On the Blind Identifiability of Orthogonal Space–Time Block Codes From Second-Order Statistics

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***Abstract*—In this paper, the conditions for blind identifiability from second-order statistics (SOS) of multiple-input multiple-output (MIMO) channels under orthogonal space–time block coded (OSTBC) transmissions are studied. The main contribution of the paper consists in the proof that, assuming more than one receive antenna, any OSTBC with a transmission rate higher than a given threshold, which is inversely proportional to the number of transmit antennas, permits the blind identification of the MIMO channel from SOS. Additionally, it has been proven that any real OSTBC with an odd number of transmit antennas is identifiable, and that any OSTBC transmitting an odd number of real symbols permits the blind identification of the MIMO channel regardless of the number of receive antennas, which extends previous identifiability results and suggests that any nonidentifiable OSTBC can be made identifiable by slightly reducing its code rate. The implications of these theoretical results include the explanation of previous simulation examples and, from a practical point of view, they show that the only nonidentifiable OSTBCs with practical interest are the Alamouti codes and the real square orthogonal design with four transmit antennas. Simulation examples and further discussion are also provided.**

***Index Terms*—Blind identifiability, multiple-input multiple-output (MIMO) communications, orthogonal space–time block codes (OSTBC), second-order statistics (SOS).**

I. Introduction

SINCE the pioneering work of Foschini [2] and Telatar [3], multiple transmit and receive diversity has been exploited to drastically improve the performance of wireless communication systems [4]–[8]. Specifically, since the work of Alamouti [9], and the later generalization by Tarokh *et al.* [10], space–time block coding (STBC) has emerged as one of the most promising techniques to exploit spatial diversity in multiple-input multiple-output (MIMO) systems.

Among space–time coding schemes, the orthogonal space–time block coding (OSTBC) [9], [10] is one of the most attractive because it is able to provide full diversity gain with very simple encoding and decoding. The special structure of OSTBCs implies that the optimal maximum-likelihood (ML) decoder is a simple linear receiver, which can be seen as a matched filter, followed by a symbol-by-symbol detector. This linear receiver maximizes the signal-to-noise ratio (SNR) for each data symbol [11] using the knowledge of the channel matrix. When the channel state information (CSI) is not available at the receiver, training approaches can be used to obtain an estimate of the channel [12]. However, the price to be paid is reduced bandwidth efficiency, and even inaccurate channel estimates due to the effect of the noise and the limited number of training symbols. These shortcomings suggest the use of blind or semiblind methods [13], [14], as well as techniques based on redundant linear precoding [15]–[18].

Popular approaches to avoid the penalty in bandwidth efficiency include the so-called differential space–time coding schemes [19]–[23] and unitary space–time modulation [24], [25], which do not require channel knowledge at the receiver. However, these approaches incur a penalty in performance of 3 dB (differential coding) and 2–4 dB (unitary modulation) as compared with the coherent ML receiver [24]. Moreover, the receiver complexity for the unitary scheme increases exponentially with the number of points in the unitary space–time constellation.

Recently, several methods for blind channel estimation or blind decoding have been proposed. These methods include the optimal ML blind decoder, which implies a prohibitive computational cost, as well as several suboptimal approaches. In particular, the technique proposed in [14], [26] is based on alternating minimizations over the channel and the signal estimates; whereas the methods in [27] assume BPSK or QPSK source signals and are based on a semidefinite relaxation approach (suboptimal) or the sphere decoder (optimal). Unfortunately, the computational complexity of these approaches remains relatively high.

A common assumption to alleviate the computational complexity of blind techniques consists in the relaxation of the finite alphabet property of the sources. Thus, several subspace methods have been proposed. For instance, in [28], [29], a general class of STBCs is described, and a blind receiver is proposed. However, the proposed receiver is not applicable to any rate-one OSTBC. A more interesting technique for blind channel estimation in OSTBC systems has been proposed in [30]. This method is solely based on second-order statistics (SOS), it does not require any assumption about the correlation properties of the sources, and its computational complexity reduces to the extraction of the principal eigenvector of a modified correlation matrix. Under white Gaussian noise, the method proposed in [30] is equivalent to the relaxed blind ML detector, and its performance has been evaluated by means of

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numerical examples, finding that, if $n_R > 1$ receive antennas are available, it provides accurate channel estimates in most of the cases. Unfortunately, some OSTBCs (including the Alamouti code [9]) cannot be identified with this technique. This raises the question of whether a particular OSTBC can be identified using SOS-based methods. There are some partial identifiability results in the literature, but, to the best of our knowledge, the identifiability conditions still remain unclear. The main goal of this paper is to fill this gap by presenting, in a unified manner, some new results regarding the blind identifiability conditions of OSTBCs.

Previous work on identifiability conditions for blind channel estimation or symbol detection under real OSTBC transmissions can be found in [31], [32]. Specifically, [31] deals with SOS-based blind channel estimation and two main results have been proven for the case of real OSTBCs: first, if the OSTBC symbol dimension is odd, the channel can be identified up to a scalar; and second, if the number of transmit antennas is odd, any full row rank channel matrix is identifiable (which implies that the number of transmit antennas cannot be greater than the number of receive antennas). In [32], the study is carried out considering finite alphabet constraints on the information symbols, which is used to introduce the definition of rotatable, nonrotatable and strictly nonrotatable OSTBCs. Specifically, it has been proven that OSTBCs transmitting an odd number of real information symbols, and real OSTBCs with an odd number of transmit antennas, are strictly nonrotatable. Thus, exploiting the finite alphabet property of the sources, the channel can be extracted up to a sign change. Unfortunately, in many practical cases, these conditions are not satisfied, and the identifiability properties of a large number of codes have been obtained by means of numerical examples [30], [32].

The main contribution of this work is based on the definition of identifiable and nonidentifiable OSTBCs, and consists in the proof that any OSTBC transmitting at a rate higher than a given threshold, which is inversely proportional to the number of transmit antennas, permits the blind channel identification for any number of receive antennas $n_R > 1$. Additionally, we have found that any OSTBC transmitting an odd number of real symbols permits the blind identification of the MIMO channel regardless of the number of receive antennas, which extends to complex OSTBCs the first result in [31]. As a by-product, this result suggests that by slightly reducing its code rate, any nonidentifiable OSTBC can be made identifiable [1]. Moreover, it has been proven that any real OSTBC with an odd number of transmit antennas is identifiable, which explains some of the results in [32] avoiding the finite alphabet constraint.

The implications of these results include the explanation of the simulation examples in [30]; the generalization of the identifiability conditions in [31] and [32], and finally, that the only nonidentifiable OSTBCs with practical interest are the Alamouti code and the real OSTBC with $n_T = 4$ transmit antennas and transmission rate $R = 1$.

The organization of this paper is as follows. In Section II, the notation and OSTBC data model are presented. Section III summarizes the blind identification method proposed in [30] and presents some previous results on blind identifiability of MIMO channels under OSTBC transmissions. The main contribution of the paper is presented in Section IV. Section V introduces some properties of skew-Hermitian matrices, which are exploited in Section VI to prove the results in Section IV. Finally, additional discussions and numerical examples are presented in Section VII, and the main conclusions are summarized in Section VIII.

II. Some Background on OSTBCs

A. Notation

1) Vectors/Matrices: Throughout this paper, we will use bold-faced upper case letters to denote matrices, e.g., $\mathbf{X}$, with elements $x_{i,j}$; bold-faced lower case letters for column vector, e.g., $\mathbf{x}$, and light-face lower case letters for scalar quantities. The superscripts $(\cdot)^T$ and $(\cdot)^H$ denote transpose and Hermitian, respectively. The real and imaginary parts will be denoted as $\Re(\cdot)$ and $\Im(\cdot)$, and superscript $\hat{(\cdot)}$ will denote estimated matrices, vectors or scalars. The trace, range (or column space) and Frobenius norm of matrix $\mathbf{A}$ will be denoted as $\mathrm{Tr}(\mathbf{A})$, $\mathrm{range}(\mathbf{A})$ and $\|\mathbf{A}\|$, respectively. The notation $\mathbf{A} \in \mathbb{C}^{M \times N}$ and $\mathbf{A} \in \mathbb{R}^{M \times N}$ will be used to denote that $\mathbf{A}$ is a complex or real matrix of dimension $M \times N$. Finally, the identity and zero matrices of dimensions $p \times p$ will be denoted as $\mathbf{I}_p$ and $\mathbf{0}_p$ (although the subindex will be omitted when confusion is not possible), $E[\cdot]$ will denote the expectation operator and $\lceil q \rceil$ will denote the smallest integer greater or equal than q.

2) MIMO Parameters: In this paper, a flat-fading MIMO system with n_T transmit and n_R receive antennas is assumed. The $n_T \times n_R$ complex channel matrix is

$$\mathbf{H} = [\mathbf{h}_1 \cdots \mathbf{h}_{n_R}] = \begin{bmatrix} h_{1,1} & \cdots & h_{1,n_R} \\ \vdots & \ddots & \vdots \\ h_{n_T,1} & \cdots & h_{n_T,n_R} \end{bmatrix}$$

where $h_{i,j}$ denotes the channel response between the ith transmit and the jth receive antennas, and $\mathbf{h}_j$ contains the channel response associated with the jth receive antenna. The complex noise at the receive antennas is considered both spatially and temporally white with variance σ^2.

B. Data Model for STBCs

Let us consider a space–time block code (STBC) transmitting M symbols during L time slots and using n_T antennas at the transmitter side. The transmission rate is defined as $R = M/L$, and the number of real symbols M' transmitted in each block is

$$M' = \begin{cases} M & \text{for real constellations} \\ 2M & \text{for complex constellations.} \end{cases}$$

For a STBC, the nth block of data can be expressed as

$$\mathbf{S}[n] = \sum_{k=1}^{M'} \mathbf{C}_k s_k[n]$$

where $\mathbf{C}_k \in \mathbb{C}^{L \times n_T}$, $k = 1, \ldots, M'$, are the STBC code matrices

$$s_k[n] = \begin{cases} \Re(r_k[n]), & k \leq M \\ \Im(r_{k-M}[n]), & k > M \end{cases}$$

and $r_k[n]$ denotes the kth complex information symbol of the nth STBC block. In the case of real STBCs, the transmitted matrix $\mathbf{S}[n]$ and the code matrices $\mathbf{C}_k$ are real.

The combined effect of the STBC and the jth channel can be represented by the $L \times 1$ complex vectors

$$\mathbf{w}_k(\mathbf{h}_j) = \mathbf{C}_k\mathbf{h}_j, \qquad k = 1, \ldots, M'$$

and taking into account the isomorphism between complex vectors $\mathbf{w}_k(\mathbf{h}_j)$ and real vectors $\tilde{\mathbf{w}}_k(\mathbf{h}_j) = [\Re(\mathbf{w}_k(\mathbf{h}_j))^T, \Im(\mathbf{w}_k(\mathbf{h}_j))^T]^T$ we can define the extended code matrices with real elements

$$\tilde{\mathbf{C}}_k = \begin{bmatrix} \Re(\mathbf{C}_k) & -\Im(\mathbf{C}_k) \\ \Im(\mathbf{C}_k) & \Re(\mathbf{C}_k) \end{bmatrix}$$

which imply

$$\tilde{\mathbf{w}}_k(\mathbf{h}_j) = \tilde{\mathbf{C}}_k\tilde{\mathbf{h}}_j \tag{1}$$

with $\tilde{\mathbf{h}}_j = [\Re(\mathbf{h}_j)^T, \Im(\mathbf{h}_j)^T]^T$. The signal at the jth receive antenna is

$$\mathbf{y}_j[n] = \mathbf{S}[n]\mathbf{h}_j = \sum_{k=1}^{M'} \mathbf{w}_k(\mathbf{h}_j)s_k[n] + \mathbf{n}_j[n]$$

where $\mathbf{n}_j[n]$ is the white complex noise with variance σ^2.

