Analytical Approximations for the Capacity of Orthogonal SFBC

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***Abstract*— Orthogonal space-frequency block coding (OSFBC) combined with orthogonal frequency-division multiplexing (OFDM) has been shown to be a simple and efficient means to exploit the inherent diversity of multiple-input-multiple-output (MIMO) broadband channels. In this paper we derive simple analytical closed-form expressions for the ergodic and outage capacity of OSFBC-OFDM systems assuming that the channel is unknown at the transmitter and perfectly known at the receiver. These expressions are simple functions of the spatial correlation matrices at the channel taps. They clearly reveals the dependence of the capacity on the channel and system parameters. Numerical results show the excellent accuracy of the derived expressions.**

I. Introduction

In MIMO-OFDM systems orthogonal block codes can be applied across the transmit antennas and OFDM tones to exploit the inherent space-frequency diversity of the channel, leading to the so-called OSFBC. Different orthogonal coding strategies have been proposed in the technical literature [1] [2]. An inherent advantage of such orthogonal codes is the significant low decoding and detection complexity. In fact, in OSFBC maximum-likelihood (ML) detection is performed separately on each symbol, which leads to simple receivers. There are other more complex non-orthogonal space-frequency block coding techniques that outperform OSFBC, but at the price of higher decoding and detection complexity than in OSFBC receivers [3], [4].

OSFBC can be concatenated with outer codes providing frequency diversity and enhancing the system performance [5]. In this context, the channel capacity is a crucial performance measure to investigate the capacity-approaching capabilities of the overall system. The capacity of OFDM-based spatial multiplexing MIMO systems was analyzed in [6]. Here, we focus on the capacity of OSFBC systems when the channel is unknown at the transmitter and known at the receiver.

In the case of narrowband fading channels, a number of closed-form expressions has been proposed for the capacity of orthogonal space-time block coding [7], [8], [9], [10] [11]. On the contrary, from the author's best knowledge, there are not analytical expressions for the capacity of OSFBC in broadband MIMO channels. In this work, from a general broadband MIMO channel model model, we derive tight general closed-form expressions for the ergodic and outage capacity of OSFBC. These expressions give great insight into the influence of the channel and system parameters on the capacity.

In general, the closed-form expressions are useful in two ways. They can be used to generate performance curves (in this case ergodic and outage capacity curves) without resorting to time-consuming Monte Carlo simulations. Second and more important, they can reveal the influence of the channel and system parameters on the capacity. Based on the derived expressions we can easily analyze the dependence of the OSFBC capacity on the spatial correlation, power delay profile (PDP), number of antennas, signal-to-noise ratio (SNR), etc.

II. Broadband MIMO Channel Model

Consider a discrete-time broadband MIMO channel with n_T transmit antennas and n_R receive antennas. The MIMO channel frequency response at the OFDM tones can be expressed as follows

$$\mathbf{H}_k = \sum_{n=0}^{L-1} \mathbf{F}_n \exp\left(-j2\pi nk/K\right), \; k = 0, 1, \ldots K, \quad (1)$$

where K is the number of OFDM tones, $\mathbf{F}_n$ is a $n_R \times n_T$ matrix denoting the n-th tap of the discrete-time MIMO fading channel impulse response and L is the number of taps. The entries of each matrix $\mathbf{F}_n$ are assumed to be circular symmetric complex Gaussian random variables. In general, they are correlated according to a specific spatial covariance matrix $\mathbf{R}_n = E\left[\text{vec}\left(\mathbf{F}_n\right)\, \text{vec}^H\left(\mathbf{F}_n\right)\right]$, whose entries are given by

$$\rho_n^{ij,ks} = E\left[f_n^{ij}\left(f_n^{ks}\right)^*\right], \quad {\scriptstyle i,k=1,\ldots,n_R, \quad j,s=1,\ldots,n_T} \quad (2)$$

where f_n^{ij} is the entry of $\mathbf{F}_n$ corresponding to the jth transmit and the ith receive antennas. Note that the diagonal terms of the covariance matrices determine the PDP's of the channel. The nth term of the discrete-time PDP between the jth transmit and the ith receive antennas is given by $p_n^{ij} = \rho_n^{ij,ij}$. In general, there will be different PDP's for the different pairs of transmit-receive antennas. We assume, without loss of generality, that the channel is normalized so

$$\sum_{i=1}^{n_R}\sum_{j=1}^{n_T}\sum_{n=0}^{L-1} p_n^{ij} = \sum_{n=0}^{L-1} \text{Tr}\left(\mathbf{R}_n\right) = n_R n_T. \quad (3)$$

