Optical Sensors and Their Fusion in a Quasi-Smart Structure for Real-Time Vibration Monitoring and Predictive Maintenance of Large Power Electric Power Generators

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ABSTRACT: Optical accelerometers developed in both fiber and integrated optics technologies and several traditional sensors are integrated in a quasi-smart structure for real-time monitoring and predictive maintenance of large Electric Power Generators. The structures were properly tested in field trials.

1. INTRODUCTION

IN the electric power industry the continuity and quality of the electric power supply service to the customers is of great importance. In power plants the costs of large generators and their associated maintenance are very high. As a consequence, in general, long lifetime, minimum failure rates and maintenance costs are three of the more important requirements for these kinds of machines. Furthermore, around the world there are an important number of hydroplants in which the life of the generators is reaching the expected lifetime and, as has been stated above, it is important to have the possibility to improve their reliable life and avoid large maintenance costs. However, it is becoming more and more important to receive early warning of any problem before failure. The analysis of vibration signals from suitably selected parts of these machines is the most popular monitoring tool for their capability to detect most of the mechanical and hydraulic malfunctions (Arregui et al., 1996; Lopez-Higuera et al., 1998).

The objective of a monitoring system is to solve problems such as: instantaneous protection of the machines, prediction of catastrophic risks, real-time detection of mechanical failures generating electrical faults and early detection of possible failures. However, due to the harsh environment (high electromagnetic fields and high temperature) and the very low frequency spectrum of the vibrations, the above-mentioned machines cannot be appropriately monitored using only sensors manufactured in traditional technologies like piezoelectric sensors and so, optical sensors must be used as an alternative. Several optical sensor based approaches have already been developed either in fiber optics (Miers et al., 1987), or a combination of optical fiber with silicon microstructures (Malki et al., 1995) or in integrated optics (Hoffmann et al., 1994), but due to technical requirements (such as frequency range, thermal or transverse sensitivity, packaging) or for economic reasons, up to now, these sensors are not completely suitable for the above-mentioned applications. Taking into account these concerns, two optical fiber and integrated optics accelerometers have been fully developed in order to satisfy specifications required for these applications.

A new quasi-smart structure for vibration monitoring, which includes the above-mentioned optical sensors, was developed. This monitoring system for large hydro-generator sets is mainly made of: (a) a wide variety of sensors to measure relevant magnitudes (displacement, vibration, etc.); (b) an acquisition unit which integrates and processes the sensor signals to obtain characteristic features, capable of taking real-time decisions (alert and alarm) for instantaneous protection of the machine; (c) a host computer which stores the parameters provided by the acquisition unit and is able to take decisions regarding the long-term behaviour of the machine; for instance, the time for the next revision or the possible development of a failure.

The "smartness" of the system is distributed throughout the processing chain in the Digital Acquisition and Signal Processing Unit (DASPU) and in the host computer. The DASPU must be able to issue alert and alarm signals by comparing some of the extracted features with lower and upper

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thresholds. On the other hand, the host computer software must have the ability to learn from data, perform predictive maintenance and help the user for diagnostic purposes. The host software incorporates a neural network based module to carry out these tasks.

In this paper, the optical sensors and the quasi-smart structure, developed and tested in the framework of a European project dedicated to real-time defect detection and prediction of forthcoming failures of generator groups in an electric power plant in Spain, are presented. In Section 2, the design, fabrication and in-lab characterization of the fiber and integrated optical sensor systems are mentioned. In order to integrate the Quasi-smart structure, the fusion of all sensors is dealt with in Section 3. The results from the field trials are presented in Section 4, and finally in Section 5 the conclusions are summarised.

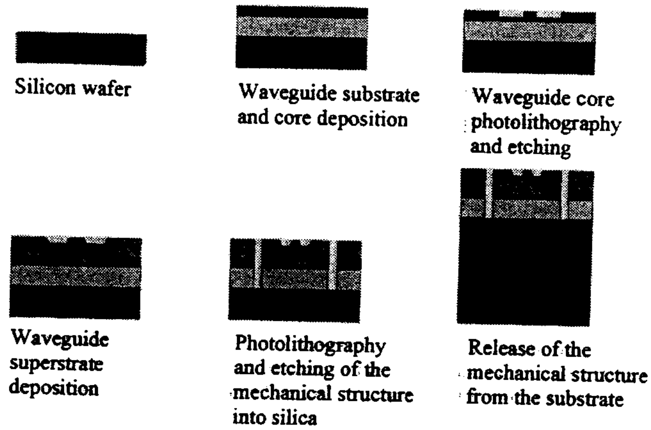

Figure 1. *Technological process flow used for the fabrication of the integrated optic transducer.*

2. SENSORS

First of all, the newly developed optical sensors are presented and the the rest of the sensors used will be mentioned.