Defining now the real vectors

$$\tilde{\mathbf{y}}_j[n] = [\Re(\mathbf{y}_j[n])^T, \Im(\mathbf{y}_j[n])^T]^T$$

and

$$\tilde{\mathbf{n}}_j[n] = [\Re(\mathbf{n}_j[n])^T, \Im(\mathbf{n}_j[n])^T]^T$$

the above equation can be rewritten as

$$\tilde{\mathbf{y}}_j[n] = \sum_{k=1}^{M'} \tilde{\mathbf{w}}_k(\mathbf{h}_j)s_k[n] + \tilde{\mathbf{n}}_j[n] = \tilde{\mathbf{W}}(\mathbf{h}_j)\mathbf{s}[n] + \tilde{\mathbf{n}}_j[n]$$

where $\mathbf{s}[n] = [s_1[n], \ldots, s_{M'}[n]]^T$ contains the M' transmitted real symbols and $\tilde{\mathbf{W}}(\mathbf{h}_j) = [\tilde{\mathbf{w}}_1(\mathbf{h}_j) \cdots \tilde{\mathbf{w}}_{M'}(\mathbf{h}_j)]$. Finally, stacking all the received signals into $\tilde{\mathbf{y}}[n] = [\tilde{\mathbf{y}}_1^T[n], \ldots, \tilde{\mathbf{y}}_{n_R}^T[n]]^T$, we can write

$$\tilde{\mathbf{y}}[n] = \tilde{\mathbf{W}}(\mathbf{H})\mathbf{s}[n] + \tilde{\mathbf{n}}[n]$$

where $\tilde{\mathbf{W}}(\mathbf{H}) = [\tilde{\mathbf{W}}^T(\mathbf{h}_1) \cdots \tilde{\mathbf{W}}^T(\mathbf{h}_{n_R})]^T$ is the equivalent channel, and $\tilde{\mathbf{n}}[n]$ is defined analogously to $\tilde{\mathbf{y}}[n]$.

If $\mathbf{H}$ is known at the receiver, and assuming a white Gaussian distribution for the noise, the coherent maximum likelihood (ML) decoder amounts to minimizing the following criterion [33]:

$$\arg\min_{\hat{\mathbf{s}}[n]} ||\tilde{\mathbf{y}}[n] - \tilde{\mathbf{W}}(\mathbf{H})\hat{\mathbf{s}}[n]||^2$$

subject to the constraint that the elements of $\hat{\mathbf{s}}[n]$ belong to a finite set $\mathcal{S}$. This is a NP-hard problem and optimal algorithms to solve it, such as *sphere decoding*, can be computationally expensive [5], [34]–[36].

C. Properties of OSTBCs

In the case of orthogonal STBCs (OSTBCs), the equivalent channel matrix $\tilde{\mathbf{W}}(\mathbf{H})$ satisfies

$$\tilde{\mathbf{W}}^T(\mathbf{H})\tilde{\mathbf{W}}(\mathbf{H}) = ||\mathbf{H}||^2\mathbf{I} \tag{2}$$

which reduces the complexity of the ML receiver to find the closest symbols to the estimated signal

$$\hat{\mathbf{s}}_{\mathrm{ML}}[n] = \frac{\tilde{\mathbf{W}}^T(\mathbf{H})\tilde{\mathbf{y}}[n]}{||\mathbf{H}||^2}.$$

The necessary and sufficient conditions on the code matrices to satisfy (2) are [33], for $k = 1, \ldots, M'$,

$$\mathbf{C}_k^H\mathbf{C}_l = \begin{cases} \mathbf{I}, & k = l \\ -\mathbf{C}_l^H\mathbf{C}_k, & k \neq l. \end{cases} \tag{3}$$

It is straightforward to prove that the above condition must be also satisfied by the real extended code matrices

$$\tilde{\mathbf{C}}_k^T\tilde{\mathbf{C}}_l = \begin{cases} \mathbf{I}, & k = l \\ -\tilde{\mathbf{C}}_l^T\tilde{\mathbf{C}}_k, & k \neq l. \end{cases}$$

The following properties are direct consequences of (2) and (3)

Property 2.1: The transmitted signals using an OSTBC satisfy $\mathbf{S}^H[n]\mathbf{S}[n] = ||\mathbf{s}[n]||^2\mathbf{I}$.

Property 2.2: Given the OSTBC code matrices $\mathbf{C}_k \in \mathbb{C}^{L \times n_T}, k = 1, \ldots, M'$, and a pair of unitary matrices $\mathbf{Q}_1 \in \mathbb{C}^{L \times L}$ and $\mathbf{Q}_2 \in \mathbb{C}^{n_T \times n_T}$, the modified matrices

$$\mathbf{A}_k = \mathbf{Q}_1\mathbf{C}_k\mathbf{Q}_2, \qquad k = 1, \ldots, M'$$

define a new OSTBC with the same parameters n_T, L and M'.

Property 2.3: Given the OSTBC code matrices $\mathbf{C}_k \in \mathbb{C}^{L \times n_T}, k = 1, \ldots, M'$, and an orthogonal matrix $\mathbf{Q} \in \mathbb{R}^{M' \times M'}$ with elements $q_{k,l}$ in its kth row and lth column, the matrices

$$\mathbf{B}_k = \sum_{l=1}^{M'} q_{k,l}\mathbf{C}_l$$

define a new OSTBC with the same parameters n_T, L and M'.

III. Previous Work on Blind OSTBC Channel Estimation

In this paper, we consider the problem of blind channel estimation from second-order statistics (SOS) without assuming any particular structure in the correlation matrix $\mathbf{R_s} = E[\mathbf{s}[n]\mathbf{s}^T[n]]$ of the sources. In the case of correlated or linearly precoded sources, it is easy to prove that the channel can be recovered up to a sign change [30], [37]. However, the transmission of correlated sources translates into a penalty in the system capacity, and the assumption of known correlation matrices introduces a noise floor in the channel estimate, which is due to the difference between theoretic and estimated statistics [30].

In the case of uncorrelated sources, or sources with an unknown correlation matrix $\mathbf{R_s}$, the channel can be estimated by means of the method proposed in [30], which is equivalent to the relaxed ML technique for joint channel estimation and symbol detection. This approach is able to blindly identify the channel, up to a real scalar ambiguity, in most of the analyzed OSTBCs when the number of receive antennas is $n_R > 1$. However, some OSTBCs (including the Alamouti code [9]) do not permit the unambiguous blind channel estimation by using this technique.

In this section, the method proposed in [30] is summarized, and the identifiability condition for a given code and channel realization is introduced. Additionally, we point out that this technique does not introduce additional ambiguities to those associated to the problem of blind channel estimation from SOS. Finally, we summarize some previous results on blind channel identifiability.

A. A SOS-Based Criterion

Given a set of N observations $\tilde{\mathbf{y}}[0],\ldots,\tilde{\mathbf{y}}[N-1]$, and assuming a Gaussian distribution for the noise, the relaxed blind ML decoder amounts to minimizing

$$\arg\min_{\hat{\mathbf{H}},\hat{\mathbf{s}}[0],\ldots,\hat{\mathbf{s}}[N-1]} \sum_{n=0}^{N-1} \|\tilde{\mathbf{y}}[n] - \tilde{\mathbf{W}}(\hat{\mathbf{H}})\hat{\mathbf{s}}[n]\|^2. \quad (4)$$

Here, it is straightforward to see that the relaxation of the finite alphabet constraint introduces a real scalar ambiguity affecting $\hat{\mathbf{H}}$ and $\hat{\mathbf{s}}[n]$, i.e., if $\hat{\mathbf{H}}, \hat{\mathbf{s}}[n]$ is a solution of (4), then $c\hat{\mathbf{H}}, c^{-1}\hat{\mathbf{s}}[n]$, with $c \neq 0$ an arbitrary real scalar, also minimizes (4). Therefore, from now on we will assume, without loss of generality

$$\tilde{\mathbf{W}}^T(\hat{\mathbf{H}})\tilde{\mathbf{W}}(\hat{\mathbf{H}}) = \|\hat{\mathbf{H}}\|^2\mathbf{I} = \mathbf{I}.$$

The solution of (4) with respect to $\hat{\mathbf{s}}[n]$ is

$$\hat{\mathbf{s}}[n] = \frac{\tilde{\mathbf{W}}^T(\hat{\mathbf{H}})\tilde{\mathbf{y}}[n]}{\|\hat{\mathbf{H}}\|^2}, \qquad n = 0,\ldots,N-1$$

and substituting in (4), the final channel estimation criterion can be rewritten as

$$\arg\max_{\hat{\mathbf{H}}} \mathrm{Tr}(\tilde{\mathbf{W}}^T(\hat{\mathbf{H}})\mathbf{R}_{\tilde{\mathbf{y}}}\tilde{\mathbf{W}}(\hat{\mathbf{H}})), \quad \text{s.t.} \quad \|\hat{\mathbf{H}}\| = 1 \quad (5)$$

where

$$\mathbf{R}_{\tilde{\mathbf{y}}} = \frac{1}{N}\sum_{n=0}^{N-1} \tilde{\mathbf{y}}[n]\tilde{\mathbf{y}}^T[n]$$

can be interpreted as the finite sample estimate of the correlation matrix associated to the observations $\tilde{\mathbf{y}}[n]$.

Assuming a persistently exciting source signal, i.e., considering that the matrix

$$[\mathbf{s}[0],\ldots,\mathbf{s}[N-1]]$$

is full row rank, the theoretical[1] M' principal eigenvectors of $\mathbf{R}_{\tilde{\mathbf{y}}}$ span the subspace defined by the columns of $\tilde{\mathbf{W}}(\mathbf{H})$, and then it is easy to prove [30] that the theoretical solutions to (5) satisfy

$$\mathrm{range}(\tilde{\mathbf{W}}(\hat{\mathbf{H}})) = \mathrm{range}(\tilde{\mathbf{W}}(\mathbf{H})). \quad (6)$$

[1]These are the principal eigenvectors in the absence of noise and asymptotically when $N \to \infty$.

Finally, it is interesting to point out that the proposed criterion is deterministic, i.e., considering a noise-free situation, and assuming that (6) is only satisfied by $\hat{\mathbf{H}} = c\mathbf{H}$ (being c a real scale factor), the method is able to exactly recover the channel up to a real scalar within a finite number of received blocks. Furthermore, it can be easily proven that, although in the derivation of the relaxed blind ML decoder we have assumed a Gaussian noise distribution, the criterion in (5) is able to recover the channel for any uncorrelated noise distribution [30].