Since the $\mathbf{F}_n$ are zero-mean Gaussian matrices, the power correlation between the elements of $\mathbf{F}_n$ can be expressed as follows [12]

$$E\left[\left|f_n^{ij}\right|^2\left|f_n^{ks}\right|^2\right] = \left|\rho_n^{ij,ks}\right|^2 + \rho_n^{ij,ij}\rho_n^{ks,ks}. \quad (4)$$

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We assume that the matrices $\mathbf{F}_n$ at different taps are uncorrelated. Therefore

$$E\left[f_n^{ij} f_m^{ks}\right] = 0,\ n \neq m. \tag{5}$$

Although the above assumption is not exact due to the finite bandwidth of the receiver, this is commonly accepted in discrete-time broadband channel models [5], [6], [13]. Since the f_n^{ij} are Gaussian, they are independent. Then

$$E\left[\left|f_n^{ij}\right|^2 \left|f_m^{ks}\right|^2\right] = \rho_n^{ij,ij} \rho_m^{ks,ks},\ n \neq m. \tag{6}$$

Let γ_k denotes the squared Frobenius norm of the channel response at the kth tone: $\gamma_k = \|\mathbf{H}_k\|_F^2$. Considering (1) and (5), it is straightforward to show that the γ_k's are identically distributed. The mean, and covariances of the γ_k's are given by

$$\mu_\gamma = \sum_{n=0}^{L-1} \mathrm{Tr}\left(\mathbf{R}_n\right), \tag{7}$$

$$\sigma_\gamma^{k,s} = \sum_{n=0}^{L-1} \sum_{m=0}^{L-1} \mathrm{Tr}\left(\mathbf{R}_n \mathbf{R}_m^H\right) e^{-j2\pi(k-s)\ (n-m)/K}. \tag{8}$$

In particular, setting $k = s$, the variance of the γ_k's is given by

$$\sigma_\gamma^2 = \mathrm{var}\left[\gamma_k\right] = \sum_{n=0}^{L-1} \sum_{m=0}^{L-1} \mathrm{Tr}\left(\mathbf{R}_n \mathbf{R}_m\right) = \|\mathbf{R}_S\|_F^2, \tag{9}$$

where $\mathbf{R}_S = \sum_{n=0}^{L-1} \mathbf{R}_n$ is the sum of the correlation matrices at the channel taps.

III. Capacity of OSFBC-OFDM systems

Assuming that the channel is unknown at the transmitter, the total available power is allocated uniformly across all the transmit antennas and the OFDM subchannels. In OSFBC, the space-frequency codeword (of size $n_T \times K$) comprises a number (N) of different orthogonal block codewords of size $n_T \times L_C$, where L_C is the number of adjacent OFDM tones involved in each block. To exploit the frequency selectivity of the channel, each block can be repeated S times, where $S \leq L$ [2]. Then, the total number of OFDM tones is $K = NL_C S$. As example (10) shows the codeword in the simple case of Alamouti coding ($n_T = 2$, $L_C = 2$), $N = 2$, $S = 2$ and $K = 8$.

$$\mathbf{C} = \begin{bmatrix} s_1 & -s_2^* & s_3 & -s_4^* & s_1 & -s_2^* & s_3 & -s_4^* \\ s_2 & s_1^* & s_4 & s_3^* & s_2 & s_1^* & s_4 & s_3^* \end{bmatrix}. \tag{10}$$

In general the code rate will be R/S, where $R = n_S/L_C$ is the code rate of the individual block codewords, and n_S is the number of symbols involved in the block. In the case of Alamouti coding $n_S = L_C = 2 \Rightarrow R = 1$.

