ADAPTIVE MODULATION AND POWER IN WIRELESS COMMUNICATION SYSTEMS WITH DELAY CONSTRAINTS

Jesus Perez, Jesus Ibañez

Dept. of Communications Engineering, University of Cantabria, Santander, Spain
Email:{jperez, jesus}@gtas.dicom.unican.es

ABSTRACT

This paper considers the problem of adaptive modulation and power in wireless systems with a strict delay constraint. Assuming perfect causal channel state information at the transmitter and the receiver, the modulation and power are dynamically adapted to minimize the outage probability for a fixed data rate. A slow frequency-flat channel is assumed and a discrete-time stationary Markov chain is used to model the time-varying channel. The problem is formulated as a finite-horizon discrete dynamic programming problem. The solution is a set of power/modulation allocation policies to be used during the transmission, as a function of the channel and system state. Numerical results show the gain of such adaptation policies in terms of average outage probability.

Index Terms— Adaptive modulation, adaptive power, fading channels, outage probability, dynamic programming.

1. INTRODUCTION

Adaptive power and modulation have revealed as efficient strategies to mitigate the channel-quality fluctuations in wireless communications. Most of the adaptive power and/or modulation schemes in the technical literature are designed for systems without delay constraints. In these works the aim was to maximize the average throughput, to minimize the long-term transmit power or to minimize the probability of error by adapting the transmission resources (power, modulation or coding) to the channel conditions. Unfortunately, these schemes are not valid for applications with hard delay constrains. In these cases the resources adaptation has to be optimized to minimize the outage probability for a fixed data rate [1], where outage probability is defined as the probability of making a transmission error or not being able to meet the delay constraint. Much less work have be done in this field. In [2] two schemes have been proposed to minimize the outage probability by means of an adaptive quadrature amplitude (QAM) modulation scheme. The modulation is adapted for each symbol which is hard for practical systems where the receiver has to inform the transmitter about the channel state. In [3] the authors presented a power adaptation algorithm, based on dynamic programming [4], in order to obtain the delay-constrained outage capacity. They assume Gaussian codebook and block-fading channels with independent channel power gains of the blocks, which is not a realistic assumption in most practical cases where the channels exhibit time correlation.

In this work we adapt both the power and modulation to minimize the outage probability, taking into account the time-correlation of the channel. Unlike [2], the adaptation is made for each frame, so power and modulation remain constant during the transmission of a frame. We assume slow frequency-flat time-varying channels where the channel remains constant during the transmission of a frame and can vary for consecutive frames. To derive the adaptation algorithm, a finite-state first-order stationary Markov chain is used to model the time variations of the channel. Finite-state first-order stationary Markov chains have been widely used to model flat-fading slow-varying channels (see for example [5] and references therein). Then, the power/modulation adaptation problem is formulated as a finite-horizon discrete dynamic programming (DP) problem [4]. The solution is a set of power/modulation adaptation policies to be used for each frame during the transmission. These policies are functions of the the channel state and of the system state: the available power and the number of frames still pending for transmission. By using the Markov model, the resulting policies take into account the channel correlation, improving the adaptation to the time-varying channel. Some numerical results are presented to illustrate the advantages of the power and rate adaptation in terms of average outage probability.

The paper is organized as follows. The system and channel models are presented in Section II. In section III we present the algorithm to obtain the optimal power and modulation adaptation policies. Numerical results are presented in section IV. Finally section V shows the main conclusions of the work.

