approximation of the best unit-norm projections of the data sets $\tilde{\mathbf{X}}_k(\hat{L})$. Consequently, this ensures that (12) is the best linear combination of the equalizer outputs.

IV. Influence of the Channel Order on the Identification and Equalization Cost Functions

The identification and equalization cost functions described in Sections II and III, respectively, are the terms that compose the overall cost function used for order estimation. In order to understand the rationale for this new cost function, in this section, we first study the influence of the estimated channel order $\hat{L}$ on the LS identification and equalization cost functions.

A. *Influence of $\hat{L}$ on the Identification Cost Function*

Assuming that the exact channel order L is known in advance, it has been proved in [2] that the sufficient and necessary conditions for blind multichannel identification can be stated as follows.

1) The SIMO channel satisfies the *length-and-zero condition*.
2) The linear complexity [2] of $s[n]$, defined as the maximum order r which provides a full column rank Hankel matrix

$$\mathbf{S}_r = \begin{bmatrix} s[r-1] & \cdots & s[0] \\ \vdots & \ddots & \vdots \\ s[P-1] & \cdots & s[P-r] \end{bmatrix}$$

for some P, satisfies

$$\mathcal{C}\left\{s[n]\right\} \geq L + 1 + \lceil L/M - 1 \rceil.$$

Based on these conditions, and assuming a noiseless system, the effect of the channel order estimate can be easily analyzed.

1) $\hat{L} < L$: In this case, channel identification is not possible, (1) cannot be satisfied and $J_{id}(\hat{L}) > 0$.
2) $\hat{L} = L$: This is the case analyzed in [2]. The identification method perfectly recovers, up to a scale factor, the SIMO channel, i.e., $J_{id}(\hat{L}) = 0$.
3) $\hat{L} > L$: In this case the *length-and-zero condition* is violated $(h_k[\hat{L}] = 0)$, which implies that the solutions of (1) are any set of channels

$$\hat{\mathbf{h}}_k(\hat{L}) = \mathbf{h}_k(L) * \mathbf{c}(\hat{L} - L), \quad k = 1, \ldots, M$$

$$\tilde{\mathbf{R}}(\hat{L}) = \begin{bmatrix} \mathbf{0} & \tilde{\mathbf{R}}_{01}(\hat{L}) & \cdots & \tilde{\mathbf{R}}_{0(K+\hat{L})}(\hat{L}) \\ \tilde{\mathbf{R}}_{10}(\hat{L}) & \mathbf{0} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \tilde{\mathbf{R}}_{(K+\hat{L}-1)(K+\hat{L})}(\hat{L}) \\ \tilde{\mathbf{R}}_{(K+\hat{L})0}(\hat{L}) & \cdots & \tilde{\mathbf{R}}_{(K+\hat{L})(K+\hat{L}-1)}(\hat{L}) & \mathbf{0} \end{bmatrix}$$

where $\mathbf{c}(\hat{L}-L)=[c[0],\ldots,c[\hat{L}-L]]$ is some nonzero vector. In other words, when the channel order is overestimated, there is a linear space of possible, although undesirable, solutions; for all of them: $J_{id}(\hat{L})=0$.

If we now consider the effect of the system noise, it can be proved in a straightforward manner that, for $\hat{L}_1 > \hat{L}_2$, $\min\{J_{id}(\hat{L}_1)\} \leq \min\{J_{id}(\hat{L}_2)\}$. Therefore the identification cost function is a monotonically decreasing function with the estimated channel order.

B. Influence of $\hat{L}$ on the Equalization Cost Function

Let us introduce the following theorem.

Theorem 1: In the absence of noise, and if the linear complexity satisfies

$$\mathcal{C}\{s[n]\} \geq K+L+2$$

then (7) is only satisfied if $\hat{L} \leq L$ and then

$$\tilde{\mathcal{T}}\hat{\mathbf{W}}(\hat{L})=\sum_{k=0}^{L-\hat{L}} a_k \mathbf{F}_k(\hat{L}) \tag{13}$$

where a_k is some constant and $\mathbf{F}_k(\hat{L})$ is a $(K+L+1)\times(K+\hat{L}+1)$ matrix with ones along its kth lower diagonal and zeros elsewhere.