2.1 Optical Sensors

The optical fiber and integrated optical sensors are composed of a transducer head and an analog electronic unit linked by optical fibers. Both sensors are based on the same modulation technique, but the transducers are developed in fiber optic technology for low frequency vibrations (shaft bearing) and in integrated optics technology for the medium frequency range (winding).

These sensors detect vibration levels by means of a moving fiber or a moving waveguide in cantilever configuration that modulates the transmitted light intensity through two output fibers or two integrated waveguides respectively, according to the acceleration of the place where they are located. The acceleration produces a proportional displacement of the fiber or waveguide end, thus modulating the coupled optical power in a differential way. The two simultaneous output light signals coming from the transducers are transmitted to the optoelectronic unit using optical fibers and then, both are detected and electronically processed to obtain the applied acceleration. After the corresponding modelling (Morante, 1996; Lopez-Higuera, 1997), the optical sensor heads or optical transducers were fabricated. The optical path and the seismic mass of the integrated transducer were made of the same silica material using LETIS's integrated optics technology. The main steps of the technological process are shown in Figure 1.

Three silica layers with different phosphorus doping levels were used to define the waveguide structure. The device was fabricated in five main steps (Ollier et al., 1997a, 1997b). The first step is the optical path fabrication. It is performed using a reactive ion etching technology. The second step is the mechanical structure fabrication.

This step is divided into two parts: RIE anisotropic etching of the three silica layers (about 15 microns thick) and etching of silicon using an isotropic reactive ion etching process. This step makes the mechanical structure free standing, freed from the substrate (Ollier et al., 1997b). Grooves for sturdy fiber connection were simultaneously etched in front of the input and output waveguides.

Two SEM photographs of the sensitive part of the integrated transducer and the groves for optical fiber-integrated waveguide optical inter-connection are shown in Figures 2 and 3. The holes into the butterfly-like seismic mass allow the structure to be freed from the substrate. The fiber ends of the optical channel are efficiently positioned and glued in these accurate and precisely etched grooves (Ollier et al., 1998).

The main components of the optical fiber transducer head are the input optical fiber cantilever, the two receiving fibers and the housing, as shown in Figure 4.

The optical fiber transducer head has been fabricated using only insulated materials. The optical fibers (100/140 type) were positioned on an alumina plate that had accurate laser micromachined grooves suitably dimensioned and aligned, and they were led to their exact place with the help of a four-degree-of-freedom micropositioning system. They were fixed by means of a low shrinkage optical adhesive that meets U.S. Federal Specification Mil-A-3920. As well as curing them with ultraviolet light, the optical heads were appropriately packaged to avoid the effect of humidity, and were subject to temperature shocks in an ACS Hygros-15 climatic chamber, in order to reduce the infantile life period as much as possible. The return and the illuminating fibers exit and enter through the same place in the head. So, only one optical cable with three fibers is needed for each optical fiber transducer.

The optoelectronic unit is the part of the sensor system that generates, receives and processes the optical signals occurring in the acceleration transduction. It communicates with the optical head by means of a fiber transducer operating in a harsh environment. The basic functions are: generation of the

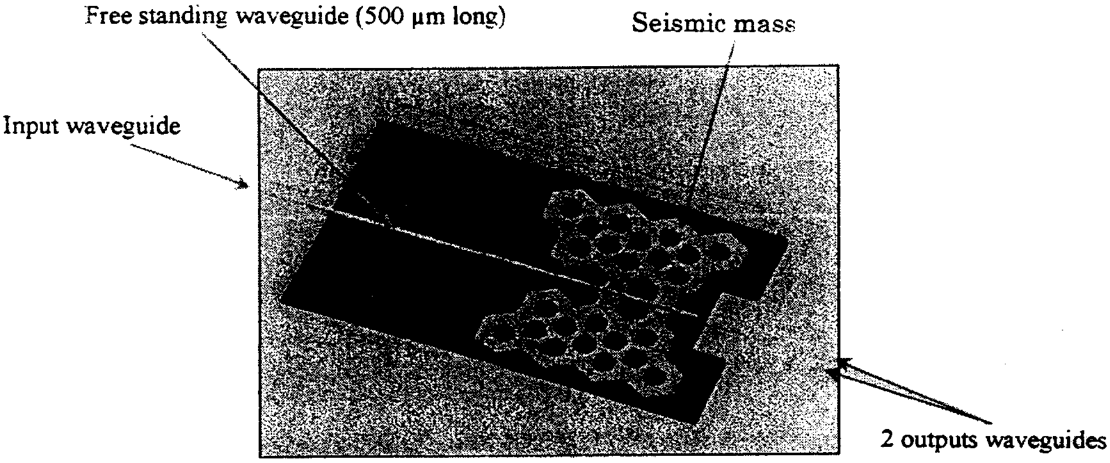

Figure 2. A SEM photograph of the optical transducer developed in integrated optic technology.

optical signal sent to the transducer, detection and preamplification of the return optical signals, differential processing, band-pass filtering, and conditioning of the output signal to its output scale. The light generation subsystem consists of a optical source and its driving electronics. An LED diode and a laser diode were chosen as the light sources for the optical fiber and the integrated optic sensor respectively.