2. CHANNEL MODEL

We consider a point-to-point frequency-flat block-fading channel, where the channel remains constant during the transmission of a block, and can change for consecutive blocks. Therefore, we assume that the duration of each block (T_B) is less than the coherence time of the channel. The channel power gain at the kth block is denoted by γ_k. In general, channel responses at different blocks are time correlated. To model this correlation we use a discrete-time first-order Markov chain with time discretized to T_B. The fading range $0 \leq \gamma < \infty$ is discretized into N_γ regions so that the jth region is defined as $R_j = \{\gamma : A_j \leq \gamma < A_{j+1}\}$, where $A_1 = 0$ and $A_{N_\gamma+1} = \infty$. The channel for the kth block is in state j if $\gamma_k \in R_j$. Let us denote the integer set of the possible channel states by $S_\gamma = \{1, 2, ..., N_\gamma\}$. We define the integer variables $\bar{\gamma}_k \in S_\gamma$ so $\bar{\gamma}_k = j \Leftrightarrow \gamma_k \in R_j$. Given N_γ and $\{A_j\}$, the Markov model is defined by its transition probability matrix $\mathbf{T}$ of size $N_\gamma \times N_\gamma$, where $[\mathbf{T}]_{i,j} = T_{i,j} = \text{Prob}\{\bar{\gamma}_k = j/\bar{\gamma}_{k-1} = i\}$. The transition probabilities also depend on the normalized Doppler frequency $f_d T_B$, which determines the rate of variation of the channel with respect the duration of the blocks. Although the physical wireless channel is inherently non-Markovian, it has been shown that a stationary first-order Markov chain can capture the essence of the channel dy-

This work was supported by the Spanish Government, Ministerio de Ciencia e Innovación (MICINN), under projects MultiMIMO (TEC2007-68020-C04-02), COSIMA (TEC2010-19545-C04-03) and COMONSENS (CSD2008-00010, CONSOLIDER-INGENIO 2010 Program).

namics when the number of regions/states is low and the channel fades slow enough (see for example [5] and references therein). The noise power at the receiver is absorbed into the channel gain so the received SNR for block kth is $\gamma_k p_k$, where p_k is the power employed to transmit the block kth.

3. SYSTEM AND APPLICATION MODEL

We consider a block coded communication system where the transmitter is able to change the transmit power and the modulation from block to block. The transmit power and the modulation employed for the kth block are denoted by p_k and m_k, respectively. We assume that the transmitter knows the channel gain (γ_k) at the beginning of each block, so it is able to set p_k and m_k accordingly. In general $m_k \in S_M$, where S_M is a finite set with the available modulations.

In practical systems, the transmitter cannot transmit with arbitrary power so only a finite set S_P of transmission power levels uses to be available: $p_k \in S_P$. We also consider the possibility to postpone the transmission during a block period (or an integer number of block periods) if the channel and/or system state are not suitable for transmission. In those cases $p_k = 0$ and we say that a non-data block has been transmitted.

We assume that the blocks are of identical length, so all blocks comprise the same number of coded symbols regardless the modulation employed. We also assume that there is not channel coding adaptation so a single AWGN channel code is used all the time. Since the time required for a codeword transmission depends on the modulation, blocks with different modulations comprise different number of codewords. We denote the number of codewords in the kth block by $n_c(m_k)$. In fact, the number of codewords in a block is proportional to the number of bits per symbol of the modulation. Figure 1 shows an example where $n_c(4QAM) = 1, n_c(16QAM) = 2$ and $n_c(64QAM) = 3$. When a non-data block is transmitted we assume that $n_c = 0$.

In a delay constrained application a given number of information bits has to be transmitted in a given number of block periods N subject to a power constraint $P_T \geq \sum_{k=1}^{N} p_k$, where P_T is the total available power. We denote the average transmit power by $\bar{P} = P_T/N$Since a single channel code is used, all codewords have the same number of information bits so the number of information bits of the kth block will be proportional to $n_c(m_k)$. Therefore, the delay constraint requirement can be formulated as follows: A given number of codewords (N_c) has to be transmitted in the N block periods, being $N_c \leq \sum_{k=1}^{N} n_c(m_k)$.

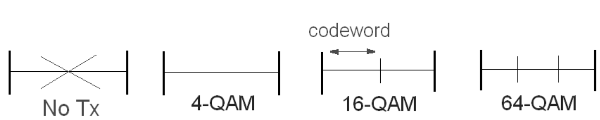

Fig. 1. Types of blocks for different modulations.