B. Algorithm Implementation-PCA Equivalence

Let us start by rewriting the channel identification criterion (5) as

$$\arg\max_{\hat{\mathbf{H}}} \sum_{k=1}^{M'} \tilde{\mathbf{w}}_k^T(\hat{\mathbf{H}})\mathbf{R}_{\tilde{\mathbf{y}}}\tilde{\mathbf{w}}_k(\hat{\mathbf{H}}) \quad \text{s.t.} \ \|\hat{\mathbf{H}}\| = \|\hat{\hat{\mathbf{h}}}\| = 1$$

where $\tilde{\mathbf{w}}_k(\hat{\mathbf{H}}) = [\tilde{\mathbf{w}}_k^T(\hat{\mathbf{h}}_1),\ldots,\tilde{\mathbf{w}}_k^T(\hat{\mathbf{h}}_{M'})]^T$ is the kth column of $\tilde{\mathbf{W}}(\hat{\mathbf{H}})$ and $\hat{\hat{\mathbf{h}}} = [\hat{\hat{\mathbf{h}}}_1^T,\ldots,\hat{\hat{\mathbf{h}}}_{n_R}^T]^T$.

Taking (1) into account, the channel estimation criterion can be rewritten as

$$\arg\max_{\hat{\hat{\mathbf{h}}}} \hat{\hat{\mathbf{h}}}^T\tilde{\mathbf{Z}}^T\tilde{\mathbf{Z}}\hat{\hat{\mathbf{h}}} \quad \text{s.t.} \ \|\hat{\hat{\mathbf{h}}}\| = 1 \quad (7)$$

where the modified data matrix is defined as $\tilde{\mathbf{Z}} = [\tilde{\mathbf{Z}}[0]^T \cdots \tilde{\mathbf{Z}}[N-1]^T]^T$, and

$$\tilde{\mathbf{Z}}[n] = \begin{bmatrix} \tilde{\mathbf{y}}_1^T[n]\tilde{\mathbf{C}}_1 & \cdots & \tilde{\mathbf{y}}_{n_R}^T[n]\tilde{\mathbf{C}}_1 \\ \vdots & \ddots & \vdots \\ \tilde{\mathbf{y}}_1^T[n]\tilde{\mathbf{C}}_{M'} & \cdots & \tilde{\mathbf{y}}_{n_R}^T[n]\tilde{\mathbf{C}}_{M'} \end{bmatrix}$$

i.e., the criterion (5) has been reduced to a principal component analysis (PCA) problem. This reformulation permits a straightforward derivation of adaptive versions of the algorithm [30], for instance, by direct application of the Oja's rule [37], [38]. Finally, we must note that, once the channel has been extracted, and under white Gaussian noise, the relaxed ML estimate of the signal is

$$\hat{\mathbf{s}}[n] = \tilde{\mathbf{Z}}[n]\hat{\hat{\mathbf{h}}}.$$

C. Indeterminacy Problems

As previously pointed out, the relaxation of the finite alphabet constraint translates into a real scalar ambiguity in the channel estimate. Fortunately, this ambiguity is not important in practice, and it can be resolved in a later step, for instance, taking into account the total transmitted energy. Thus, we can assume $\|\hat{\mathbf{H}}\| = \|\mathbf{H}\| = 1$.

A more important indeterminacy is revealed by (6), which implies that, if there exists an estimated channel $\hat{\mathbf{H}} \neq c\mathbf{H}$ such that its associated equivalent channel matrix $\tilde{\mathbf{W}}(\hat{\mathbf{H}})$ spans the same subspace as $\tilde{\mathbf{W}}(\mathbf{H})$, then the MIMO channel cannot be unambiguously recovered by the method proposed in [30]. An interesting question is whether this ambiguity problem is associated to this specific SOS-based estimation method, i.e., if the channel could be unambiguously estimated by a different SOS-based technique. Here, we prove that, unlike other approaches [14],

[29], [31], the criterion in (7) does not introduce additional ambiguities to those associated to the problem of blind channel estimation from SOS.

Let us start by rewriting (6) as $\tilde{\mathbf{W}}(\hat{\mathbf{H}}) = \tilde{\mathbf{W}}(\mathbf{H})\mathbf{Q}(\mathbf{H})$, where $\mathbf{Q}(\mathbf{H})$ is an orthogonal (i.e., real and unitary) matrix of dimensions $M' \times M'$, and introducing the following lemma.

Lemma 3.1: In OSTBC systems, the MIMO channel $\mathbf{H}$ can be identified up to a real scalar based only on second-order statistics iff the equality

$$\tilde{\mathbf{W}}(\hat{\mathbf{H}}) = \tilde{\mathbf{W}}(\mathbf{H})\mathbf{Q}(\mathbf{H}) \tag{8}$$

where $\mathbf{Q}(\mathbf{H})$ is an orthogonal matrix, is only satisfied by $\hat{\mathbf{H}} = \pm\mathbf{H}$ and $\mathbf{Q}(\mathbf{H}) = \pm\mathbf{I}$.

Proof: From (6) it is clear that, if (8) is only satisfied by $\hat{\mathbf{H}} = \pm\mathbf{H}$ and $\mathbf{Q}(\mathbf{H}) = \pm\mathbf{I}$, the channel can be estimated up to a real scalar ambiguity by means of the criterion in (7). Assuming that (8) is satisfied by $\hat{\mathbf{H}} \neq \pm\mathbf{H}$ and $\mathbf{Q}(\mathbf{H}) \neq \pm\mathbf{I}$, then we can define $\hat{\mathbf{s}}[n] = \mathbf{Q}^T(\mathbf{H})\mathbf{s}[n]$ such that

$$\begin{aligned}\tilde{\mathbf{y}}[n] &= \tilde{\mathbf{W}}(\mathbf{H})\mathbf{s}[n] + \tilde{\mathbf{n}}[n] \\ &= \tilde{\mathbf{W}}(\mathbf{H})\mathbf{Q}(\mathbf{H})\mathbf{Q}^T(\mathbf{H})\mathbf{s}[n] + \tilde{\mathbf{n}}[n] \\ &= \tilde{\mathbf{W}}(\hat{\mathbf{H}})\hat{\mathbf{s}}[n] + \tilde{\mathbf{n}}[n],\end{aligned}$$

which implies that the observation vector $\tilde{\mathbf{y}}[n]$ could be the result of a channel $\hat{\mathbf{H}}$ and a signal $\hat{\mathbf{s}}[n]$ instead of the true channel $\mathbf{H}$ and signal $\mathbf{s}[n]$. □

The above lemma implies that if the method proposed in [30] is not able to identify the channel, it is because the channel cannot be unambiguously identified without exploiting other properties of $\mathbf{s}[n]$, such as its belonging to a finite alphabet, or a known and colored correlation matrix $\mathbf{R_s}$ [1], [30], [37]. Finally, from a practical point of view, the indeterminacy in (6) translates into a multiplicity $P > 1$ of the largest eigenvalue of the matrix $\tilde{\mathbf{Z}}^T\tilde{\mathbf{Z}}$ in (7) [30].

D. Previous Work on Blind Identifiability

The ambiguity problems associated to the blind channel estimation problem have been observed in several works. For instance, in [14] the authors propose a method similar to the relaxed blind ML estimator. In particular, the finite alphabet constraint is relaxed, the channel is expressed as a function of the information symbols, and the estimated symbols are obtained by means of an eigenvalue problem similar to (7). The identifiability problem in [14] is again related with the multiplicity of the largest eigenvalue of the associated eigenvalue problem. However, in [14] the identifiability analysis is reduced to ensure that the OSTBCs with $M = n_T$ are not identifiable, which we will prove is wrong.

Recently, some other works have tried to analyze the identifiability conditions of OSTBCs. In [29] the authors have pointed out that it is impossible to achieve blind equalization for the Alamouti code [9] without using some precoding or assuming a correlation matrix $\mathbf{R_s}$ with nonequal eigenvalues. In [31], it has been proven that, for real OSTBCs, if the symbol dimension M is odd, or if the number of transmit antennas is odd and the channel matrix $\mathbf{H}$ is full row rank ($n_R \geq n_T$), then the channel is identifiable up to a scalar ambiguity based solely on SOS. In [32], the author studies the identifiability conditions considering the blind ML decoder and assuming BPSK or QPSK signals. Specifically, the concepts of nonrotatable and strictly nonrotatable codes are introduced, and some new identifiability results have been obtained for the particular cases of real OSTBCs with an odd number of transmit antennas, and OSTBCs with an odd number of real information symbols. However, to the best of our knowledge, the question of why some particular OSTBCs provoke the ambiguity, while others not, remains unclear. In other words: What are the conditions, related to the underlying structure of the OSTBC, which yield a nonidentifiable code? The goal of this paper is to shed some light on this point.

IV. New Results on Blind Identifiability of OSTBCS

In this section, we establish sufficient conditions for the blind identifiability of MIMO channels under OSTBC transmissions. These conditions validate the experimental results in [30], and they relate the identifiability properties of the code with its transmission rate. Specifically, we derive a threshold in the transmission rate, which decreases with the number of transmit antennas, and ensure that all the OSTBCs transmitting with a higher rate are identifiable. Interestingly, the conditions include, as a particular case, the results in [31], and in some cases they are closely related to the conditions in [32], where the identifiability analysis is based on the finite alphabet properties of the sources.

A. Main Results

Let us start by introducing the following definition.

Definition 4.1 (Identifiable OSTBCs): An OSTBC is said to be identifiable iff there exists at least one channel $\mathbf{H}$ such that the equality

$$\tilde{\mathbf{W}}(\hat{\mathbf{H}}) = \tilde{\mathbf{W}}(\mathbf{H})\mathbf{Q}(\mathbf{H})$$

with $\mathbf{Q}(\mathbf{H})$ an orthogonal matrix, is only satisfied by $\hat{\mathbf{H}} = \pm\mathbf{H}$ and $\mathbf{Q}(\mathbf{H}) = \pm\mathbf{I}$. Otherwise, the OSTBC is said to be nonidentifiable.

The following theorem ensures blind channel identifiability for any number of receive antennas.

Theorem 4.1: If an OSTBC transmits an odd number of real symbols (M' odd), then the channel can be identified, from SOS, regardless of the number of receive antennas.

The following theorems state sufficient conditions for an OSTBC to be identifiable.

Theorem 4.2: All the real OSTBCs with an odd number of transmit antennas n_T are identifiable.

Theorem 4.3: If an OSTBC with n_T transmit antennas, and transmitting M' real symbols over L slots satisfies

$$\frac{M'}{L} > \frac{2}{\lceil \frac{n_T}{2} \rceil}$$

then, the code is identifiable.