Commercial low cost PIN photo-detectors, pre-amplifiers, processing, filtering, and other electronic components were used for the implementation of the opto-electronic unit. Due to the optical structure of the transducer head, added to the fact that the three optical fibers per sensor head are enclosed together inside each optical pigtail, and because of the differential nature of the processing carried out on the two optical intensities, the output signal (voltage following the acceleration on the transducer) is compensated against environmental and other undesirable perturbations and source drifts or ageing.

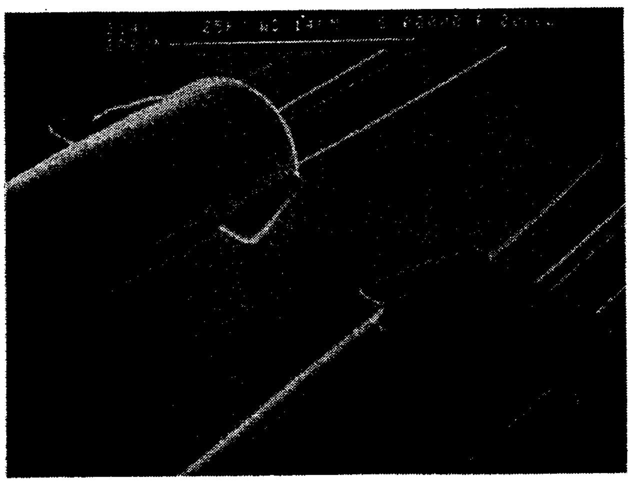

Figure 3. View of two etched grooves. On the right, free of fiber, and on the left, containing a positioned optical fiber.

In order to facilitate the real installation in the power plant together with the overall set of traditional sensors, six optical fiber sensor heads for low frequency shaft-bearing and three integrated optics sensor heads for winding vibration monitoring were fabricated and their associated electronic boards, including their power supply, were integrated in a 19″ rack. In Figure 5, a view of one multi-channel optoelectronic unit is shown. A custom-made optical cable that meets the requirements of international specifications such as B56724 and VDE-0207 was built to link the 19″ rack with the optical distribution boxes at each turbine level, and to connect the sensor heads to these boxes.

The system was fully tested and calibrated in the laboratory prior to its integration in the quasi-smart structure. Samples of the amplitude response and frequency response of an optical fiber accelerometer, and the transverse sensitivity of the integrated sensor head are shown in Figures 6 and 7 respectively. In Table 1 a summary of the main technical characteristics given by both the calibrated integrated and fiber optics sensors is shown.

2.2 Other Sensors

The system includes six commercial eddy current displacement sensors from Micro-Epsilon to measure the

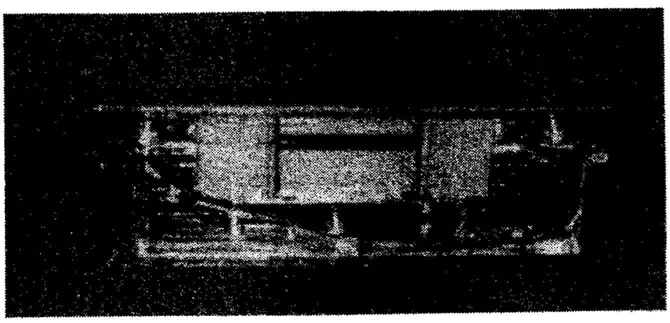

Figure 4. Photograph of a transducer developed in fiber technology.

Figure 5. *A view of one multi-channel optoelectronic unit.*

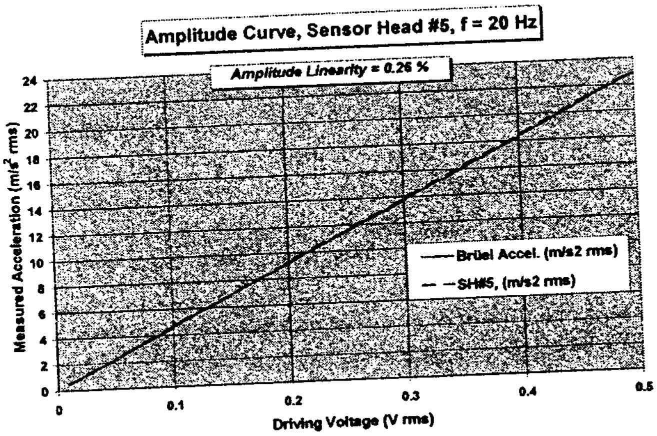

Figure 6. *Calibrated amplitude response accelerometer system developed in fiber technology.*