The outage probability is defined as the probability of making one or more transmission errors after decoding, subject to the delay constraint. Let us denote the probability of a codeword error by $P_e(\gamma, m, p)$, which, for a specific channel coding, is a function of the channel gain, the modulation employed and the transmit power. The specific form of $P_e(\gamma, m, p)$ depends on the channel code used. If the decision errors on codewords are independent, the probability of the kth block is decoded without errors will be $(1 - P_e(\gamma_k, m_k, p_k))^{n_c(m_k)}$. Then, the outage probability

$$P_{out} = 1 - \prod_{k=1}^{N} (1 - P_e(\gamma_k, m_k, p_k))^{n_c(m_k)}. \qquad (1)$$

It depends on the channel gains for the N blocks $\boldsymbol{\gamma} = [\gamma_1 \gamma_2 ... \gamma_N]^T$, on the transmit powers $\mathbf{p} = [p_1 p_2 ... p_N]^T$ and on the modulations $\mathbf{m} = [m_1 m_2 ... m_N]^T$ assigned to each block, where $m_k \in S_M$ and $p_k \in S_P$.

4. OPTIMUM POLICY FOR MINIMUM AVERAGE OUTAGE PROBABILITY

Because the randomness of the channel, P_{out} is a random variable and cannot meaningfully optimized. Therefore, we formulate the problem as a minimization of the expected probability of outage, where the expectation is with respect the joint distribution of the block channel states, and where the minimization is over the modulation and power assigned to each block

$$\begin{aligned} \min_{\mathbf{m},\mathbf{p}} \quad & \underset{\bar{\boldsymbol{\gamma}}}{E} \left\{ 1 - \prod_{k=1}^{N} (1 - P_e(\bar{\gamma}_k, m_k, p_k))^{n_c(m_k)} \right\}, \\ \text{s.t.} \quad & N_c \leq \sum_{k=1}^{N} n_c(m_k), \quad P_T \geq \sum_{k=1}^{N} p_k, \end{aligned} \qquad (2)$$

where $\bar{\boldsymbol{\gamma}} = [\bar{\gamma}_1 \bar{\gamma}_2 ... \bar{\gamma}_N]^T$ contains the sequence of channel states of the blocks. Note that, according to the channel Markov model, we consider the channel states instead of the channel values, where the probability of codeword error for the jth channel state is

$$P_e(\bar{\gamma}_j, m, p) = \frac{\int_{\gamma_j}^{\gamma_{j+1}} P_e(\gamma, m, p) f_\gamma(\gamma) d\gamma}{\text{Prob}\{\gamma_j \leq \gamma < \gamma_{j+1}\}}, \qquad (3)$$

being $f_\gamma(\gamma)$ the probability density function (pdf) of the channel power gain.

An important remark is needed here: modulation and power for each block have to be assigned as a function of the channel state of the past and current blocks (causal channel knowledge). In other words, power and modulation have to be adapted sequentially without knowledge of future channel states.

To formulate (2) as a stochastic finite-horizon DP problem we consider the function $F = \ln(1 - P_{out}) \Leftrightarrow P_{out} = 1 - e^F$. Since F is a monotonically decreasing function of P_{out}, minimization of P_{out} is equivalent to maximization of F

$$\begin{aligned} \max_{\mathbf{m},\mathbf{p}} \; \underset{\bar{\boldsymbol{\gamma}}}{E} & \left\{ \sum_{k=1}^{N} n_c(m_k) \ln(1 - P_e(\bar{\gamma}_k, m_k, p_k)) \right\} \\ & + g_{N+1}(l_{N+1}, h_{N+1}), \end{aligned} \qquad (4)$$

where we have included an additional term g_{N+1} which is equivalent to the constraints in (2)

$$g_{N+1}(l_{N+1}, h_{N+1}) = \begin{cases} -\infty, & l_{N+1} < N_c \text{ or } h_{N+1} > P_T \\ 0, & \text{otherwise}, \end{cases} \qquad (5)$$

where l_{N+1} and h_{N+1} denote the number of codewords and the amount of power transmitted after the last block, respectively.