Proof: Defining $\hat{\mathbf{g}}_k(\hat{L})=\tilde{\mathcal{T}}\hat{\mathbf{w}}_k(\hat{L})$ as the combined channel-equalizer response, (7) can be rewritten as

$$\tilde{\mathbf{S}}_k\hat{\mathbf{g}}_k(\hat{L})=\tilde{\mathbf{S}}_l\hat{\mathbf{g}}_l(\hat{L}), \quad k,l=0,\ldots,K+\hat{L}$$

or, equivalently

$$\tilde{\mathbf{S}}_k\hat{\mathbf{g}}_k(\hat{L})=\tilde{\mathbf{S}}_{k+1}\hat{\mathbf{g}}_{k+1}(\hat{L}), \quad k=0,\ldots,K+\hat{L}-1.$$

Taking into account that $\mathcal{C}\{s[n]\} \geq K+L+2$, it follows that the $K+L+2$ different columns in $\tilde{\mathbf{S}}_k$ and $\tilde{\mathbf{S}}_{k+1}$ are linearly independent and then

$$\left.\begin{array}{l}\hat{\mathbf{g}}_k(\hat{L})=\mathbf{J}_1\hat{\mathbf{g}}_{k+1}(\hat{L})\\ \hat{\mathbf{g}}_{k+1}(\hat{L})=\mathbf{J}_1^T\hat{\mathbf{g}}_k(\hat{L})\end{array}\right\} \quad k=0,\ldots,K+\hat{L}-1$$

where $\mathbf{J}_1$ is a $(K+L+1)\times(K+L+1)$ shifting matrix with ones along its first upper diagonal. Finally, it is straightforward to show that $\tilde{\mathcal{T}}\hat{\mathbf{W}}(\hat{L})=[\hat{\mathbf{g}}_0(\hat{L})\cdots\hat{\mathbf{g}}_{K+\hat{L}}(\hat{L})]$ is a Toeplitz matrix whose first column is given by

$$\hat{\mathbf{g}}_0(\hat{L})=[a_0,\ldots,a_{L-\hat{L}},\underbrace{0,\ldots,0}_{K+\hat{L}}]^T$$

and whose first row is $[a_0,0,\ldots,0]$, which implies (13). ■

Based on the above theorem, the effect of the channel order estimate can be easily analyzed.

1) $\hat{L}>L$: Channel equalization is not possible, (7) cannot be exactly satisfied, and $J_{eq}(\hat{L})>0$.
2) $\hat{L}=L$: In this case the proposed equalization method perfectly equalizes, up to a scale factor, the SIMO channel ($\tilde{\mathcal{T}}(K)\hat{\mathbf{W}}(\hat{L},K)=a_0\mathbf{I}$), $J_{eq}(\hat{L})=0$.
3) $\hat{L}<L$: There exists an infinite number of solutions to (7) that do not satisfy the zero intersymbol interference (ISI) equalization objective ($\tilde{\mathcal{T}}\hat{\mathbf{W}}(\hat{L})=a_0\mathbf{I}$). The ISI cannot be completely removed, and the estimated signal is $\hat{s}[n]=\mathbf{a}*s[n]$, where $\mathbf{a}=[a_0,\ldots,a_{L-\hat{L}}]$. $J_{eq}(\hat{L})=0$.

V. A New Channel Order Estimation Criterion

The proposed channel order estimation criterion is based on the joint identification and equalization of the SIMO channel. The selection of the restrictions (3) and (9) ensures that $J_{id}(\hat{L})$ and $J_{eq}(\hat{L})$ are bounded between zero and one, which allows us to define the total combined cost function as

$$J(\hat{L})=J_{id}(\hat{L})+J_{eq}(\hat{L}). \tag{14}$$

The proposed channel order estimation algorithm is based on the combination of the identifiability and equalization conditions.

1) A maximum possible channel order $\hat{L}_{\max}$ is previously known or estimated from the SIMO channel properties. This condition is not very restrictive in practice.
2) The SIMO channel satisfies the *length-and-zero condition*.
3) The equalizer length parameter satisfies
$$K \geq \frac{\hat{L}_{\max}+1-M}{M-1}.$$
4) The linear complexity satisfies
$$\mathcal{C}\{s[n]\} \geq K+\hat{L}_{\max}+2.$$

Using the above conditions and considering the effect of the channel order estimate over $J_{id}(\hat{L})$ and $J_{eq}(\hat{L})$, it can be deduced that, in a noiseless case

$$\begin{cases} J(\hat{L})=0, & \text{if } \hat{L}=L\\ J(\hat{L})>0, & \text{if } \hat{L}\neq L.\end{cases}$$

