A Simple Expression for the Optimization of Spread-Spectrum Code Acquisition Detectors Operating in the Presence of Carrier-Frequency Offset

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***Abstract*—In this letter, we present a simple expression for the optimization of the threshold detection performance for direct-sequence spread-spectrum code acquisition in the presence of carrier-frequency offset. The proposed scheme divides the total integration time into subintervals, and the results of the coherent integrations performed over these subintervals are noncoherently combined prior to detection. The proposed expression allows obtaining the optimum number of coherent-integration subintervals for a given total integration time.**

***Index Terms*—Code acquisition, direct sequence, frequency offset, noncoherent integration, spread spectrum.**

I. Introduction

CODE synchronization between the received pseudonoise (PN) code and the local despreading code is the first and one of the most important tasks in any spread-spectrum system. This process is generally carried out in two steps: acquisition and tracking. In this letter, we focus on code acquisition. There has been considerable research on this issue [1]–[3]. In a nutshell, code acquisition is achieved by performing a search over the code-phase uncertainty region, advancing the code phase in fractions of code chips. For each code-phase offset, test statistics are derived and a detection strategy is applied to them in order to distinguish synchronous conditions, where a signal component is present, from asynchronous conditions, exclusively determined by noise. Apart from the chosen search strategy, acquisition performance crucially depends on the detection performance.

Carrier-frequency offset, e.g., due to Doppler shift or oscillator frequency drift, is one of the most important causes of detection failure and, thus, acquisition performance degradation. The problem is studied in [4], where a square-law noncoherent combining detector with partial correlation is proposed to alleviate the effect of Doppler shift. The same approach was also considered in [5] to reduce the effect of data modulation. In this letter, we develop a simple expression aiding the design of the direct-sequence spread-spectrum (DS/SS) code-acquisition scheme proposed in [4] and [5] in the presence of carrier-frequency offset. The total correlation time T_o is partitioned into subintervals, and the results of the coherent integrations performed over these subintervals are noncoherently combined to form detection test statistics. The design objective is to find the optimum number of subintervals N, since we are reducing the effect of the carrier-frequency offset at the price of a noncoherent combining loss, increasing with N. In Section II, we study the noncoherent combining detector, evaluating its probabilities of detection and false alarm. Then, in Section III, we derive both a lower bound as well as the proposed expression for the optimum number of subintervals.

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II. Code-Acquisition Scheme

Code acquisition is assumed to be based on the distinction between "signal" and "no-signal" test statistics by comparing the output of a square-law noncoherent combining receiver with a threshold, adjusted to achieve a particular false-alarm probability for "no-signal" statistics. This detector is shown in Fig. 1. The incoming signal is

$$r(t) = \sqrt{2P}c(t+\tau_1)\cos\left(2\pi(f_c+\Delta f)t+\theta\right)+n(t)$$

where P is the transmitter signal power, $c(t)$ is the PN signal, and f_c is the carrier frequency. For the received signal, τ_1 denotes the code-phase offset, while θ is the random carrier phase. $n(t)$ is additive white Gaussian noise (AWGN) with two-sided power spectral density $N_o/2$. The carrier-frequency offset Δf includes both the effect of the Doppler shift and the oscillator frequency drift. We assume that symbol transitions will not occur during the coherent integration time. After down-conversion to baseband in-phase (I) and quadrature-phase (Q) components, the received signal is despread. The total correlation time T_o is partitioned into N subintervals of mT_c seconds each, where m is a positive integer and T_c is the chip duration of the PN code. To optimize the number of coherent-integration subintervals, we assume that a correct local code phase $\tau_2 = \tau_1$ is selected, and that the code-frequency offset is so small that it can be neglected [5]. Since the integrators eliminate the high-frequency terms, the integration outputs for the quadrature components are found as

$$y_I = \sqrt{\frac{P}{2}}\int_0^{mT_c}\cos(2\pi\Delta f t-\theta)dt+n_I$$

$$y_Q = \sqrt{\frac{P}{2}}\int_0^{mT_c}\sin(2\pi\Delta f t-\theta)dt+n_Q$$

where n_I and n_Q can be shown to be independent Gaussian random variables with zero mean and variance