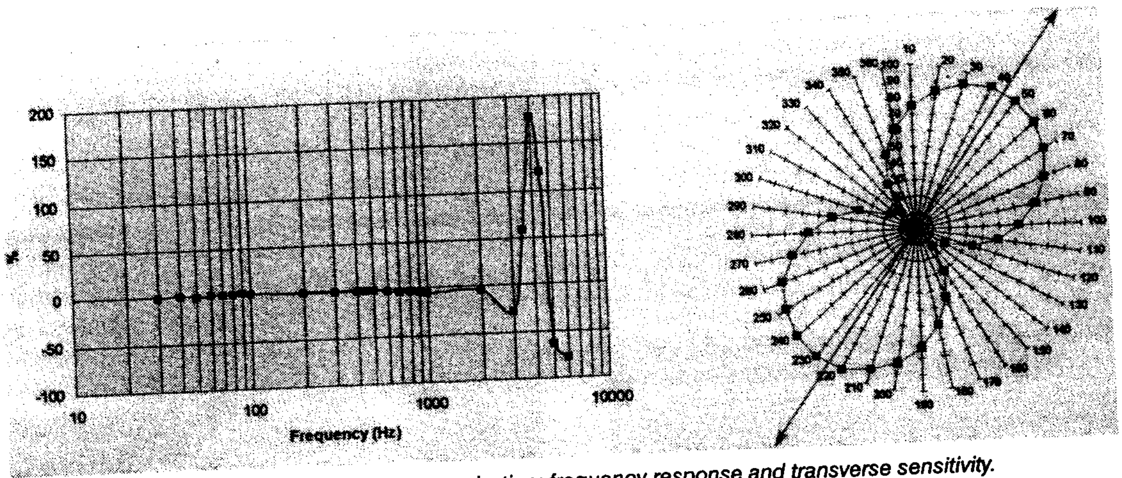

Figure 7. *Examples of in-lab characterization: frequency response and transverse sensitivity.*

Table 1. Summary of the final behaviours of the two sensor systems developed.

Characteristics	Integrated Optic	Fiber Optic
Frequency range	30–2000 Hz	0.2–140 Hz
Frequency response	<±5%	<±5%
Amplitude range	0.5–600 m/s^2	0.025–10 m/s^2 RMS
Amplitude linearity	<1%	0.04% (0 . . . 1 G)
Resolution	0.5 m/s^2	0.02 m/s^2 RMS
Transverse sensitivity	<5%	<4%
Temperature range	10 . . . 90°C	10 . . . 75°C
Temperature sensitivity	—	0.09%/°C

shaft's displacement. Their main characteristics are the following: measuring range from 0 to 2 mm, 1% of linearity and 0.1% of resolution. Each couple of sensors was orthogonally mounted (0° and 90°) at three different levels: two below the upper bearing oil carter, two below the lower bearing and two fixed to the upper crosspiece. Additionally, the system has a keyphasor installed at the upper level, next to the brushes, to give one pulse per revolution. This signal is used to transform the acquired signals and to obtain synchoronous parameters.

3. SENSOR FUSION IN A QUASI-SMART STRUCTURE

The developments carried out in order to reach the quasi-smart structure will be presented in the following.

3.1 Sensor Integration

As monitoring electrical machines is a complex task, in order to reach a quasi-smart structure, the "harmonic integration" of many different sensors, each one having different characteristics, is necessary. Moreover, electrical machines are quite large; therefore, sensors can be placed far away from each other, and even in different levels of the building. Finally, electric and magnetic fields can be strong; thus impairing the transmission of data to a remote control room. To avoid some of these drawbacks, the proposed monitoring system integrates the measurements provided by the sensors by means of a hierarchical and distributed system such as the one shown in Figure 8.

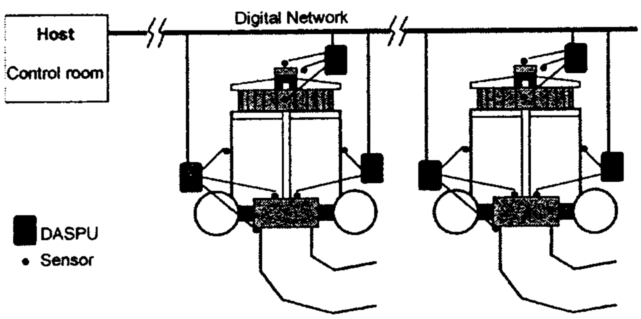

Figure 8. Architecture of the proposed monitoring system.