From Definition 4.1, we know that for any identifiable OSTBC there exists at least one channel $\mathbf{H}$ such that the

TABLE I
IDENTIFIABILITY CHARACTERISTICS FOR THE MOST COMMON OSTBCs

Constellation	n_T	M	L	$R = M/L$	R_{th}	Identifiable	Design	Multiplicity ($n_R = 1$)	Multiplicity ($n_R > 1$)
real	2	2	2	1	2	No	Alamouti	2	2
real	3	4	4	1	1	Yes	gen. ort	2	1
real	4	4	4	1	1	No	gen. ort	4	4
real	5	8	8	1	2/3	Yes	gen. ort	2	1
real	6	8	8	1	2/3	Yes	gen. ort	2	1
real	7	8	8	1	1/2	Yes	gen. ort	2	1
real	8	8	8	1	1/2	Yes	gen. ort	2	1
complex	2	2	2	1	1	No	Alamouti	4	4
complex	3	4	8	1/2	1/2	Yes	gen. ort	2	1
complex	4	4	8	1/2	1/2	No	gen. ort	4	4
complex	5	8	16	1/2	1/3	Yes	gen. ort	2	1
complex	6	8	16	1/2	1/3	Yes	gen. ort	2	1
complex	7	8	16	1/2	1/4	Yes	gen. ort	2	1
complex	8	8	16	1/2	1/4	Yes	gen. ort	2	1
complex	3	3	4	3/4	1/2	Yes	amicable	2	1
complex	4	3	4	3/4	1/2	Yes	amicable	2	1
complex	5	4	8	1/2	1/3	Yes	amicable	1	1
complex	6	4	8	1/2	1/3	Yes	amicable	1	1
complex	7	4	8	1/2	1/4	Yes	amicable	1	1
complex	8	4	8	1/2	1/4	Yes	amicable	1	1

criterion in (7) is able to recover the channel with the only ambiguity of a real scale factor. Additionally, taking into account Definition 4.1 it is easy to prove the following theorem:

Theorem 4.3: For a MIMO system transmitting with an identifiable OSTBC and a full row rank channel matrix $\mathbf{H}$ $(n_R \geq n_T)$, the channel can be extracted, up to a real scalar, from SOS.

The above theorem constitutes a sufficient condition for blind channel identifiability based on SOS. However, simulation results have shown that the full row rank condition on the channel matrix is not necessary for channel identification (see Section VII and [30]). In order to shed some light into the cases with $n_R < n_T$, we introduce the following condition:

Condition 4.1: The MIMO channel matrix $\mathbf{H}$ is complex circular Gaussian distributed, and the correlation matrix $E[\tilde{\mathbf{h}}_j \tilde{\mathbf{h}}_j^T]$ associated to all the multiple-input–single-output (MISO) channels $\tilde{\mathbf{h}}_j$ is full rank regardless of the value of the remaining MISO channels.

Note that the above condition includes, as a particular case, the popular independent and identically distributed (i.i.d.) Rayleigh channel model. This is a sufficient, but not necessary condition for the proof of the following theorem.

Theorem 4.5: Consider an identifiable OSTBC and a MIMO channel $\mathbf{H}$ such that the multiplicity of the largest eigenvalue of (7) is $P > 1$. Then, under Condition 4.1, the addition of a new receive antenna will decrease the multiplicity P of the new blind channel estimation problem with probability one.

As a direct consequence of the previous theorem we can state the following corollary.

Corollary 4.1: Consider an identifiable OSTBC such that, for $n_R = 1$, the multiplicity of the blind channel estimation problem is P with probability one. Then, for $n_R \geq P$, the MIMO channel can be extracted, up to a real scalar, with probability one.

The relevance of the above results is increased by the following conjecture, which has been validated by means of numerical results (see Table I and [30]):

Conjecture 4.1: Consider an identifiable OSTBC and a MISO channel satisfying Condition 4.1, then, the multiplicity of the largest eigenvalue of the PCA problem in (7) is $P \leq 2$ with probability one.

Thus, we can state the following corollary.

Corollary 4.2: Assuming an identifiable OSTBC, and under Condition 4.1, if the number of receive antennas is $n_R > 1$, the channel can be extracted from SOS, and up to a real scalar, with probability one.

Finally, the proof of these results is provided in Section VI.

B. Further Discussion and Relationship With Other Results

Here we analyze, in more detail, the identifiability results.

- Theorem 4.1 extends to complex OSTBCs the first result in [31] reducing the ambiguity to a real scalar. Here it is important to note that the complex scalar ambiguity introduced by the algorithm in [31] translates into a non trivial indeterminacy in the decoded information symbols. On the other hand, in [32] the author has proven that when M' is odd, the code is a nonintersecting subspace OSTBC, which ensures blind identifiability when the blind ML decoder is used. Therefore, Theorem 4.1 proves this result avoiding the finite alphabet constraint.
- Theorem 4.2, in combination with Theorem 4.4, is equivalent to the second result in [31]. However, in combination with Theorem 4.5 or Corollary 4.2, the identifiability of the channel is explained for cases with $1 < n_R < n_T$. Analogously, in [32] it has been proven that real OSTBCs with odd n_T are strictly nonrotatable, which ensures blind identifiability, for any number of receive antennas, by exploiting the finite alphabet property of the sources. Theorem 4.2 can be seen as the SOS counterpart of that result.
- Theorem 4.3 is completely new. Interestingly, this result could seem rather counterintuitive, since the more redundant information (the less symbol rate) should promise the better identifiability of the code. However, we must take into account that Theorem 4.3 does not imply that OSTBCs with low rates are nonidentifiable. The right interpretation of this result, which can be clarified by the proof

in Section VI, is that nonidentifiable OSTBCs should have a very special structure, and in order to design OSTBCs with those properties, the transmission rate should remain under the given threshold. On the other hand, it seems sensible to think that, in the case of low transmission rates, the large number of degrees of freedom for the code design can be also exploited to obtain identifiable OSTBCs.

- Finally, taking into account Definition 4.1 it is straightforward to prove that identifiable codes are also nonrotatable [32], which, under mild assumptions, ensures the blind channel identifiability, for any number of receive antennas, by exploiting the finite alphabet property of the sources.

V. Preliminaries: Properties of Skew-Hermitian Matrices

In this section, some properties of skew-Hermitian and skew-symmetric matrices are introduced. They will be used in the next section to prove the main results of the paper. Let us start by defining skew-Hermitian and skew-symmetric matrices.

Definition 5.1 (Skew-Hermitian): A square matrix $\mathbf{A}$ with complex entries is skew-Hermitian, iff $\mathbf{A}^H = -\mathbf{A}$.

Definition 5.2 (Skew-symmetric): A square matrix $\mathbf{A}$ with real entries is skew-symmetric, iff $\mathbf{A}^T = -\mathbf{A}$.

Definition 5.3 (Unitarily/Orthogonally Equivalent [39]): A square matrix $\mathbf{A}$ is said to be *unitarily equivalent* to $\mathbf{B}$ if there is a unitary matrix $\mathbf{U}$ such that $\mathbf{B} = \mathbf{U}^H\mathbf{A}\mathbf{U}$. If $\mathbf{U}$ may be taken to be real (and hence is orthogonal), then $\mathbf{A}$ is said to be *orthogonally equivalent* to $\mathbf{B}$.

Some well-known properties of skew-symmetric and skew-Hermitian matrices are the following.

Property 5.1: All eigenvalues of skew-Hermitian matrices are purely imaginary or zero.

Property 5.2: If $\mathbf{A}$ is skew-symmetric the elements along its main diagonal are zero: $a_{i,i} = 0, \forall i$. Consequently, $\mathrm{Tr}(\mathbf{A}) = 0$.

A Proof of Properties 5.1 and 5.2 can be found in [39]. For unitary matrices it is easy to prove the following properties.

Property 5.3: The eigenvalue decomposition of a unitary skew-Hermitian matrix $\mathbf{A}$ can be written as $\mathbf{A} = \mathbf{X}\boldsymbol{\Sigma}\mathbf{X}^H$, where the eigenvalues in $\boldsymbol{\Sigma}$ are $+j$ or $-j$, and $\mathbf{X}$ is a unitary matrix.

Proof: Assuming the eigenvalue decomposition $\mathbf{A} = \mathbf{X}\boldsymbol{\Sigma}\mathbf{X}^{-1}$ and taking into account $\mathbf{A}^H\mathbf{A} = -\mathbf{A}\mathbf{A} = \mathbf{I}$ we can write

$$\mathbf{A}\mathbf{A} = \mathbf{X}\boldsymbol{\Sigma}\mathbf{X}^{-1}\mathbf{X}\boldsymbol{\Sigma}\mathbf{X}^{-1} = \mathbf{X}\boldsymbol{\Sigma}^2\mathbf{X}^{-1} = -\mathbf{I}$$

which implies $\mathbf{X}\boldsymbol{\Sigma}^2 = -\mathbf{X}$ and $\boldsymbol{\Sigma}^2 = -\mathbf{I}$. Hence, the eigenvalues of $\mathbf{A}$ are $\pm j$. Denoting the orthonormal basis of the eigenvectors with eigenvalues $+j$ and $-j$ as $\mathbf{X}_1$ and $\mathbf{X}_2$, respectively, we can write $\mathbf{A}\mathbf{X}_1 = j\mathbf{X}_1, \mathbf{A}\mathbf{X}_2 = -j\mathbf{X}_2$ and then

$$\begin{aligned}\mathbf{X}_1^H\mathbf{X}_2 &= \mathbf{X}_1^H(\mathbf{A}^H\mathbf{A})\mathbf{X}_2\\ &= (j\mathbf{X}_1)^H(-j\mathbf{X}_2) = -\mathbf{X}_1^H\mathbf{X}_2 = \mathbf{0}\end{aligned}$$

i.e., the eigenvectors of $\mathbf{A}$ can be chosen to form an orthonormal basis. □

Property 5.4: Any orthogonal skew-symmetric matrix has even order and the same number of $+j$ and $-j$ eigenvalues.

Proof: Considering Properties 5.2 and 5.3, and taking into account that the trace of a matrix is equal to the sum of its eigenvalues, it is clear that an orthogonal skew-symmetric matrix must have the same number of $+j$ and $-j$ eigenvalues, and hence, its order must be even. □

Property 5.5: Any pair of unitary skew-Hermitian matrices with the same eigenvalues is unitarily equivalent.

Proof: Let $\mathbf{A}_1$ and $\mathbf{A}_2$ be two skew-Hermitian matrices with the same eigenvalues. Writing their eigenvalue decompositions as $\mathbf{A}_1 = \mathbf{X}_1\boldsymbol{\Sigma}\mathbf{X}_1^H$ and $\mathbf{A}_2 = \mathbf{X}_2\boldsymbol{\Sigma}\mathbf{X}_2^H$, it is easy to prove that

$$\mathbf{A}_1 = \mathbf{B}\mathbf{A}_2\mathbf{B}^H$$

where $\mathbf{B} = \mathbf{X}_1\mathbf{X}_2^H$ is a unitary matrix. □

Property 5.6: Any pair of orthogonal skew-symmetric matrices is orthogonally equivalent.

Proof: This is a direct consequence of Properties 5.4 and 5.5. □

In other words, Property 5.6 states that there exists an orthonormal change of basis between any pair of orthogonal skew-symmetric matrices.

VI. Proof of the Main Theorems

The organization of this section is as follows: firstly, the properties of the indeterminacy matrices are studied, which is used to prove Theorem 4.1. Secondly, (8) is rewritten as a linear set of equations relating $\hat{\mathbf{H}}$ and $\mathbf{H}$, and the nonidentifiable OSTBCs are analyzed, proving Theorems 4.2 and 4.3. Finally, the identifiable codes are studied, and Theorems 4.4 and 4.5 are proven.

A. Properties of the Indeterminacy Matrices: Proof of Theorem 4.1

We first extend Lemma 3.1 by showing that the orthogonal matrix $\mathbf{Q}(\mathbf{H})$ in (8) must also be skew-symmetric, i.e., $\mathbf{Q}^T(\mathbf{H}) = -\mathbf{Q}(\mathbf{H})$.