Therefore, the proposed cost function attains its minimum value at the true channel order. In order to find it, we evaluate the cost function (14) for orders in the range $0 \leq \hat{L} \leq \hat{L}_{\max}$. For each estimated order, the evaluation of the cost function requires one to solve the identification and equalization GEV problems of dimensions $M(\hat{L}+1)$ and $M(K+1)(K+\hat{L}+1)$, respectively. Taking into account that the computational cost of a GEV problem of size N is $\mathcal{O}(N^3)$, the complexity of one step of the proposed method is $\mathcal{O}(M^3\hat{L}^3+M^3K^3(K+\hat{L})^3)$, and considering that $K \geq (\hat{L}_{\max}+1-M)/(M-1)$, the total cost is $\mathcal{O}((M^3+\hat{L}_{\max}^3)\hat{L}_{\max}^4)$, which implies a higher computational complexity than previously proposed methods [9], [15], [27]. However, unlike other order estimation methods, we also obtain the channel estimate and a set of FIR equalizers to restore the source signal. The overall channel order estimation procedure is summarized in Algorithm 1.

Algorithm 1: Proposed algorithm for blind SIMO channel order estimation.

Select $\hat{L}_{\max}$ and $K \geq (\hat{L}_{\max} + 1 - M)/(M-1)$.

for $\hat{L} = 1, \ldots, \hat{L}_{\max}$ **do**

Obtain $J_{id}(\hat{L})$ and $\hat{\mathbf{h}}(\hat{L})$ by means of (6).

Obtain $J_{eq}(\hat{L})$ and $\hat{\mathbf{w}}(\hat{L})$ by means of (11).

Obtain $J(\hat{L}) = J_{id}(\hat{L}) + J_{eq}(\hat{L})$.

endfor

Obtain the final estimate $\hat{L}$ which minimizes $J(\hat{L})$.

As we will see in the simulations section, the proposed method is very robust in noisy situations. On the other hand, the deterministic framework in which the method has been derived provides accurate estimates for small data lengths (N) and even for colored signals. In the following section, the performance of the proposed algorithm is compared with other blind channel order estimation techniques.

A. Comparison With Other Techniques

1) Methods Based on Information Theoretic Criteria: Information theoretic criteria such as the MDL and the AIC have been widely applied to determine the dimension of a certain signal subspace [27]. The main drawbacks of these techniques are their strong assumptions. For instance, they assume that successive data vectors are i.i.d. zero-mean Gaussian random vectors, which is not true in blind channel identification problems due to the shift property. Furthermore, both methods tend to overestimate the channel order at high SNRs and, although the MDL is asymptotically consistent [27], it is strongly affected when the channel impulse response has small head and tail coefficients. A detailed analysis of the behavior of AIC and MDL for blind channel order estimation can be found in [10]. In this paper, it is shown that when the noise eigenvalues are not clustered sufficiently close, both the AIC and the MDL methods tend to overestimate the channel order.

2) Effective Channel Order Determination Algorithm: This method has been proposed by Liavas *et al.* in [9] and is based on the ratio of two consecutive eigenvalues of the autocorrelation matrix. Although this method solves the overestimation problem of information theoretic criteria at high SNR as well as for channels with small tails, it suffers from poor performance at low SNRs. Furthermore, this algorithm is based on the assumption of white source signals, which implies a performance degradation for colored signals.

3) Joint Order Detection and Channel Estimation by Least Squares Smoothing: The deterministic method for blind channel order estimation proposed in [15] requires three sequential singular value decompositions. Without noise, this method can recover exactly the SIMO channel; however, its performance is rather poor in noisy scenarios or when the channel impulse response has small head or tail taps.

TABLE I
IMPULSE RESPONSES OF THE ONE-INPUT THREE-OUTPUT SIMO CHANNEL

n	$\mathbf{h}_1[n]$	$\mathbf{h}_2[n]$	$\mathbf{h}_3[n]$
0	$1.7491 - j0.9173$	$0.9323 - j0.7836$	$1.0488 + j0.2484$
1	$0.1326 - j1.1061$	$1.1647 + j0.2133$	$1.4886 + j0.0596$
2	$0.3252 + j0.8106$	$-2.0457 + j0.7879$	$1.2705 + j1.3766$
3	$-0.7938 + j0.6985$	$-0.6444 + j0.8967$	$-1.8561 - j1.0830$
4	$0.3149 - j0.4016$	$1.7411 - j0.1869$	$2.1343 + j1.0354$
5	$-0.5273 + j1.2688$	$0.4868 + j1.0132$	$1.4358 + j1.5854$