Actually, two different kinds of signals must be acquired: very low frequency signals or static signals, e.g., active and reactive power, temperatures, flow rate, levels, etc; and dynamic signals such as the shaft's displacement at the guide bearings level or the acceleration of the bearings support or the winding. Usually, static and dynamic signals are acquired using two different units. In this paper, we mainly consider the characteristics of the DASPU, which only processes dynamic signals.

The proposed DASPU is a stand-alone unit that acquires and processes 16 channels with a bandwidth of up to 450 Hz. It deals with signal conditioning, analog to digital conversion, implements several digital signal processing algorithms to extract relevant parameters for machine monitoring and, finally, it is able to issue alert and alarm signals when these parameters are outside the admissible range. In the next subsection we describe with some detail the digital signal processing algorithms implemented in the DASPU.

3.2 Signal Processing in the DASPU

The problem of rotating machine vibration analysis requires performing some specific processing (see Figure 9). For instance, it is often necessary to obtain orbits describing the displacement (or vibration) of the shaft centreline. This requires the use of a pair of orthogonally-mounted displacement or vibration sensors and estimating their amplitudes and phases at integer multiples of the running speed. For each pair of sensors, the amplitudes and phases estimated at each integer multiple of the fundamental frequency compose an elliptic figure denoted as an orbit.

To get useful information from the orbits at different bearing levels, all the sensors must be simultaneously sampled. However, this cannot be accomplished by typical Analog to Digital Converters (ADC) since they multiplex the input channels before sampling. The developed monitoring system includes a digital signal processing algorithm in the DASPU to correct for the delay among channels introduced by the

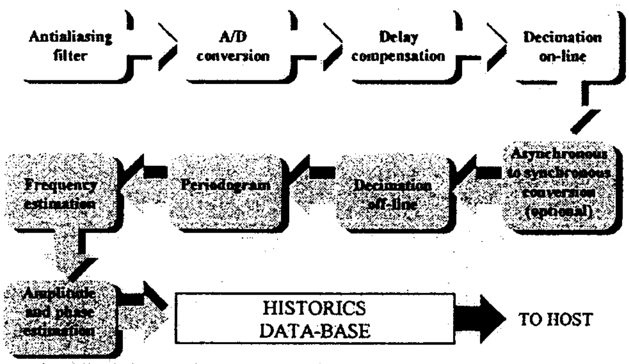

Figure 9. *DASPU processing chain.*

ADC (Luengo et al., 1997). Basically, this method samples the input channels using a high oversampling ratio and then applies a linear interpolation algorithm to get samples at the same time for all the channels. Using a sufficiently high oversampling ratio, the error introduced by the non-ideal linear interpolation can be made lower than half the quantization interval; then we have the same error as real (hardware) simultaneous sampling.

In this monitoring system, the signals requiring simultaneous sampling, (low frequency accelerometers and displacement sensors), have a bandwidth of 140 Hz and we use a sampling frequency of 15625 Hz, therefore the oversampling ratio is slightly over 55. This high oversampling ratio, besides making feasible the use of a simple linear interpolator to achieve simultaneous sampling, allows an easy design of the analog antialiasing input filters.

After the phase correction stage the signals are decimated to reduce the data rate. The system uses two decimation stages (11 and 4, respectively), achieving a final data sampling rate of 355 samples/sec.

The frequencies, amplitudes and phases are estimated from the largest peaks of the spectrum of each channel. To obtain the spectrum we store a time window of 4096 samples and then apply a well-known Fast Fourier Transform (FFT) algorithm (Oppenheim and Schafer, 1988). The frequencies of the more important harmonics are estimated directly as the largest peaks of the spectrum, while the amplitudes and phases are estimated using the method described in (Santamaría et al., 1998).

Finally, the DASPU includes a signal processing algorithm denoted as asynchronous to synchronous conversion, which is indicated as optional in Figure 9. This method uses a tachometer (sometimes denoted as keyphasor), which gives one pulse per revolution of the shaft to obtain exactly the same number of samples per revolution, i.e., synchronous data. The signal of the tachometer, with pulses indicating the start and end point of each revolution, is used to resample the signals by means of linear interpolation techniques.

After this transformation, the samples of the displacement and vibration signals correspond to the same physical positions of the shaft. Working with synchronous data has three advantages: first, we have a phase reference, allowing us to orientate the orbits obtained using synchronous data. Second, we can average the acquired time registers reducing noise without distorting useful information. Finally, as is discussed in (Santamaría et al., 1998), the use of synchronous data avoids the interference among harmonics which appears when Fourier-based spectral estimation techniques are applied over finite data registers (this kind of interference is known in the signal processing literature as "leakage").