Lemma 6.1: In OSTBC systems, the MIMO channel $\mathbf{H}$ can be identified up to a real scalar based only on second-order statistics iff the equality

$$\tilde{\mathbf{W}}(\hat{\mathbf{H}}) = \tilde{\mathbf{W}}(\mathbf{H})\mathbf{Q}(\mathbf{H}) \tag{9}$$

cannot be satisfied for any orthogonal skew-symmetric matrix $\mathbf{Q}(\mathbf{H})$ of dimensions $M' \times M'$.

Proof: Rewriting (8) as

$$\tilde{\mathbf{W}}(\hat{\mathbf{h}}_j) = \tilde{\mathbf{W}}(\mathbf{h}_j)\mathbf{Q}(\mathbf{H}), \qquad j = 1, \ldots, n_R$$

and multiplying from the left by $\tilde{\mathbf{W}}^T(\mathbf{h}_j)$ we obtain

$$\tilde{\mathbf{W}}^T(\mathbf{h}_j)\tilde{\mathbf{W}}(\hat{\mathbf{h}}_j) = \|\mathbf{h}_j\|^2\mathbf{Q}(\mathbf{H}), \qquad j = 1, \ldots, n_R.$$

Taking into account that $\tilde{\mathbf{W}}(\mathbf{h}_j) = [\tilde{\mathbf{w}}_1(\mathbf{h}_j) \cdots \tilde{\mathbf{w}}_{M'}(\mathbf{h}_j)]$ and $\tilde{\mathbf{w}}_k(\mathbf{h}_j) = \tilde{\mathbf{C}}_k \tilde{\mathbf{h}}_j$, we can write the element $q_{k,l}$ in the kth row and lth column of $\mathbf{Q}(\mathbf{H})$ as

$$q_{k,l} = \frac{\tilde{\mathbf{w}}_k^T(\mathbf{h}_j)\tilde{\mathbf{w}}_l(\hat{\mathbf{h}}_j)}{\|\mathbf{h}_j\|^2} = \frac{\tilde{\mathbf{h}}_j^T \tilde{\mathbf{C}}_k^T \tilde{\mathbf{C}}_l \hat{\tilde{\mathbf{h}}}_j}{\|\mathbf{h}_j\|^2}, \qquad j = 1, \ldots, n_R$$

and considering that, for $k \neq l, \tilde{\mathbf{C}}_k^T \tilde{\mathbf{C}}_l = -\tilde{\mathbf{C}}_l^T \tilde{\mathbf{C}}_k$, the above equation implies

$$q_{k,l} = \begin{cases} \frac{\tilde{\mathbf{h}}_j^T \hat{\tilde{\mathbf{h}}}_j}{\|\mathbf{h}_j\|^2} & k = l, \\ -q_{l,k} & k \neq l, \end{cases} \qquad j = 1, \ldots, n_R.$$

Therefore, it can be easily proven that $\mathbf{Q}(\mathbf{H})$ can be written as

$$\mathbf{Q}(\mathbf{H}) = \alpha \mathbf{I} + \sqrt{1-\alpha^2}\mathbf{Q}^{\perp}(\mathbf{H})$$

where $\alpha = \tilde{\mathbf{h}}_j^T \hat{\tilde{\mathbf{h}}}_j / \|\mathbf{h}_j\|^2$, $j = 1, \ldots, n_R$, and $\mathbf{Q}^{\perp}(\mathbf{H})$ is an orthogonal and skew-symmetric matrix. Thus, (8) yields

$$\tilde{\mathbf{W}}(\hat{\mathbf{H}}) = \alpha \tilde{\mathbf{W}}(\mathbf{H}) + \sqrt{1-\alpha^2}\tilde{\mathbf{W}}(\mathbf{H})\mathbf{Q}^{\perp}(\mathbf{H})$$

which implies that, assuming $\hat{\mathbf{H}} \neq \pm \mathbf{H}$, i.e., $\alpha \neq \pm 1$, we can find a channel $\mathbf{H}^{\perp} = \frac{\hat{\mathbf{H}} - \alpha \mathbf{H}}{\sqrt{1-\alpha^2}}$ satisfying

$$\tilde{\mathbf{W}}(\mathbf{H}^{\perp}) = \tilde{\mathbf{W}}(\mathbf{H})\mathbf{Q}^{\perp}(\mathbf{H}).$$

In other words, we can assume, without loss of generality, that in (8), matrix $\mathbf{Q}(\mathbf{H})$ is orthogonal and skew-symmetric, and $\hat{\mathbf{H}}$ satisfies $\tilde{\mathbf{h}}_j^T \hat{\tilde{\mathbf{h}}}_j = 0$ for $j = 1, \ldots, n_R$. □

The combination of Lemma 6.1 and Property 5.4 yields the Proof of Theorem 4.1.

Proof: (Theorem 4.1): The proof proceeds by contradiction. Let us assume that the channel cannot be unambiguously identified for an OSTBC transmitting an odd number of real symbols (M' odd), then from Lemma 6.1 there exists an orthogonal and skew-symmetric matrix $\mathbf{Q}(\mathbf{H})$ of dimensions $M' \times M'$ relating $\tilde{\mathbf{W}}(\hat{\mathbf{H}})$ and $\tilde{\mathbf{W}}(\mathbf{H})$. From Property 5.4 it is clear that an orthogonal skew-symmetric matrix of odd order cannot exist and therefore the MIMO channel can be identified up to a real scalar. □

Theorem 4.1 establishes that any OSTBC transmitting an odd number of real symbols permits the blind identification of the MIMO channel. For real OSTBCs, this results is only of limited value from a practical standpoint since most of the useful codes transmit an even number of real symbols (see [30], [33]). On the other hand, for complex OSTBCs we obviously have $M' = 2M$, where M is the number of complex symbols. Therefore M' is always even and the theorem does not apply. However, an interesting idea derived from this theorem is that a nonidentifiable complex OSTBC can be made identifiable simply by not transmitting one real symbol (either the real or imaginary part of a symbol in the case of complex OSTBCs) [1]. Obviously, the price we pay is a reduction in the code rate: for a complex OSTBC transmitting $M = 4$ symbols the original code rate would be reduced by $7/8$. This idea will be illustrated by computer simulations in Section VII-B.

B. Nonidentifiable OSTBCs: Proofs of Theorems 4.2 and 4.3

In this section, we study the ambiguity conditions in the case of nonidentifiable OSTBCs. We will start by taking into account $\tilde{\mathbf{W}}(\mathbf{H}) = [\tilde{\mathbf{W}}^T(\mathbf{h}_1) \cdots \tilde{\mathbf{W}}^T(\mathbf{h}_{n_R})]^T$ and $\tilde{\mathbf{w}}_k(\mathbf{h}_j) = \tilde{\mathbf{C}}_k \tilde{\mathbf{h}}_j$, which can be used to easily prove that the relationship in (9) is equivalent to

$$\begin{bmatrix} \tilde{\mathbf{C}}_1 \\ \vdots \\ \tilde{\mathbf{C}}_{M'} \end{bmatrix} \tilde{\mathbf{H}} = \tilde{\mathbf{P}}_{\mathbf{Q}(\mathbf{H})} \begin{bmatrix} \tilde{\mathbf{C}}_1 \\ \vdots \\ \tilde{\mathbf{C}}_{M'} \end{bmatrix} \hat{\tilde{\mathbf{H}}} \tag{10}$$

where $\tilde{\mathbf{H}} = [\Re(\mathbf{H})^T \quad \Im(\mathbf{H})^T], \tilde{\mathbf{P}}_{\mathbf{Q}(\mathbf{H})} \in \mathbb{R}^{2LM' \times 2LM'}$ is a block matrix given by

$$\tilde{\mathbf{P}}_{\mathbf{Q}(\mathbf{H})} = \begin{bmatrix} q_{1,1}\mathbf{I}_{2L} & \cdots & q_{1,M'}\mathbf{I}_{2L} \\ \vdots & \ddots & \vdots \\ q_{M',1}\mathbf{I}_{2L} & \cdots & q_{M',M'}\mathbf{I}_{2L} \end{bmatrix}$$

and $q_{k,l}$ are the elements of $\mathbf{Q}(\mathbf{H})$, which implies that $\tilde{\mathbf{P}}_{\mathbf{Q}(\mathbf{H})}$ is also orthogonal and skew-symmetric. Furthermore, considering the structure of the matrices $\tilde{\mathbf{C}}_k$ and $\tilde{\mathbf{P}}_{\mathbf{Q}(\mathbf{H})}$, (10) can be rewritten as

$$\begin{bmatrix} \mathbf{C}_1 \\ \vdots \\ \mathbf{C}_{M'} \end{bmatrix} \mathbf{H} = \mathbf{P}_{\mathbf{Q}(\mathbf{H})} \begin{bmatrix} \mathbf{C}_1 \\ \vdots \\ \mathbf{C}_{M'} \end{bmatrix} \hat{\mathbf{H}} \tag{11}$$

where $\mathbf{P}_{\mathbf{Q}(\mathbf{H})} \in \mathbb{R}^{LM' \times LM'}$ is an orthogonal and skew-symmetric matrix with $L \times L$ blocks defined as

$$\mathbf{P}_{\mathbf{Q}(\mathbf{H})} = \begin{bmatrix} q_{1,1}\mathbf{I}_{L} & \cdots & q_{1,M'}\mathbf{I}_{L} \\ \vdots & \ddots & \vdots \\ q_{M',1}\mathbf{I}_{L} & \cdots & q_{M',M'}\mathbf{I}_{L} \end{bmatrix}.$$

As a direct consequence of (11), and taking into account that $\mathbf{C}_k^H \mathbf{C}_k = \mathbf{I}$, we obtain $\hat{\mathbf{H}} = \mathbf{U}(\mathbf{H})\mathbf{H}$, where

$$\mathbf{U}(\mathbf{H}) = \frac{1}{M'} \begin{bmatrix} \mathbf{C}_1 \\ \vdots \\ \mathbf{C}_{M'} \end{bmatrix}^H \mathbf{P}_{\mathbf{Q}(\mathbf{H})}^T \begin{bmatrix} \mathbf{C}_1 \\ \vdots \\ \mathbf{C}_{M'} \end{bmatrix}$$

and (11) yields

$$\begin{bmatrix} \mathbf{C}_1 \\ \vdots \\ \mathbf{C}_{M'} \end{bmatrix} \mathbf{H} = \mathbf{P}_{\mathbf{Q}(\mathbf{H})} \begin{bmatrix} \mathbf{C}_1 \\ \vdots \\ \mathbf{C}_{M'} \end{bmatrix} \mathbf{U}(\mathbf{H})\mathbf{H}.$$

In the case of nonidentifiable OSTBCs, the above equation must hold even for full row rank channel matrices, which implies that the dependency on $\mathbf{H}$ can be eliminated, i.e.

$$\begin{bmatrix} \mathbf{C}_1 \\ \vdots \\ \mathbf{C}_{M'} \end{bmatrix} = \mathbf{P}_{\mathbf{Q}} \begin{bmatrix} \mathbf{C}_1 \\ \vdots \\ \mathbf{C}_{M'} \end{bmatrix} \mathbf{U}. \tag{12}$$

Here, it is interesting to point out that (12) constitutes a necessary and sufficient condition for nonidentifiability of an

OSTBC, which will be used later to prove Theorem 4.4. Now, left-multiplying (12) by $[\mathbf{C}_1^H \cdots \mathbf{C}_{M'}^H]\mathbf{P}_{\mathbf{Q}}^T$ we can conclude that $\mathbf{U}$ is a unitary and skew-Hermitian matrix, which can be exploited to directly prove Theorem 4.2.