Now, (4) has the structure of a finite-horizon stochastic DP problem where the optimization stages are the blocks. At stage kth, the control vector is $\mathbf{u}_k = [m_k \; p_k]^T$, and the state vector is $\mathbf{x}_k = [\bar{\gamma}_k \; l_k \; h_k]^T$, where l_k and h_k denotes the number of codewords and the power transmitted until the kth block

$$l_{k+1} = l_k + n_c(m_k), \quad h_{k+1} = h_k + p_k, \quad l_1 = h_1 = 0. \tag{6}$$

The transition probabilities among the states are determined by the channel transition probability matrix $\mathbf{T}$

$$\text{Prob}\{\mathbf{x}_{k+1} = [j \; a \; b]^T / \mathbf{x}_k = [i \; c \; d]^T\} = \delta_{a,b}\delta_{c,d}T_{i,j}, \tag{7}$$

where δ is the Kronecker delta.

Using the DP algorithm we can compute recursively the optimum policy functions for modulation and power adaptations at each block, as a function of the state: $M_k(\mathbf{x}_k) = m_k$ and $P_k(\mathbf{x}_k) = p_k$. The algorithm proceeds backward from the last block to the first as follows

$$J_{N+1}(\bar{\gamma}_{N+1}, l_{N+1}, h_{N+1}) = g_{N+1}(l_{N+1}, h_{N+1}), \tag{8}$$

$$J_k(\bar{\gamma}_k, l_k, h_k) = \max_{\substack{m_k \in S_m \\ p_k \in S_p}} \Bigg\{ n_c(m_k) \ln(1 - P_e(\bar{\gamma}_k, m_k, p_k)) + \sum_{j=1}^{N_\gamma} T_{\bar{\gamma}_k, j} \; J_{k+1}(j, l_k + n_c(m_k), h_k + p_k) \Bigg\}, k = N, \ldots, 1.$$

The optimum policies are the solutions of the maximization problems of (8). Note that these can be easily solved by direct search over the finite sets of the possible modulations S_m and transmit powers S_p.

Note that the adaptation policies $M_k(\mathbf{x}_k) = M_k()$ and $P_k(\mathbf{x}_k) = P_k()$ are obtained offline from the channel statistics and from the system parameters. Once the adaptation policies are obtained, they are used to assign power and modulation to each block online as a function of the current channel state and the total amount of power and number of codewords transmitted in previous blocks.

5. RESULTS AND CONCLUSIONS

In this section we show some numerical simulation results to illustrate the performance gain of the adaptation policies derived from the proposed algorithm. In the following results we consider Reed-Solomom $(63, 47)$ coding and three possible modulations: $S_m = \{4 - QAM, 16 - QAM, 64 - QAM\}$. Figure 2 shows the corresponding codeword error probability P_e curves as a function of the SNR ($= p\,\gamma$). Any other channel coding and set of modulations could be used.

As example, we use the Markov channel model proposed in [6] for Rayleigh fading channels. To obtain realizations of the Rayleigh channel process γ we use the algorithm proposed in [7]. The pdf of a Rayleigh fading channel is $f_\gamma(\gamma) = e^{-\gamma}$, where we assume, without loss of generality, that the channel is normalized so $E\{\gamma\} = 1$. Then, the average SNR at the receiver is always $\bar{P}$. Any other channel model could be used e.g. Rice, Nakagami; for which channel Markov models are also available [5].

Unless otherwise indicated, other parameters of the simulations are: $f_d T_B = 0.02$, $N_\gamma = 8$, $N = N_c = 50$, $SNR = 15dB$. In the case of fixed modulation we always use 4-QAM, and in the case of adaptive power we always consider the following set of possible transmit powers: $S_p = \{0, \bar{P}, 2\bar{P}\}$.