The DASPU obtains spectral information (frequencies, amplitudes and phases) from both asynchronous and synchronous data. These data, as well as the time data and the overall spectra, are transferred to the host for storage and further processing. In particular they are used for data trend analysis in the Neural Network module.

3.3 Host Computer Software

The host computer is a PC running under Microsoft Windows NT Operating System. The monitoring software processes the information from all the DASPUs and does the following main tasks: system configuration, data transfer from the DASPUs, data storage in a signal data base, statistical data analysis, and display of signals, alerts, alarms and other parameters.

The software continuously monitors the status of the DASPUs, by means of a master-slave communication protocol. A set of very short messages allows us to know if there is some variable outside the alert/alarm ranges in each DASPU. When an alert/alarm condition occurs, a flashing and acoustic warning will appear in the operator terminal.

The host computer software can further process the data in order to obtain diagrams, to plot orbits and to perform data trends (short, medium and long term).

3.4 Neural Networks for Predictive Maintenance

The frequencies, amplitudes and phases for both asynchronous and synchronous data can be used as trend parameters to detect and prevent the development of a fault. These parameters must be correlated with some static variables such as the temperature and pressure at different points of the machine, the active and reactive power, etc; which define the operating condition of the machine.

Feed-forward neural networks have been used in a wide variety of classification and prediction problems, including classification of shaft loading condition (McCormick and Nandi, 1997), and vibration monitoring (Lui and Mengel, 1992). Their ability to approximate any nonlinear function (Haykin, 1994) and their capability to learn from the data, extracting hidden correlation among the input parameters, make neural networks a good candidate for performing data trend and predictive maintenance. The neural network module uses a Multilayer Perceptron (MLP) trained with the back-propagation algorithm (Haykin, 1994).

Although the DASPU is continuously acquiring data to perform real-time monitoring, the data and the parameters obtained are transferred to the host only at periodical intervals. Typically, in steady-state the host collects data every hour. These data, as well as the static data defining the condition of the machine, are used to train the MLP. The working mode of our MLP module is as follows: (a) a dynamic parameter, which will be predicted by the MLP, was selected. This parameter can be the amplitude or phase of one of the largest frequencies provided by the DASPU, or the root mean square value of a given sensor. (b) A large set of data from the base is selected to train the MLP as a predictor. The input pattern is composed of previous values of the variable to be predicted, as well as static variables such as the load, the active and reactive power, hydraulic load, temperatures. The input variables are normalized before training. (c) Once the network is trained, for each newly acquired register the MLP performs a prediction of the selected parameter. This prediction is compared with the true measured parameter giving a prediction error. Since the network is trained with data corresponding to safe behaviour, the development of a fault will produce an increase in the error variance. (d) The evolution of the prediction error variance is used to issue alert and alarm levels and also to predict the possible development of a fault, based on preset thresholds. A key step in this method is the selection of the thresholds, which can be different for different power generators and must be obtained using a large amount of past data (over 1 year of data). For long-term predictive maintenance, we use the evolution of the parameters over a long period of time and predict this signal some steps ahead (extrapolate). A simple linear least squares fit can be used to extract the trend of this signal and decide whether the generator is developing a failure or not. In this way we can anticipate a catastrophic failure.

4. FIELD TRIALS

After the laboratory check and the calibration of overall quasi-smart structure the system was installed and fixed for its operation in the hydroelectric plant, and it has been working continuously for a year. The electronics and optoelectronics set of the overall structure were mounted in a 19″ rack and placed about 40 m from the hydrogenerator. A view of this rack is shown in Figure 10.

The sensor heads were enclosed in holders and tightly fastened to suitable places in the machine. For instance, the 6 fiber optics and the 6 eddy current displacement sensors were placed at three different bearing levels. At each level, two vibration and two displacement sensors were mounted at 0° and 90°. Using this configuration and the information given by the keyphasor, it is possible to construct and orientate orbits, which can be employed for detection of malfunctions. On the other hand, the 3 integrated optic transducers were placed in the winding. Each sensor is set in each electric phase with the objective of measuring the tangential vibration when the rotor is turning.

The transducers-electronics rack connections were done optically and electrically by means of the required optical and electrical connectorised cables. The signals are acquired and processed by the DASPU and transmitted to a remote host computer by means of a digital communications network.

The system was periodically checked to assure its correct operation, and some minor malfunctions have been amended. Data, such as the sensor spectral characteristics which are processed by the DASPU and transmitted to the host, are used to take decisions regarding the short-term and long-term behaviour of the machine. In Figure 11 four examples of the spectral responses acquired from the optical sensors and the two of the traditional sensors are shown.