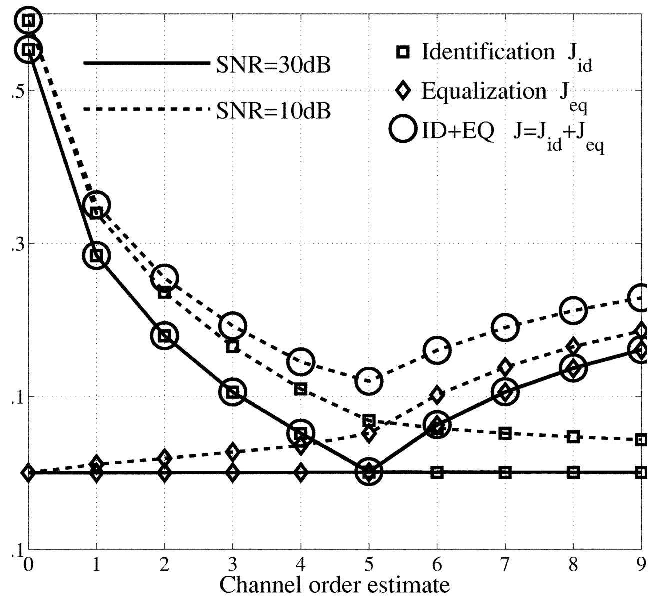

Fig. 3. Cost functions: $J_{id}(\hat{L})$ (squares), $J_{eq}(\hat{L})$ (diamonds), and $J(\hat{L}) = J_{id}(\hat{L}) + J_{eq}(\hat{L})$ (circles), for SNR $= 30$ dB (solid line) and SNR $= 10$ dB (dashed line).

VI. SIMULATION RESULTS

In this section, the performance of the proposed algorithm is evaluated using source signals of length $N = 100$ and the SIMO channel with the impulse response shown in Table I. We compare the performance of the proposed method (referred to as $\mathrm{ID} + \mathrm{EQ}$) with the AIC method, the MDL method, the effective channel order determination technique proposed by Liavas *et al.* in [9] (referred to as Liavas), and the least squares smoothing method proposed by Tong and Zhao in [15] (LSS in the figures). For the proposed method, the maximum possible order has been selected as $\hat{L}_{\max} = 9$. All the examples are based on the averaged results of 3000 independent realizations.

In the first example, an i.i.d. quadrature phase-shift keying (QPSK) signal is distorted by the SIMO channel and corrupted by zero-mean white Gaussian noise. Fig. 3 shows the averaged cost functions for high and moderate SNRs. We can see that for both SNRs, the minimum appears at the effective channel order (we have assumed that, for this channel and SNRs, the exact and effective channel orders coincide and are equal to $L = 5$). Fig. 4 shows the probability density functions of the channel order estimates for SNR $= 10$ dB for the proposed $\mathrm{ID} + \mathrm{EQ}$ method and the rest of order estimation techniques used in the

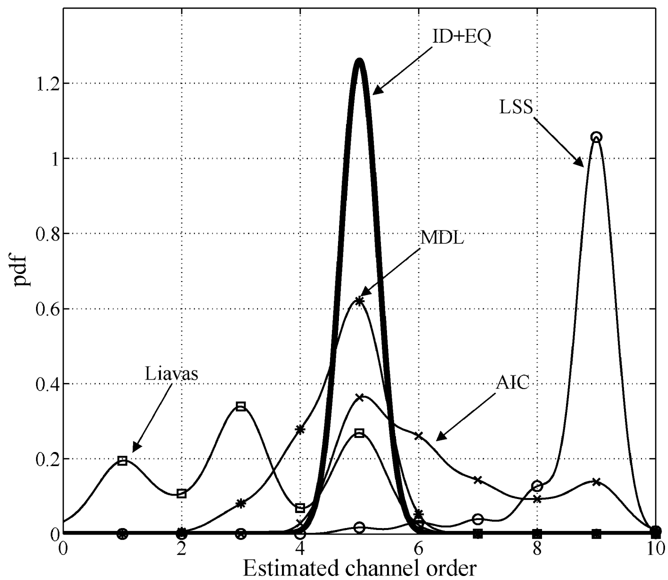

Fig. 4. Probability density functions (estimated using the Parzen method) of the order estimate for the proposed method, the joint LSS [15], the method proposed in [9], AIC, and MDL.