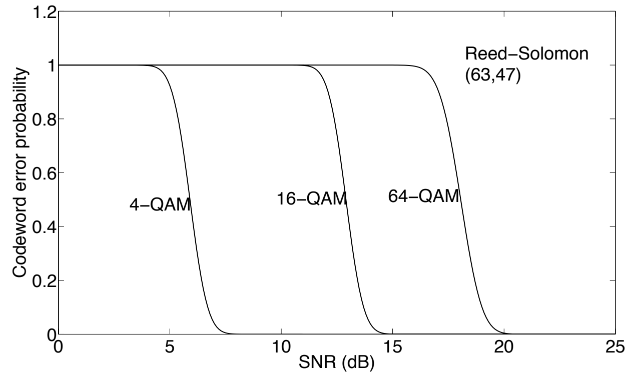

Fig. 2. Codeword error probability.

The adaptive scheme is useful if the channel exhibits enough variations during the transmission of the blocks. This can be observed in figure 3 where a fixed scheme is compared with the adaptive power and modulation, as a function of the normalized Doppler frequency ($f_d \; T_B$). Note that the higher the channel variability, the higher the gap between the adaptive and the fixed scheme.

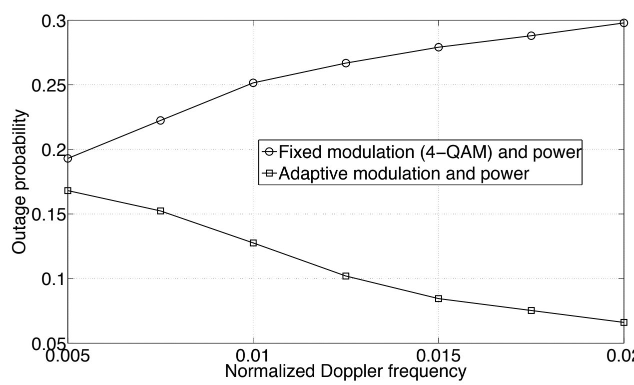

Fig. 3. Outage probability as a function of the normalized Doppler frequency $f_d T_B$ for fixed and adaptive modulation and power

Figure 4 shows the outage probability as a function of the average SNR for several cases: fixed transmit power and modulation, adaptive modulation with fixed power and adaptive power and modulation. There are curves for two Markov channel models with $N_\gamma = 8$ and $N_\gamma = 16$ states. The graph shows that the gain of the adaptive schemes increases with the average SNR. Also, the number of states of the Markov chain does not have significant influence on the outage probability, except in the case of adaptive power in the high SNR regime.

In general, the gain of the adaptation increases with the number of block periods N. As example, figure 5 compares the outage probability of adaptive and fixed modulation for several values of N, and two values of average SNR: 15 dB and 20 dB. As it is expected, for fixed modulation the probability of outage increases linearly with the number of blocks. In the case of adaptive modulation, there are

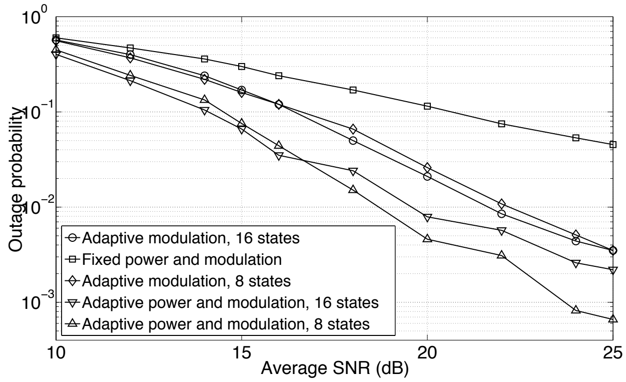

Fig. 4. Outage probability of fixed 4-QAM modulation and adaptive modulation using Markov models with $N_\gamma = 8$ and 16 states.