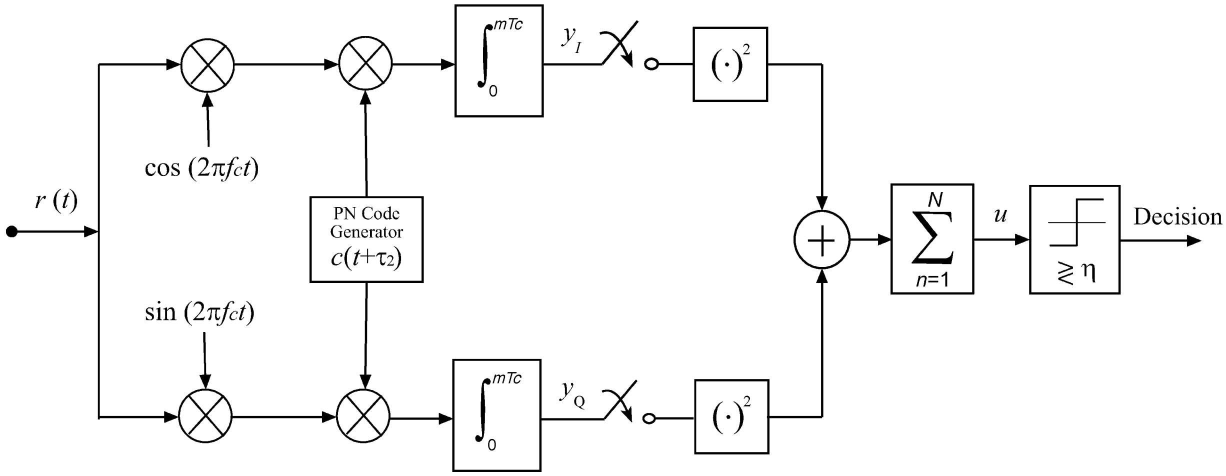

Fig. 1. Noncoherent receiver block diagram.

$\sigma_{\mathrm{IQ}}^2 = N_o m T_c/4$. The N subintegration results of the square-law detector are then noncoherently combined, yielding

$$u=\sum_{n=1}^{N}(y_{I_n})^2+\sum_{n=1}^{N}(y_{Q_n})^2 \tag{1}$$

where y_{I_n} and y_{Q_n} are the integration outputs associated with the nth subinterval. The probability density function (PDF) of the decision variable (1) is given by [6]

$$f_U(u)=\frac{1}{2\sigma_{\mathrm{IQ}}^2}\left(\frac{u}{\lambda}\right)^{\frac{N-1}{2}}\exp\left(-\frac{u+\lambda}{2\sigma_{\mathrm{IQ}}^2}\right)\mathrm{I}_{N-1}\left(\frac{\sqrt{\lambda u}}{\sigma_{\mathrm{IQ}}^2}\right)$$

where $u \geq 0$, and $\mathrm{I}_{N-1}(\cdot)$ is the modified Bessel function of the first kind of order $N-1$. This PDF is a noncentral chi-squared with $2N$ degrees of freedom (DOFs) and with the following normalized noncentrality parameter [6]:

$$\frac{\lambda}{2\sigma_{\mathrm{IQ}}^2}=\frac{\frac{2N^2E_o}{N_o}}{(2\pi\Delta f T_o)^2}\left[1-\cos\left(\frac{2\pi\Delta f T_o}{N}\right)\right] \tag{2}$$

where $E_o = T_o P$ is the signal energy associated with the total observation time. The probabilities of detection and false alarm are given by

$$P_D=\int_{\eta}^{\infty} f_U(u)du = Q_N\left(\frac{\sqrt{\lambda}}{\sigma_{\mathrm{IQ}}},\frac{\sqrt{\eta}}{\sigma_{\mathrm{IQ}}}\right) \tag{3}$$

$$P_{\mathrm{FA}}=\exp\left(-\frac{\eta}{2\sigma_{\mathrm{IQ}}^2}\right)\sum_{n=0}^{N-1}\frac{1}{n!}\left(\frac{\eta}{2\sigma_{\mathrm{IQ}}^2}\right)^n \tag{4}$$

where η is the detection threshold, and $Q_N(x,y)$ is Marcum's generalized Q-function [6].