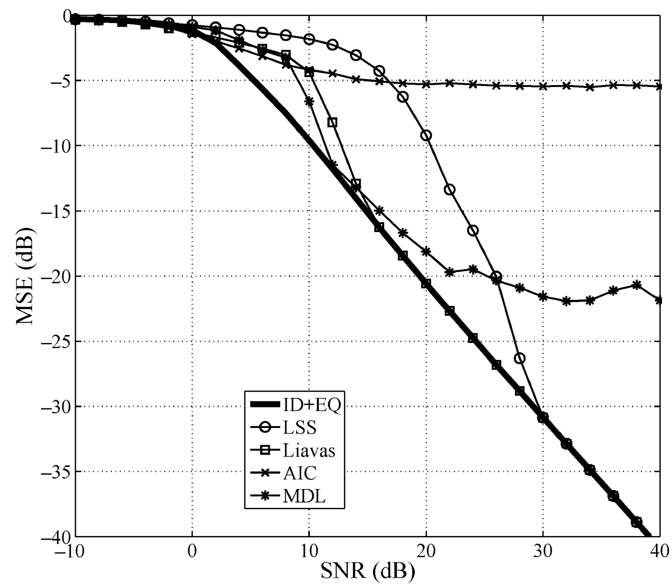

Fig. 6. Final MSE after equalization for $\mathrm{ID} + \mathrm{EQ}$, LSS, Liavas, AIC, and MDL.

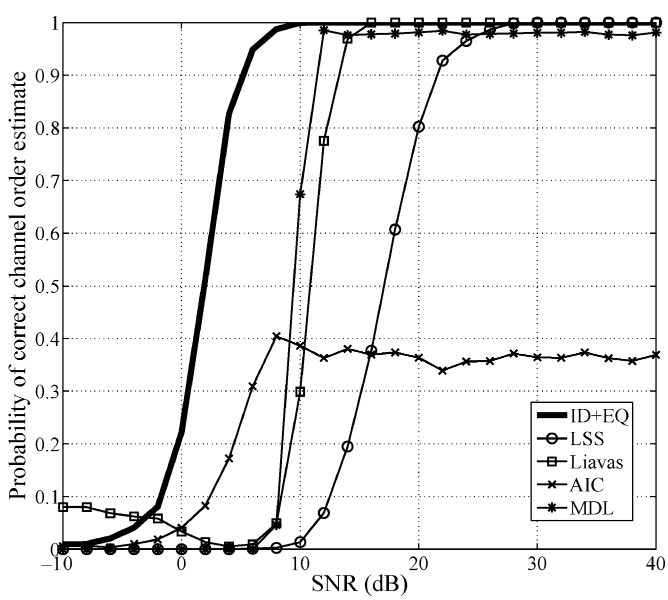

Fig. 5. Probability of correct channel order detection for $\mathrm{ID} + \mathrm{EQ}$, LSS, Liavas, AIC, and MDL.

comparison. Although obviously the probability density function (pdf) for the estimated order is discrete, for representation purposes we have estimated it using the Parzen windowing method [28]; therefore a continuous pdf results. We can see that for a moderate SNR, the proposed method obtains the best results (the correct channel order is detected in all the examples), whereas the LSS and AIC methods tend to overestimate the true channel order and the method by Liavas and the MDL tend to underestimate it. Note that, as we have pointed out before, for higher SNRs the information-theoretic criteria-based methods will tend to overestimate the channel order.

On the other hand, the probability of correct channel order detection as a function of the SNR is shown in Fig. 5: here we can see that the $\mathrm{ID} + \mathrm{EQ}$ method clearly outperforms the other blind channel order determination algorithms for a wide range of SNRs. Furthermore, although the $\mathrm{ID} + \mathrm{EQ}$ method is not robust against high noise levels ($\mathrm{SNR} \leq 0$ dB), in this situation the existing estimation/equalization algorithms fail regardless of the estimated channel order. In other words, the robustness problem of the proposed algorithm could be addressed by replacing the proposed LS methods by some robust channel estimation/equalization techniques.