Proof: (Theorem 4.2): The proof proceeds by contradiction. Since $\mathbf{P}_{\mathbf{Q}}$ is a real matrix, and real OSTBCs are defined by real code matrices $\mathbf{C}_k \in \mathbb{R}^{L \times n_T}, k = 1, \ldots, M'$, we can conclude that, if a real OSTBC is nonidentifiable, then there exists an orthogonal and skew-symmetric matrix $\mathbf{U}$ of dimensions $n_T \times n_T$. From Property 5.4 it is clear that an orthogonal skew-symmetric matrix of odd order cannot exist and therefore, any real OSTBC with an odd number of transmit antennas is identifiable. □

Considering now an even number M' of real symbols,[2] and taking into account that $\mathbf{Q}$ is an orthogonal skew-symmetric matrix, Property 5.6 implies that $\mathbf{Q}$ can be rewritten as

$$\mathbf{Q} = \mathbf{T} \begin{bmatrix} \mathbf{0}_{M'/2} & -\mathbf{I}_{M'/2} \\ \mathbf{I}_{M'/2} & \mathbf{0}_{M'/2} \end{bmatrix} \mathbf{T}^T$$

where $\mathbf{T}$ is an orthogonal matrix with elements $t_{i,j}$ in its ith row and jth column. Thus, we obtain

$$\mathbf{P}_{\mathbf{Q}} = \mathbf{P}_{\mathbf{T}} \begin{bmatrix} \mathbf{0} & -\mathbf{I}_{LM'/2} \\ \mathbf{I}_{LM'/2} & \mathbf{0} \end{bmatrix} \mathbf{P}_{\mathbf{T}}^T$$

where $\mathbf{P}_{\mathbf{T}}$ is an orthogonal block matrix given by

$$\mathbf{P}_{\mathbf{T}} = \begin{bmatrix} t_{1,1}\mathbf{I}_L & \cdots & t_{1,M'}\mathbf{I}_L \\ \vdots & \ddots & \vdots \\ t_{M',1}\mathbf{I}_L & \cdots & t_{M',M'}\mathbf{I}_L \end{bmatrix}.$$

Therefore, (12) can be rewritten as

$$\mathbf{P}_{\mathbf{T}}^T \begin{bmatrix} \mathbf{C}_1 \\ \vdots \\ \mathbf{C}_{M'} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & -\mathbf{I}_{LM'/2} \\ \mathbf{I}_{LM'/2} & \mathbf{0} \end{bmatrix} \mathbf{P}_{\mathbf{T}}^T \begin{bmatrix} \mathbf{C}_1 \\ \vdots \\ \mathbf{C}_{M'} \end{bmatrix} \mathbf{U}$$

and Property 2.3 implies that there exists an OSTBC with code matrices $\mathbf{B}_k \in \mathbb{C}^{L \times n_T}$ $(k = 1, \ldots, M')$ given by

$$\begin{bmatrix} \mathbf{B}_1 \\ \vdots \\ \mathbf{B}_{M'} \end{bmatrix} = \mathbf{P}_{\mathbf{T}}^T \begin{bmatrix} \mathbf{C}_1 \\ \vdots \\ \mathbf{C}_{M'} \end{bmatrix}$$

which satisfy

$$\begin{bmatrix} \mathbf{B}_1 \\ \vdots \\ \mathbf{B}_{M'} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & -\mathbf{I}_{LM'/2} \\ \mathbf{I}_{LM'/2} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{B}_1 \\ \vdots \\ \mathbf{B}_{M'} \end{bmatrix} \mathbf{U}$$

or equivalently, for $k = 1, \ldots, M'/2$

$$\begin{aligned} \mathbf{B}_k &= -\mathbf{B}_{k+M'/2}\mathbf{U} \\ \mathbf{B}_k\mathbf{U} &= \mathbf{B}_{k+M'/2}. \end{aligned} \tag{13}$$

[2]The case of odd M' is explained by Theorem 4.1.

Applying now Property 5.3 we can write

$$\mathbf{U} = \mathbf{V} \begin{bmatrix} j\mathbf{I}_p & \mathbf{0} \\ \mathbf{0} & -j\mathbf{I}_q \end{bmatrix} \mathbf{V}^H$$

where $\mathbf{V}$ is a unitary matrix and $p + q = n_T$. Thus, defining the OSTBC matrices $\mathbf{A}_k = \mathbf{B}_k\mathbf{V}$ (see Property 2.2), (13) yields

$$\mathbf{A}_k \begin{bmatrix} j\mathbf{I}_p & \mathbf{0} \\ \mathbf{0} & -j\mathbf{I}_q \end{bmatrix} = \mathbf{A}_{k+M'/2}, \qquad k = 1, \ldots, M'/2$$

and decomposing these code matrices as $\mathbf{A}_k = [\mathbf{F}_k \quad \mathbf{G}_k]$, where $\mathbf{F}_k$ and $\mathbf{G}_k$ are $L \times p$ and $L \times q$ matrices, respectively, it is straightforward to prove that the orthogonality condition $\mathbf{A}_k^H\mathbf{A}_l = -\mathbf{A}_l^H\mathbf{A}_k$ implies

$$\begin{aligned} \mathbf{F}_k^H\mathbf{F}_l &= -\mathbf{F}_l^H\mathbf{F}_k \\ \mathbf{G}_k^H\mathbf{G}_l &= -\mathbf{G}_l^H\mathbf{G}_k \end{aligned}$$

for $k \neq l$ and $k, l = 1, \ldots, M'/2$. Analogously, $\mathbf{A}_k^H\mathbf{A}_{l+M'/2} = -\mathbf{A}_{l+M'/2}^H\mathbf{A}_k$ implies

$$\begin{aligned} \mathbf{F}_k^H\mathbf{F}_l &= \mathbf{F}_l^H\mathbf{F}_k \\ \mathbf{G}_k^H\mathbf{G}_l &= \mathbf{G}_l^H\mathbf{G}_k \end{aligned}$$

and then, we can conclude that

$$\begin{aligned} \mathbf{F}_k^H\mathbf{F}_l &= \mathbf{0} \\ \mathbf{G}_k^H\mathbf{G}_l &= \mathbf{0} \end{aligned}$$

for $k \neq l$ and $k, l = 1, \ldots, M'/2$. Thus, taking into account that $\mathbf{A}_k^H\mathbf{A}_k = \mathbf{I}$ implies $\mathbf{F}_k^H\mathbf{F}_k = \mathbf{I}$ and $\mathbf{G}_k^H\mathbf{G}_k = \mathbf{I}$, we can prove Theorem 4.3 in a straightforward manner.

Proof (Theorem 4.3): Assuming a nonidentifiable OSTBC and considering, without loss of generality, $p \geq q$, it is easy to realize that the columns of the matrix $[\mathbf{F}_1 \cdots \mathbf{F}_{M'/2}]$ define an orthogonal basis of a subspace of dimension $pM'/2$ into a space of dimension L, which implies $M'/L \leq 2/p$. Finally, taking into account $p = \max(p, q) \geq \lceil \frac{n_T}{2} \rceil$, we conclude that the transmission rate of any nonidentifiable OSTBC satisfies

$$\frac{M'}{L} \leq \frac{2}{\lceil \frac{n_T}{2} \rceil}.$$

Conversely, if the transmission rate satisfies

$$\frac{M'}{L} > \frac{2}{\lceil \frac{n_T}{2} \rceil}$$

then the OSTBC is identifiable. □

Theorem 4.3 relates the underlying structure of the OSTBC codes with their identifiability properties and, as an example, it ensures that the real OSTBC with $n_T = M = L = 8$ [33] is identifiable, which contradices the hypothesis in [14] and validates the experimental results in [30] and [40].

C. Identifiable OSTBCs: Proofs of Theorems 4.4 and 4.5

We start this section by introducing the Proof of Theorem 4.4, which extends the partial result for real OSTBCs and an odd number of transmit antennas n_T stated in [31].

Proof: (Theorem 4.4): For full row rank channel matrices $\mathbf{H}$, the indeterminacy condition in (9) can be rewritten as (12), which implies that if a full row rank channel matrix cannot be unambiguously identified by means of SOS, then the OSTBC is nonidentifiable. Equivalently, if the OSTBC is identifiable and the channel matrix $\mathbf{H}$ is full row rank, then the channel can be unambiguously extracted, up to a real scalar, by means of SOS. □

Theorem 4.4 establishes a sufficient condition for blind MIMO-OSTBC channel identification from SOS. Although it is based on the constraint $n_R \geq n_T$, which can be very restrictive, it ensures, in combination with Theorems 4.2 and 4.3, that most of the practical OSTBCs permit the blind channel identification without exploiting the finite alphabet properties or the higher-order statistics (HOS) of the signals.

Let us now define, for each orthogonal and skew-symmetric matrix $\mathbf{Q} \in \mathbb{R}^{M' \times M'}$, the subspace $\mathcal{G}_{\mathbf{Q}}^{||}$ (and its complementary $\mathcal{G}_{\mathbf{Q}}^{\perp}$) of multiple-input–single-output (MISO) channels $\tilde{\mathbf{h}}_{\mathbf{Q}}^{||} \in \mathbb{R}^{2n_T \times 1}$ such that there exists an estimated channel $\hat{\tilde{\mathbf{h}}}_{\mathbf{Q}}^{||}$ satisfying

$$\tilde{\mathbf{W}}(\tilde{\mathbf{h}}_{\mathbf{Q}}^{||})\mathbf{Q} = \tilde{\mathbf{W}}(\hat{\tilde{\mathbf{h}}}_{\mathbf{Q}}^{||}), \quad \tilde{\mathbf{h}}_{\mathbf{Q}}^{||} \in \mathcal{G}_{\mathbf{Q}}^{||}.$$

Then, we can introduce the following lemma.

Lemma 6.2: If the projection of a MISO channel $\tilde{\mathbf{h}} \in \mathbb{R}^{2n_T \times 1}$ onto the subspace $\mathcal{G}_{\mathbf{Q}}^{\perp}$ is not null, then we cannot find an estimated channel $\hat{\tilde{\mathbf{h}}}$ satisfying the indeterminacy equation $\tilde{\mathbf{W}}(\tilde{\mathbf{h}})\mathbf{Q} = \tilde{\mathbf{W}}(\hat{\tilde{\mathbf{h}}})$.