III. Expression for the Optimum Number of Subintervals

In the proposed code-acquisition scheme, results of coherent integrations performed over small subintervals are noncoherently combined prior to detection. In this way, the effect of the carrier-frequency offset is reduced at the price of a loss due to the noncoherent combining. For a fixed total integration time, the loss due to the carrier-frequency error dominates for a small number of coherent integration subintervals, while the noncoherent combining loss dominates if N is large. Therefore, an optimum number of subintervals N_{opt} exists. A lower bound for N_{opt} can be derived, considering the optimum integration time mT_c when noncoherent combination is not used, i.e., $N=1$, $T_o = mT_c$, and $E_o = mT_cP$. In this case, for a given probability of false alarm, the integration time that maximizes the probability of detection (3) is the one that maximizes the normalized noncentrality parameter $\lambda/(2\sigma_{\mathrm{IQ}}^2)$. Taking the derivative of (2) with respect to the integration time, equating it to zero, and solving numerically, we obtain

$$(mT_c)_{\mathrm{opt}_{N=1}}=\frac{r_1}{2\pi\Delta f} \tag{5}$$

where $r_1 = 2.331$. This result sets an upper bound on mT_c and, therefore, for a given T_o, a lower bound on N, according to

$$N_{\mathrm{opt}} \geq \frac{2\pi\Delta f T_o}{r_1}. \tag{6}$$

To verify this fact, it is just necessary to realize that, in the general case, any increase in mT_c over (5) will degrade the coherent-integration result and, due to the fixed T_o, also decrease the number of noncoherent combinations N, reducing the probability of detection. However, a higher number of subintegrations could provide even better results, and the next objective is to find, given a total observation time, the optimum mT_c or, equivalently, the optimum N that maximizes the probability of detection, if the effect of noncoherent integration is fully considered.

To obtain an analytic expression for this general case, it would be necessary to take the derivative of (3) with respect to mT_c or N. Because this does not seem feasible, we have developed an approximate expression, based on observations, which we have validated through exhaustive numerical evaluations. From (2)–(4), it can be inferred that the N_{opt} which maximizes the probability of detection depends only on the signal energy-to-noise ratio E_o/N_o, the product $\Delta f T_o$, and the probability of false alarm P_{FA}. It was concluded empirically that N_{opt} primarily depends on $\Delta f T_o$ and less pronounced on E_o/N_o. Fig. 2

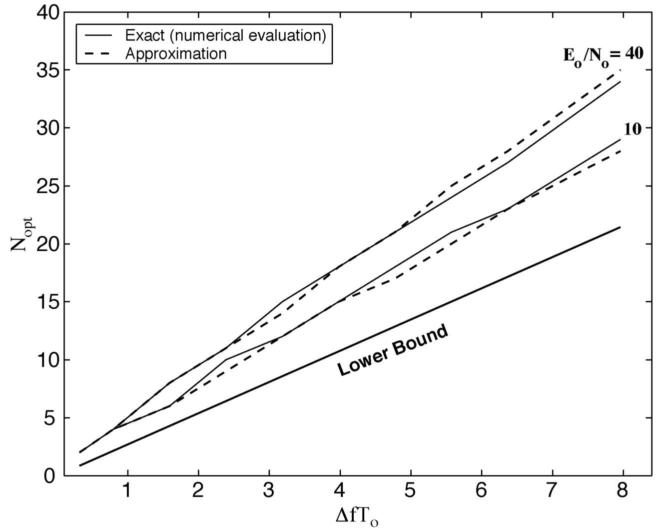

Fig. 2. Optimum number of subintervals as a function of $\Delta f T_o$ ($P_{\mathrm{FA}} = 10^{-5}$).