It is remarkable that very small vibration signals have been encountered at some of the points, even below the minimum acceleration foreseen by the initial specifications (0.025 m/s^2 rms). It must be noted that in the fiber optic sensors the noise level is below 0.01 m/s^2 rms, which means that the system can resolve vibration amplitudes three times smaller than the specifications. Furthermore, the value is in good agreement with the laboratory results taken prior to its installation in the plant. Finally, the long-term stability of the system has been surveyed. The signals from the sensors have been analyzed during a long period in which the generator group operated in similar conditions, and no significant time drift has been found (Figure 12). The results obtained support the validation of the system in field operation in a harsh

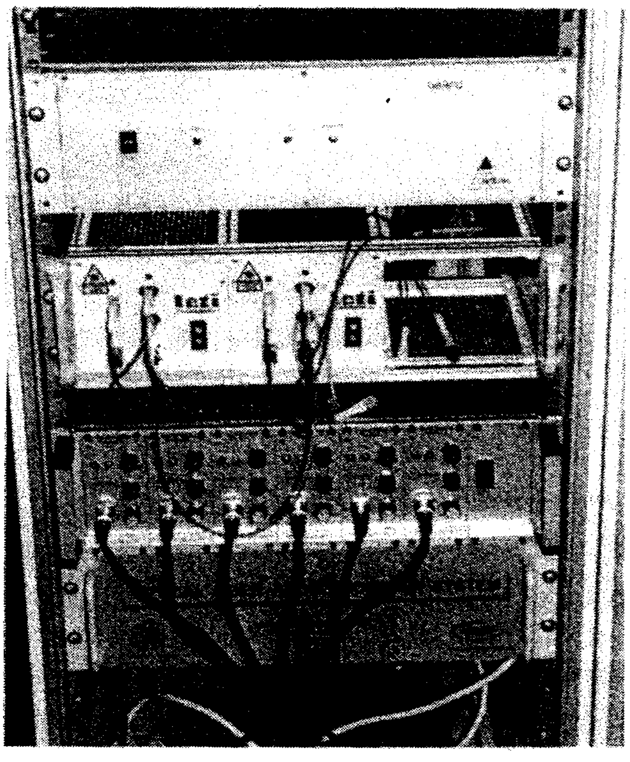

Figure 10. General view of the electronics rack installed in the electric power plant.