Proof: The proof proceeds by contradiction. Writing $\tilde{\mathbf{h}} = \tilde{\mathbf{h}}_{\mathbf{Q}}^{||} + \tilde{\mathbf{h}}_{\mathbf{Q}}^{\perp}$ in terms of its projections onto $\mathcal{G}_{\mathbf{Q}}^{||}$ and $\mathcal{G}_{\mathbf{Q}}^{\perp}$ we obtain

$$\begin{aligned} \tilde{\mathbf{W}}(\tilde{\mathbf{h}})\mathbf{Q} &= \tilde{\mathbf{W}}(\hat{\tilde{\mathbf{h}}}) \\ \tilde{\mathbf{W}}(\tilde{\mathbf{h}}_{\mathbf{Q}}^{||} + \tilde{\mathbf{h}}_{\mathbf{Q}}^{\perp})\mathbf{Q} &= \tilde{\mathbf{W}}(\hat{\tilde{\mathbf{h}}}) \\ \tilde{\mathbf{W}}(\tilde{\mathbf{h}}_{\mathbf{Q}}^{||})\mathbf{Q} + \tilde{\mathbf{W}}(\tilde{\mathbf{h}}_{\mathbf{Q}}^{\perp})\mathbf{Q} &= \tilde{\mathbf{W}}(\hat{\tilde{\mathbf{h}}}) \\ \tilde{\mathbf{W}}(\tilde{\mathbf{h}}_{\mathbf{Q}}^{\perp})\mathbf{Q} &= \tilde{\mathbf{W}}(\hat{\tilde{\mathbf{h}}}) - \tilde{\mathbf{W}}(\hat{\tilde{\mathbf{h}}}_{\mathbf{Q}}^{||}) \\ \tilde{\mathbf{W}}(\tilde{\mathbf{h}}_{\mathbf{Q}}^{\perp})\mathbf{Q} &= \tilde{\mathbf{W}}(\hat{\tilde{\mathbf{h}}} - \hat{\tilde{\mathbf{h}}}_{\mathbf{Q}}^{||}) \qquad \# \end{aligned}$$

which cannot be possible since $\tilde{\mathbf{h}}_{\mathbf{Q}}^{\perp} \notin \mathcal{G}_{\mathbf{Q}}^{||}$. □

Using this lemma, and assuming that Condition 4.1 is satisfied, we can prove Theorem 4.5.

Proof: (Theorem 4.5): Consider an identifiable OSTBC and a MIMO channel $\mathbf{H}$ such that the largest eigenvalue of (7) has multiplicity $P > 1$. This implies the existence of $P-1$ spurious channels $\hat{\mathbf{H}}^{(k)}$, and orthogonal skew-symmetric matrices $\mathbf{Q}_k$, satisfying

$$\tilde{\mathbf{W}}(\mathbf{H})\mathbf{Q}_k = \tilde{\mathbf{W}}(\hat{\mathbf{H}}^{(k)}), \qquad k = 1, \ldots, P-1.$$

If the multiplicity P does not decrease with the addition of a new receive antenna, then the new MISO channel $\mathbf{h}_{n_R+1}$ satisfies

$$\tilde{\mathbf{W}}(\mathbf{h}_{n_R+1})\mathbf{Q}_k = \tilde{\mathbf{W}}(\hat{\mathbf{h}}_{n_R+1}^{(k)}), \qquad k = 1, \ldots, P-1$$

and taking into account Lemma 6.2, this implies

$$\tilde{\mathbf{h}}_{n_R+1} \in \mathcal{G}_{\mathbf{Q}_k}^{||}, \qquad k = 1, \ldots, P-1.$$

Assuming an identifiable OSTBC we know that $\mathcal{G}_{\mathbf{Q}_k}^{\perp} \neq \emptyset$. Therefore, if $\tilde{\mathbf{h}}_{n_R+1}$ is a Gaussian random vector with full rank correlation matrix $E[\tilde{\mathbf{h}}_{n_R+1}\tilde{\mathbf{h}}_{n_R+1}^T]$, then the probability of $\tilde{\mathbf{h}}_{n_R+1} \in \mathcal{G}_{\mathbf{Q}_k}^{||}$ is zero, and the multiplicity P decreases with probability one. □

Finally, we must take into account that the results of Theorem 4.5 will happen with probability one. This means that there exist degenerated cases, for which the addition of a new receive antenna does not guarantee the reduction of the multiplicity P. These cases are those satisfying $\tilde{\mathbf{h}}_{n_R+1} \in \mathcal{G}_{\mathbf{Q}_k}^{||}$ ($k = 1, \ldots, M'$) and, fortunately, they define a set of measure zero.

VII. Discussion and Computer Simulations

In this Section, the main results are validated by means of some simulation examples. Specifically, the new theorems allow us to shed some light into the numerical examples presented in [30]. Additionally, we propose a new OSTBC transmission technique which ensures blind channel identifiability with one single receive antenna (see also [1]). Finally, some discussions about the obtained results are presented, showing that blind channel identifiability based on SOS is still an open issue, and pointing out further research lines.

A. Interpretation of the Results in [30]

The combination of Theorems 4.2, 4.3 and 4.5 allows us to explain previous results obtained by others authors. Table I shows the main results in [30], where we have added a column with the transmission rate $R = M/L$ and the threshold derived from Theorem 4.3

$$R_{\text{th}} = \begin{cases} \frac{2}{\lceil n_T/2 \rceil} & \text{for real OSTBCs} \\ \frac{1}{\lceil n_T/2 \rceil} & \text{for complex OSTBCs} \end{cases}$$

which ensures that any OSTBC transmitting at a higher rate is identifiable. The results of Table I have been obtained using the most common OSTBCs based on the generalized orthogonal designs [10] and amicable designs [33]. The elements $h_{i,j}$ of the channel matrices $\mathbf{H}$ have been generated as independent complex zero-mean Gaussian random variables. As can be seen, any OSTBC transmitting at a rate $R > R_{\text{th}}$ permits the channel identification with $n_R > 1$ receive antennas, as predicted by Theorem 4.3 and Corollary 4.2. Furthermore, we must note that the condition on the transmission rate is very restrictive and there are only six OSTBC examples with $R \leq R_{\text{th}}$, which are the following.

- **Alamouti codes**: As pointed out in [29], it is impossible to achieve blind identification for the Alamouti code without using some precoding or assuming a correlation matrix $\mathbf{R_s}$ with nonequal eigenvalues.

- **Real OSTBC** ($n_T = M = L = 4$): Analogously to the Alamouti code, this is a nonidentifiable code with practical application.
- **Real OSTBC** ($n_T = 3, M = L = 4$): In this case, Theorem 4.2 implies that the code is identifiable, and Corollary 4.2 explains the blind identifiability of the channel with $n_R > 1$.
- **Complex OSTBC** ($n_T = 4, M = 4, L = 8$): This is a nonidentifiable code with no practical use because we can use the amicable design to transmit with the same number of transmit antennas $n_T = 4$, but at a higher rate, $M = 3$ and $L = 4$, and with a lower decoding delay.
- **Complex OSTBC** ($n_T = 3, M = 4, L = 8$): Analogously to the previous case, this is an OSTBC with no practical use because we can transmit with rate $R = 3/4$ using the amicable design. Furthermore, in case of transmitting with $R = 1/2$, we could use the real OSTBC with $n_T = 3$ to transmit $M = 2$ complex symbols ($M' = 4$) in $L = 4$ channel uses, which is an identifiable OSTBC (by direct consequence of Theorem 4.2) with the same rate $R = 1/2$ and a lower decoding delay $L = 4$.

Finally, taking into account the maximum rate designs proposed in [41], which ensure that for any number of transmit antennas there exists a real OSTBC with transmission rate $R = 1$ and a complex OSTBC with

$$R = \frac{1}{2} + \frac{1}{2\left\lceil \frac{n_T}{2} \right\rceil}$$

we can conclude that the only non identifiable OSTBCs with practical interest are the Alamouti codes (real and complex) and the real OSTBC with $n_T = M = L = 4$. Furthermore, although we could find nonidentifiable OSTBCs with a much lower decoding delay than that of the maximum rate OSTBCs, the penalty in transmission rate pointed out by Theorem 4.3 is so high that, in practice, they are useless.

B. *Application of Theorem 4.1*

In [1], we have proposed a transmission technique which ensures that the MIMO channel can be unambiguously extracted with only one receive antenna.[3] The technique is based on Theorem 4.1, and it consists in a slight reduction of the transmission rate simply by not transmitting one real symbol (either the real or imaginary part of a symbol in the case of complex OSTBCs), so that the OSTBC transmits an odd number M' of real symbols and Theorem 4.1 applies. Furthermore, by grouping B consecutive OSTBC blocks, the resulting transmission matrix can be viewed as a new OSTBC with n_T antennas transmitting BM symbols in BL time slots, and deleting one real symbol of this new OSTBC, the rate-reduction factor is

$$\beta = \frac{BM' - 1}{BM'}$$

which increases with B, and tends to one for $BM' \gg 1$.

[3]The same idea has been also exploited in [32], [42] for designing nonintersecting subspace OSTBCs.

Considering i.i.d. Gaussian noise with variance σ^2, and assuming without loss of generality that the average transmitted energy per antenna and time interval is $1/n_T$, the capacity of the OSTBC-MIMO channel $\mathbf{H}$ for unity bandwidth is [33]

$$C_{\text{OSTBC}}(R, \text{SNR}) = R \log_2 \left(1 + \frac{\text{SNR}}{R}\right)$$

where $\text{SNR} = \frac{\|\mathbf{H}\|^2}{n_T \sigma^2}$ is the received signal-to-noise ratio. In the case of the proposed technique, and assuming perfect channel estimation, the capacity reduces to

$$C_{\text{Red}} = C_{\text{OSTBC}}(\beta R, \text{SNR}).$$

On the other hand, considering the 3-dB penalty incurred by differential schemes, the capacity of a differential OSTBC is given by [33]

$$C_{\text{Diff}} = C_{\text{OSTBC}}(R, \text{SNR}/2).$$

Thus, considering $\text{SNR} \gg 1$, it can be readily proven that

$$\begin{cases} C_{\text{Red}} > C_{\text{Diff}} & \text{if } \text{SNR} < \text{SNR}_{\text{th}} \\ C_{\text{Red}} < C_{\text{Diff}} & \text{if } \text{SNR} > \text{SNR}_{\text{th}} \end{cases}$$

where SNR_{th} is a threshold given by

$$10 \log_{10}(\text{SNR}_{\text{th}}) = 10 \log_{10}(R) + \frac{3 + \beta 10 \log_{10}(\beta)}{1 - \beta}.$$

Fig. 1 shows the theoretical SNR_{th} curves for three different transmission rates ($R = 1, R = 3/4$ and $R = 1/2$). As can be seen, the curves divide the plane in two regions, in the upper region (labeled as Differential OSTBC Receiver Region) the differential scheme has more capacity than the proposed method (referred to as Rate-Reduction), whereas the converse is true in the lower region. It must be also noted that for rate-reduction factors $\beta > 0.9$, the threshold SNR_{th} is above 30 dB, which implies that in practice the proposed technique is a better approach than the differential OSTBC scheme in terms of capacity.