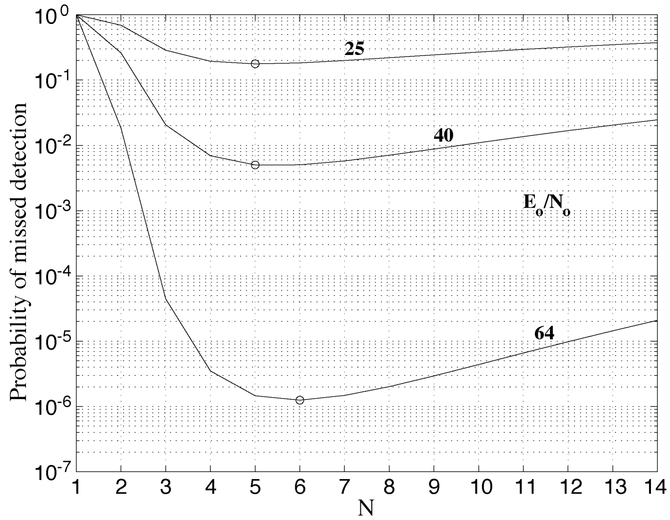

Fig. 3. Probability of missed detection as a function of the number of subintervals ($P_{\mathrm{FA}} = 10^{-5}$, $\Delta f T_o = 1$).

TABLE I
MAXIMUM ERROR ($E_{\max}$) IN P_D USING (7)

P_D	$0-0.9$	$0.9-0.99$	$0.99-0.9999$	$0.9999-1$
$E_{\max}$	$3\cdot10^{-3}$	$7\cdot10^{-4}$	$9\cdot10^{-5}$	$2\cdot10^{-6}$

shows some results obtained by numerically evaluating the theoretical probability distributions. It can be observed that N_{opt} depends almost linearly on $\Delta f T_o$. Further, we found that the slope of this linear variation depends on the natural logarithm of E_o/N_o (in linear units), and that N_{opt} shows only little dependence on P_{FA}. With these empirical evidences, we obtained by means of a curve fit the following expression for the optimum number of coherent-integration subintervals:

$$N_{\mathrm{opt}} \approx \left[\frac{\ln\left(\frac{E_o}{N_o}\right) + b}{a} 2\pi \Delta f T_o + 1 \right] \tag{7}$$

where $a = 11$, $b = 3.73$, and $[x]$ denotes the closest integer to x. The expression is applicable for values of E_o/N_o and $\Delta f T_o$ that make $N_{\mathrm{opt}} \geq 1$, and supposing that, as it was introduced above, code-frequency offset is so small that it can be ignored. In Fig. 2, we compare the results obtained using this approximation formula with those obtained by numerically evaluating the theoretical probability distributions.

Numerical evaluation of (2), (3), and (4) was also used to validate (7) and to check its accuracy. Two thousand cases were selected with P_{FA} ranging from 10^{-2} to 10^{-11}, E_o/N_o from 10 to 80 (linear units), and $\Delta f T_o$ from 0.3 to 8. For each case, N_{opt} was obtained by integrating (3) numerically. The results show that (7) gives the true optimum in 57% of the cases, and that the absolute error in the approximation of N_{opt} is never greater than two in the considered frequency-error range. The more relevant conclusion is inferred from Table I, where the maximum absolute error $E_{\max}$ in P_D as a consequence of the error in the approximation of N_{opt} using (7), is shown. This error decreases as the probability of detection increases, and it is always less than $P_M = 1 - P_D$. As an example, Fig. 3 shows the probability of missed detection P_M as a function of N for $\Delta f T_o = 1$, $P_{\mathrm{FA}} = 10^{-5}$, and E_o/N_o of 25, 40, and 64. This figure, which has been obtained by evaluating (3) numerically for these particular cases, shows that noncoherent combination is necessary to allow detection. We can see that $N_{\mathrm{opt}} = 5$ when E_o/N_o is 25 and 40, and $N_{\mathrm{opt}} = 6$ for $E_o/N_o = 64$. These results can also be obtained from (7).

IV. CONCLUSIONS

In this letter, we have analyzed a DS/SS code-acquisition scheme. Noncoherent integration was used to optimize the threshold detection performance in the presence of carrier-frequency offset. An expression to calculate the optimum number of coherent-integration subintervals has been obtained. Exhaustive numerical evaluations have shown the excellent performance of the proposed expression.

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