Channel order estimation is the first step in any blind identification/equalization technique, where the final goal is to restore the original source signal. As a by-product, the $\mathrm{ID} + \mathrm{EQ}$ method also provides a set of FIR equalizers that can be readily used to restore the source signal through (12). For the rest of the methods, we have used the channel order estimate to derive a set of equalizers using the algorithm described in [16] (which minimizes the equalization cost function described in this paper). The final mean squared error (MSE) after equalization is shown in Fig. 6, where we can see that the proposed method outperforms the rest of order estimation techniques. Note also that, for $\mathrm{SNR} = 10$ dB, the best results are obtained with the proposed method ($\hat{L} = 5$), which validates our previous assumption about the effective channel order.

In the second example, the source signal has been colored by an FIR filter with impulse response $\mathbf{b} = [1, 1]$. The final MSE after channel order detection and equalization for all the methods is shown in Fig. 7: we can see that the improvement of the $\mathrm{ID} + \mathrm{EQ}$ method over the other techniques is even larger for colored signals.

In the third example, a white QPSK signal is sent through the SIMO channel of Table I, which has now been padded to include small ending coefficients, as can be observed in Fig. 8, that represents the absolute value of the coefficients. The results for the final MSE after equalization are shown in Fig. 9, which shows that also for this type of channel, the $\mathrm{ID} + \mathrm{EQ}$ method outperforms the rest of techniques.

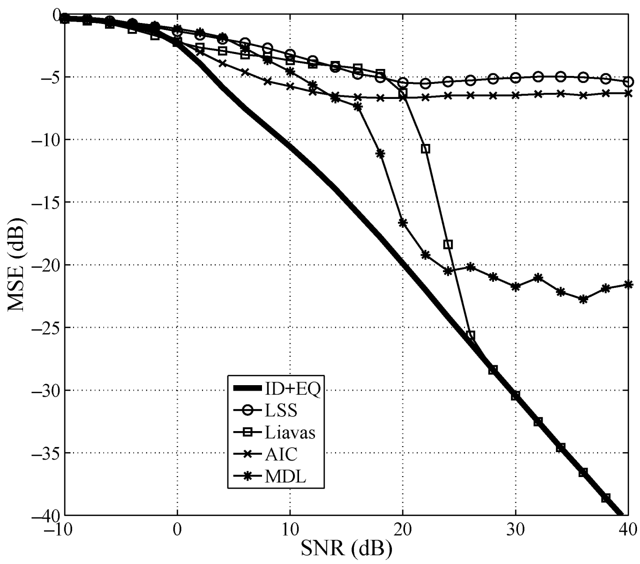

Fig. 7. Final MSE after equalization with colored inputs for $\mathrm{ID}+\mathrm{EQ}$, LSS, Liavas, AIC, and MDL.

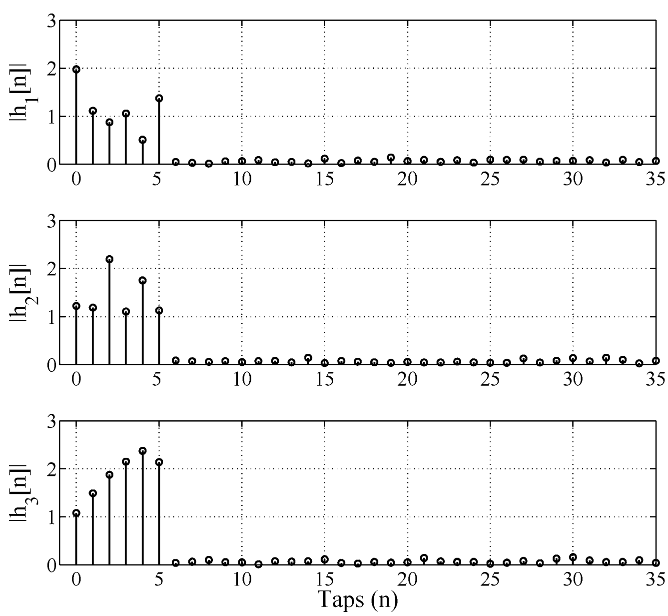

Fig. 8. Extended channel impulse responses including small ending coefficients.