Here, we must point out that the theoretical capacity analysis has been carried out considering perfect channel estimation, which is only true in a noise free situation or in the case of an infinite number of available received blocks. In order to analyze the effect of the errors in the channel estimate, the results of 1000 independent experiments have been averaged. The elements of the flat fading MIMO channels are zero-mean, circular, complex Gaussian random variables with variance $\sigma_{\mathbf{H}}^2$, the averaged transmitted energy per antenna and time interval is $1/n_T$, and the SNR at the transmitter side is defined as $10 \log_{10}(\sigma_{\mathbf{H}}^2/\sigma^2)$. We have tested the $R = 3/4$ amicable design OSTBC for $M = 3$ complex symbols, $L = 4$ time slots and $n_T = 4$ transmit antennas, which is presented in [33, eq. (7.4.10)]. The i.i.d. source signal belongs to a 16-QAM constellation and the number of receive antennas is $n_R = 1$, which provokes an ambiguity problem in the channel estimation (see Table I). The number of available OSTBC blocks at the receiver is $N = 40$ (160 time-slots), and we compare the performance of the proposed rate-reduction method, the informed ML (perfect channel state information), the differential OSTBC receiver proposed in [33] and the linear precoding technique proposed in [30] (referred to as Weighted-PCA). For the Weighted-PCA, the first $M' - 1$ weights have been selected to be equal to 1,

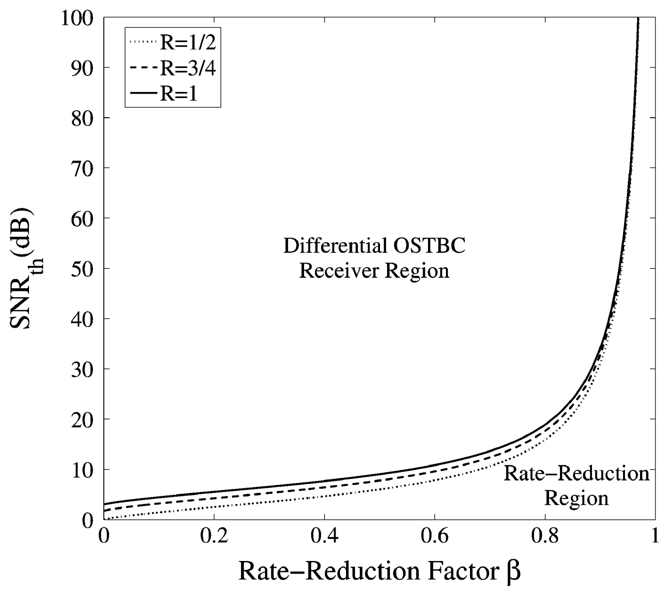

Fig. 1. Theoretical SNR threshold and capacity versus transmission rate R and rate-reduction parameter β.

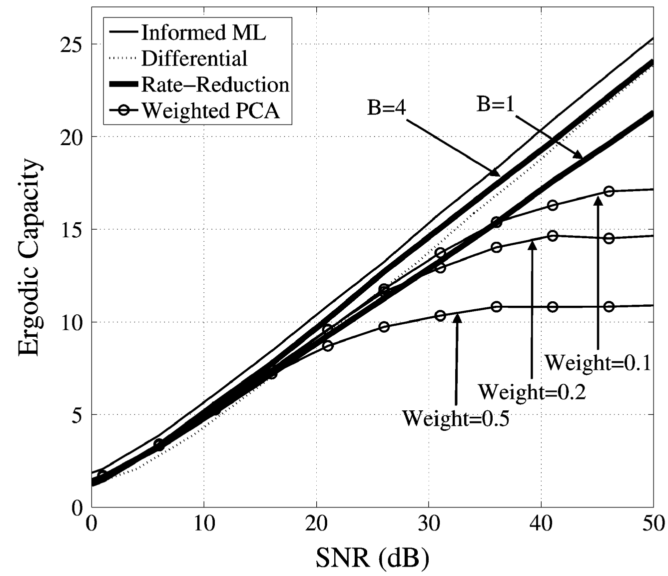

Fig. 2. Ergodic capacity including channel estimation.

and the remaining one is selected as 0.1,0.2 or 0.5 (always normalizing to transmit the same averaged energy per antenna and channel use). This means that one of the M' real source signals is transmitted with less energy than the rest. Note also that for $B = 1$ the proposed rate-reduction technique can be considered as a limiting case of the Weighted-PCA in which the weight assigned to one of the sources is zero (i.e., the symbol is not transmitted at all). For the proposed rate-reduction technique we consider $B = 1(\beta = 5/6)$ and $B = 4(\beta = 23/24)$. The estimated ergodic capacity has been obtained as the sum of the capacities of the BM' equivalent single-input single-output (SISO) channels, considering the co-channel interferences due to the error in the channel estimation as i.i.d. Gaussian noise. Fig. 2 shows the estimated ergodic capacity for the different techniques, where we can see that the proposed scheme with $B = 4$ outperforms the differential receiver for a large range of SNRs.

Finally, the tradeoff among the number of available OSTBC blocks (N), the rate-reduction parameter β (or B), the transmitted SNR, and the ergodic capacity is illustrated in Fig. 3, which shows the MSE of the channel estimate (left) and the ergodic capacity (right) for different values of B, number of available blocks and SNRs. It can be noted that, for a given N, the MSE of the channel estimate increases with B, which is due to the reduction of the number of available composite blocks (N/B). On the other hand, the increase of B yields a higher transmission rate βR, and the combination of these effects implies the existence of an optimum parameter B, which maximizes the ergodic capacity, and depends on the SNR and the number of available OSTBC blocks N.

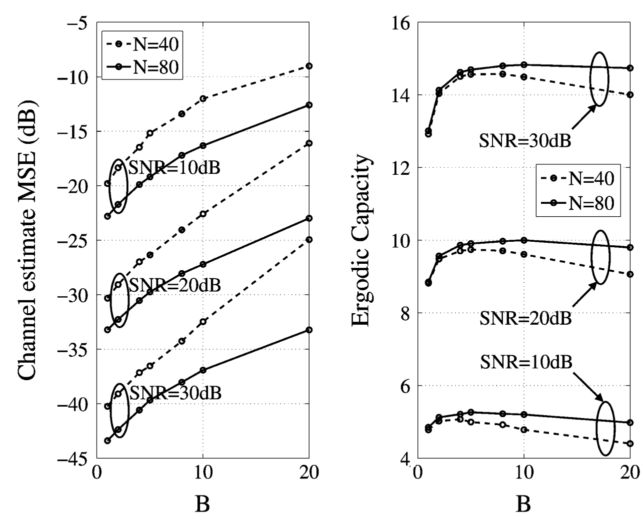

Fig. 3. Effect of the parameter B and number of available blocks N on channel estimation and ergodic capacity.

C. Additional Discussions and Further Lines

In this section, we present additional discussions about the obtained results, pointing out that the study of blind channel identifiability of MIMO-OSTBC systems is still an open issue.

1) Proof of Conjecture 4.1: Corollary 4.2 is based on Conjecture 4.1, which has been validated by means of numerical examples (see Table I). However, a theoretical proof of this conjecture has yet to be found. Furthermore, the assumption of identifiable OSTBCs does not imply multiplicity $P \leq 2$ for all the MISO channels.[4] Although in [30] the authors have concluded that the multiplicity P depends on the OSTBC and number of receive antennas n_R, but not on the specific channel realization $\mathbf{H}$ (as could be deduced from Table I), we have found, by means of numerical examples, that there exist a dependence between the channel $\mathbf{H}$ and the multiplicity order of the PCA problem (7).

2) Derivation of Tighter Transmission Rate Thresholds for Nonidentifiable Real OSTBCs: Theorem 4.3 establishes a necessary condition on the transmission rate for a nonidentifiable

[4]The multiplicity $P \leq 2$ is *only* guaranteed with probability one.

OSTBC. A logical question is: There exist nonidentifiable OSTBCs with transmission rate equal to the transmission rate threshold? For complex OSTBCs, and following in reverse order the derivation in Section VI-B, it can be easily proven that we can design nonidentifiable complex OSTBCs with transmission rate

$$R = \frac{M}{L} = \frac{1}{\lceil \frac{n_T}{2} \rceil}$$

and decoding delay $L = n_T$ for even n_T, and $L = n_T + 1$ for an odd number of transmit antennas.

In the case of real OSTBCs, the transmission rate threshold (considering that n_T must be even) is

$$R_{\text{th}} = \frac{4}{n_T}$$

and it is obvious that it cannot be achieved for $n_T = 2$. Therefore, this suggests that, for real OSTBCs, the derived threshold is not tight enough. Furthermore, following a similar procedure to the case of complex OSTBCs, and taking into account the properties of Hurwitz–Radon matrix families (see [33], [43], [44]), we can construct nonidentifiable real OSTBCs with transmission rate

$$R = \frac{M}{L} = \frac{1}{\lceil \frac{n_T}{4} \rceil}$$

and decoding delay $L = n_T$ for n_T multiple of 4, and $L = n_T + 2$ otherwise. Then, it is obvious that such constructions only achieve the transmission rate threshold for n_T multiple of 4, which yields the following open question: What is the strict transmission rate thresholds for nonidentifiable real OSTBCs?

3) Necessary Identifiability Conditions: All the theoretical results obtained in this paper constitute sufficient conditions for blind identification of MIMO channels, based on SOS, under OSTBC transmissions. However, finding necessary conditions remains as an open issue. Let us clarify the difficulty of this problem with an example: Taking into account Table I, we can consider three different complex OSTBCs with $n_T = 8$ transmit antennas and transmission rate $R = 1/2$.

- Real OSTBC with $R = 1$: Considering the real and imaginary parts of $M = 4$ complex symbols, it is easy to realize that we can construct a complex OSTBC with $M = 4$ and $n_T = L = 8$. The multiplicity associated to this OSTBC for $n_R = 1$ is $P = 2$.
- Generalized Orthogonal Design: This is an identifiable OSTBC with $n_T = M = 8$ and $L = 16$. The multiplicity associated to this OSTBC for $n_R = 1$ is $P = 2$.
- Amicable Design: In this case the OSTBC has $M = 4$ complex symbols and $n_T = L = 8$, and the multiplicity for $n_R = 1$ is $P = 1$.

As can be seen, three different OSTBCs with the same number of transmit antennas $(n_T = 8)$ and the same transmission rate $(R = 1/2)$, have different channel identifiability properties. Furthermore, whereas the amicable design permits the blind channel identification with one only receive antenna, the OSTBC constructed from the real design, which has the same parameters (M, L, and n_T) than the amicable design, does not permit the blind channel recovery when $n_R = 1$. These results suggest that the derivation of necessary identifiability conditions, and their relationship with the underlying structure of the OSTBCs, is a difficult task yet to be solved.

VIII. Conclusion

In this paper, we have presented identifiability conditions for blind multiple-input–multiple-output (MIMO) channel identification, based on second-order statistics (SOS), in orthogonal space–time block coded (OSTBC) systems. The analysis, which does not exploit possible finite alphabet constraints on the information symbols, shows that, if the OSTBC is identifiable and the number of receive antennas is greater than one, the MIMO channel can be identified with probability one by means of a single principal component analysis (PCA) problem, which is equivalent to the relaxed maximum likelihood (ML) estimator. The study reveals that the identifiability characteristics of OSTBCs are related to their underlying structure. Specifically, we have derived a threshold on the transmission rate, which is inversely proportional to the number of transmit antennas, and it has been proven that any OSTBC with a higher transmission rate is identifiable. The identifiability results include, as particular cases, some previous studies on blind channel/symbol identifiability under real OSTBCs or finite alphabet constraints. Finally, we have presented additional discussions and validated the obtained results by means of some numerical examples.

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