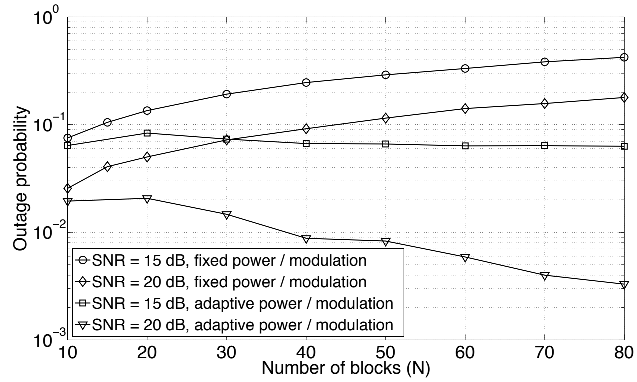

Fig. 6. Outage probability of a fixed scheme and a adaptive modulation/power scheme for different number of block periods.

two opposite trends. The higher the number of blocks, the higher the outage probability. But, on the other hand, when the number of blocks (N) is high, the channel exhibits more variability during the transmission, so there is more room to transmit high data-rate blocks (16-QAM and 64-QAM) and, consequently, more chance to transmit non-data blocks when the channel is in fading. This explains the fact that the outage probability does not increase with N, even it slightly decreases asymptotically toward the solution of the corresponding infinite-horizon DP problem ($N \to \infty$). When the number of block periods is less than $N = 10$, the adaptive modulation does not provide any gain because during the transmission the channel remains mainly constant, so the adaptive scheme does not have any chance to adapt. In these cases the outage probability is mainly determined by the initial channel value γ_1. Similar behavior is observed in 6 where we compare the fixed scheme with the adaptive power and modulation scheme.

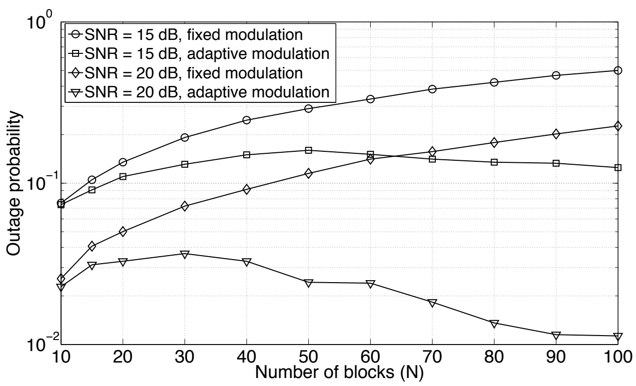

Fig. 5. Outage probability of fixed 4-QAM modulation and adaptive modulation as a function of the number of block periods.

6. REFERENCES

[1] A. Goldsmith, *Wireless Communications*, Cambridge University Press, 2005.

[2] K. M. Kamath and D. L. Goeckel, "Adaptive-modulation schemes for minimum outage probability in wireless systems," *IEEE Transactions on Communications*, vol. 52, pp. 1632–1635, October 2004.

[3] R. Negi and C. M. Cioffi, "Delay-constrained capacity with causal feedback," *IEEE Transactions on Information Theory*, vol. 48, pp. 2478–2494, September 2002.

[4] D. P. Bertsekas, *Dynamic Programming and Optimal Control. Volume I*, Athena Scientific, 2005.

[5] P. Sadeghi, R. A. Kennedy, P. B. Rapajic, and R. Shams, "Finite-state markov modeling of fading channels," *IEEE Signal Processing Magazine*, vol. 57, pp. 57–80, September 2008.

[6] H. S. Wang and N. Moayeri, "Finite-state markov channel - a useful model for radio communication channels," *IEEE Transactions on Vehicular Technology*, vol. 44, pp. 163–171, February 1995.

[7] W. C. Y. Lee, *Mobile Communications Engineering*, McGraw-Hill, 1982.