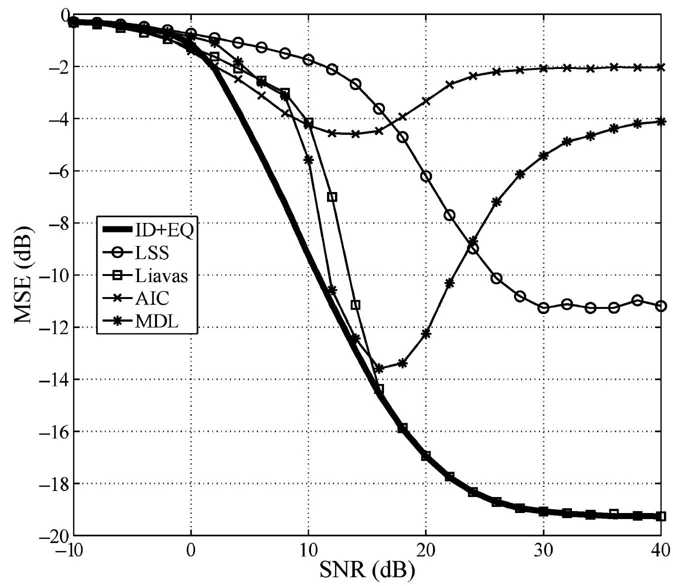

Fig. 9. Final MSE after equalization for a channel with small trailing terms.

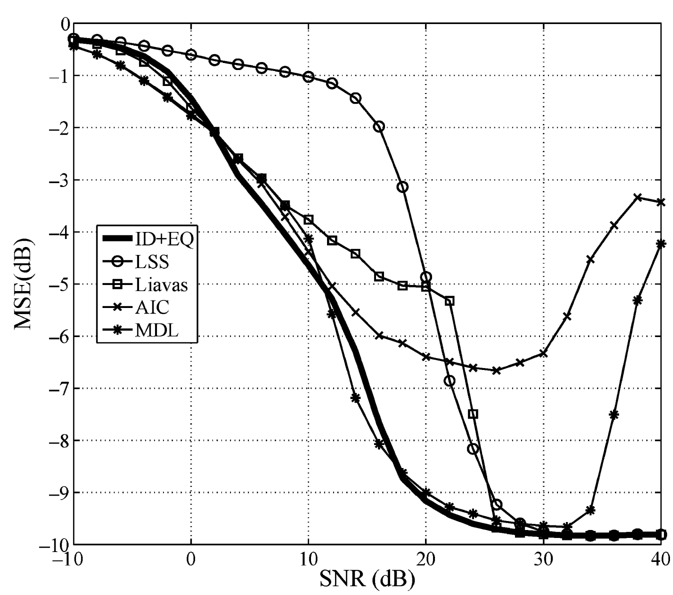

Fig. 10. Final MSE after equalization of a fractionally sampled channel.

In the final example, a white QPSK signal is transmitted using a square root raised cosine filter with rolloff factor $r = 0.5$. It is distorted by a channel with impulse response $= \delta(0) + 0.8\delta(T/2) + 0.2\delta(T) - 0.4\delta(3T/2) + 0.1\delta(2T)$, where T denotes the symbol period. The sampling period at the receiver is $T/2$, obtaining an equivalent 1×2 SIMO system. The results for the final MSE after channel order estimation and equalization are shown in Fig. 10, where we can see the good performance of the proposed method in the SNR range between 15 and 25 dB, as well as its robustness for high SNRs.

VII. Conclusion

In this paper, we have presented a new cost function for blind order determination of SIMO channels that combines an identification term with an equalization term. Specifically, the identification and equalization terms are least squares cost functions subject to a particular constraint on the energy of the output signals that yields bounded terms, thus simplifying the combination of both cost functions. The basic idea behind the method is that the LS identification term decreases with the estimated channel order, whereas the LS equalization term increases. In this way, without noise it has been proved that the combined cost function attains its minimum at the true channel order.

The method is formulated within a deterministic framework, which means that it is capable of estimating the true channel order within a finite number of samples in the absence of noise. Due to the particular form of the cost function, the method also provides an estimate of the channel impulse response as well as the equalizers. The performance of the proposed method has been compared with other channel order determination techniques through simulations, showing a better performance, mainly for colored source signals and for channels with small leading and/or ending coefficients. Further lines include the theoretical analysis of the method in the presence of noise, the improvement of the algorithm by exploiting the relationship between the estimated channels and equalizers, and, the extension of this idea to multiple-input multiple-output channels.

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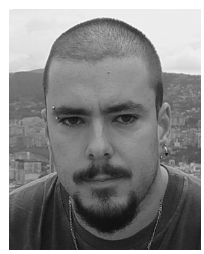

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