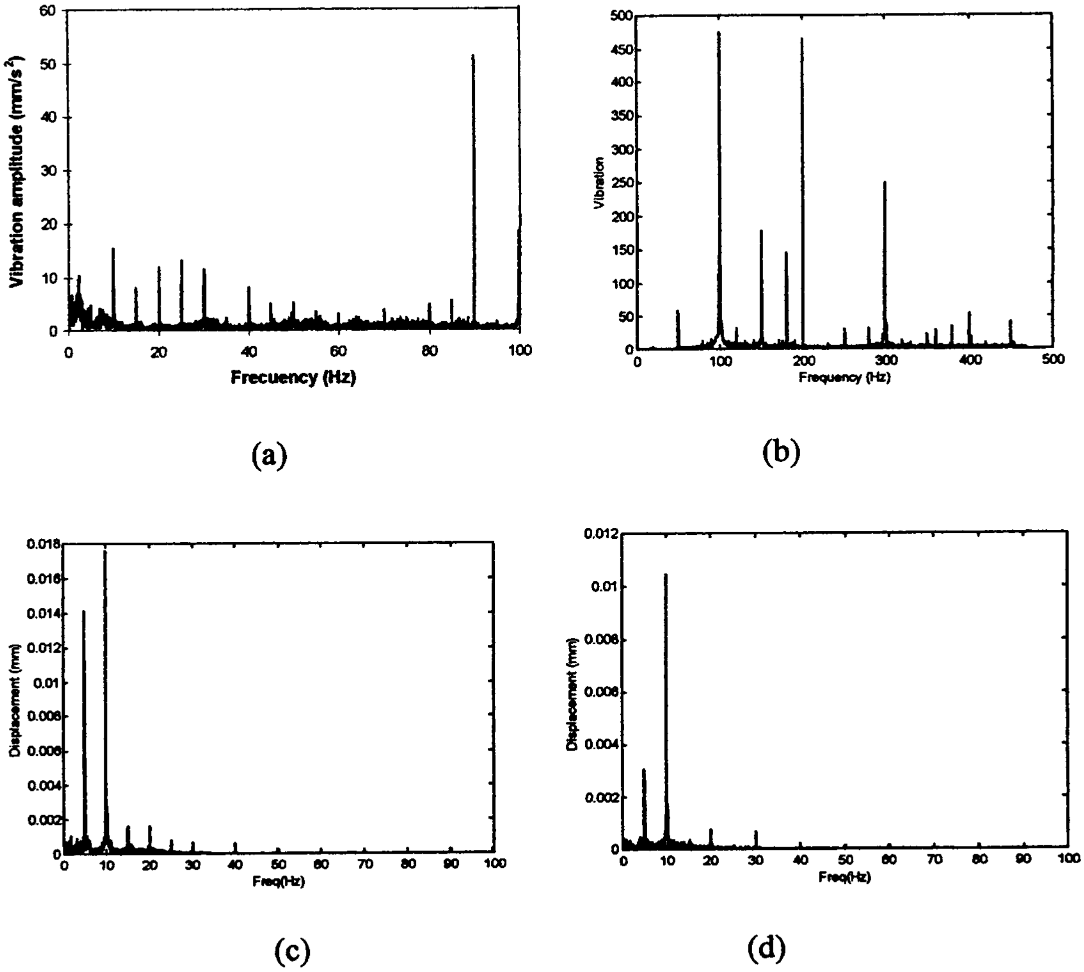

Figure 11. Examples of the spectral features of the vibrations obtained in field tests: optical sensors at the shaft bearing (a) and winding (b), and conventional displacement sensors (c), (d).

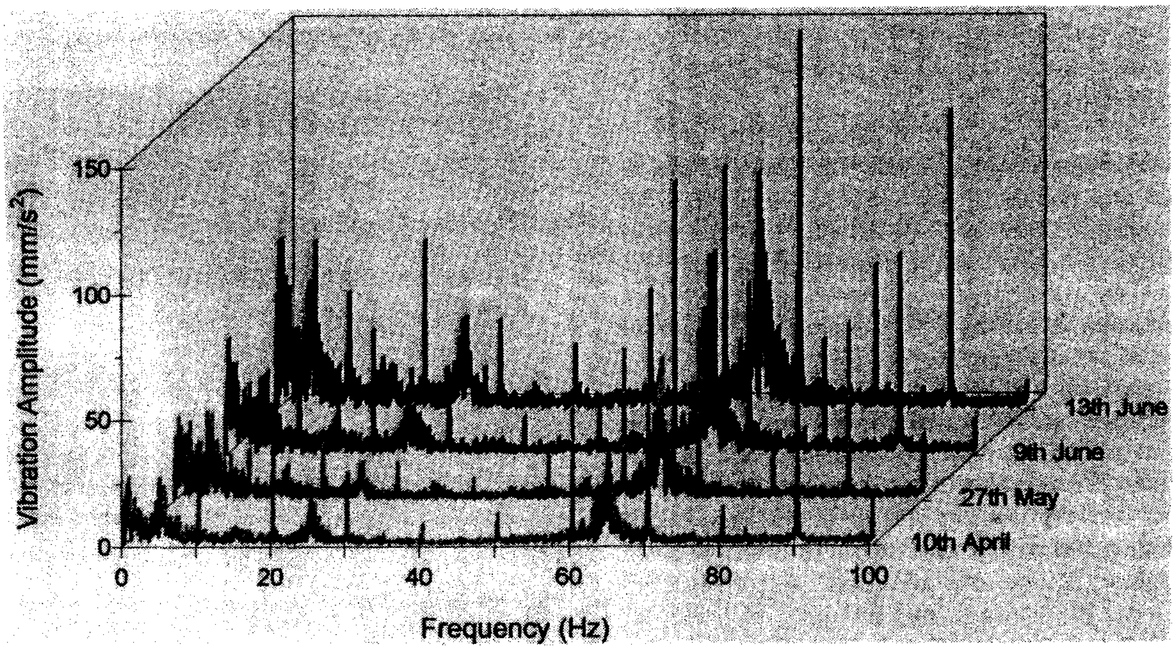

Figure 12. Time evolution of the spectral characteristic from one of the fiber optic sensors.

environment such as the one existing at an electric power station.

5. SUMMARY AND CONCLUSIONS

In the framework of one European project a quasi-smart structure for vibration monitoring and predictive maintenance of rotating machines such as electric power hydrogenerators has been developed and tested. The structure is integrated by a complex system of transducers, electronics and software.

The transducer set, among other traditional transducers, includes new optical fiber and integrated optic accelerometers which have been fully developed and tested, with very good agreement with their specifications.

The electronics include the optoelectronic and electronic units for the sensors, a Digital Acquisition and Signal Processing Unit (DASPU) and a host computer. The DASPU processes the signals and obtains relevant spectral and temporal parameters. These parameters are transmitted to a remote host, where a Neural Network module performs data trend and extracts information regarding the long-term behaviour of the machine.

The system has worked for a long period installed in a hydro-generator group operating in different conditions. The results obtained validate the quasi-smart structure for real-time field operation in a harsh environment.

Two patents related to this work are pending.

ACKNOWLEDGEMENT

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