To derive an expression for the channel mutual information, we consider the following assumptions: 1) the number of OFDM tones (K) is high so the channel response is constant at the tones involved in each orthogonal block. 2) In spite of this, the channel stays constant during the transmission of each codeword $\mathbf{C}$. 3) The OFDM uses a cyclic prefix with adequate length. Since the number of OFDM tones is high, the transmission rate penalty and the SNR penalty due to the transmission of the cyclic prefix is neglected.

Under the above assumptions, the broadband MIMO channel can be decomposed into a set of narrowband uncoupled MIMO channels, each one associated with an individual block. Moreover, due to the orthogonal coding of the individual blocks, each narrowband MIMO channel can be viewed as an effective scalar channel. Then, since each orthogonal block is repeated S times in the codeword, the broadband MIMO channel can be decomposed into a set of N effective scalar channels with signal-to-noise ratio (SNR) given by

$$\begin{aligned} SNR_n &= \frac{\rho}{Rn_T} \sum_{s=0}^{S-1} \gamma_{(n+sN-1)L_C+1} = \dots \\ &= \frac{\rho}{Rn_T} \sum_{s=0}^{S-1} \gamma_{(n+sN-1)L_C+L_C}, \end{aligned} \tag{11}$$

where ρ is the average SNR at the receive antennas (assuming AWGN noise). Each effective scalar channel transmits n_S symbols simultaneously using SL_C OFDM tones. Since the N effective channels use the K tones in each transmission, the mutual information can be expressed as follows

$$I = \frac{n_S}{K} \sum_{n=1}^{N} \log_2\left(1 + SNR_n\right), \tag{12}$$

and considering (11),

$$I = \frac{R}{K} \sum_{n=1}^{N} \sum_{l=0}^{L_C} \log_2\left(1 + \frac{\rho}{RN_T} \sum_{s=0}^{S-1} \gamma_{(n+sN-1)L_C+l}\right). \tag{13}$$

Note that the blocks repetition ($S > 1$) in (10) provides frequency diversity but at the price of a code-rate penalty by a factor of $1/S$. On the other hand, when $S = 1$, the code is full-rate but it does not provide frequency diversity [1]. Another more efficient way to exploit the inherent frequency diversity of the broadband channel is to combine an outer code with an OSFBC code [5]. Assuming that we use an outer code, a full-rate OSFBC (without repetion) is more efficient than a OSFBC with repetitions. Therefore, hereafter we will focus OSFBC codes with $S = 1$. In this case (13) reduces to

$$I = \sum_{k=0}^{K-1} I_k = \frac{R}{K} \sum_{k=0}^{K-1} \log_2\left(1 + \rho\frac{\gamma_k}{n_T R}\right). \tag{14}$$

According to (14), I is a non-linear function of the γ_k's. Since the wireless channel is random, the γ_k's are random and I will be a random variable. The first two moments of the mutual information can be approximated expanding I and I^2 in Taylor series about μ_γ and then, applying the expectation operator. The resulting expressions for the mean and variance of I are

$$\mu_I \approx R\log_2\left(1+\frac{\rho\, n_R}{R}\right) - \frac{R\rho^2\log_2 e}{2n_T^2\,(R+\rho\, n_R)^2}\ \|\mathbf{R}_S\|_F^2\,. \quad (15)$$

$$\sigma_I^2 \approx \left(\frac{R\rho\log_2 e}{n_T(R+\rho n_R)}\right)^2 \sum_{n=0}^{L-1}\|\mathbf{R}_n\|_F^2\,. \quad (16)$$

where e is the neper's number.

If the is ergodic, the capacity is given by the ensemble average of the mutual information over the channel realizations, hence it is given by (15). For quasi-static fading channels we obtain the outage capacity from a Gaussian approximation of the cumulative distribution function of I. Then, the $q\%$ outage capacity is approximated as follows

$$C_q \approx \mu_I + \sigma_I\sqrt{2}\ \mathrm{erfc}^{-1}\left(2-\frac{q}{50}\right). \quad (17)$$

where $\mathrm{erfc}(x)$ is the complementary error function. Note that, according to (15)-(17), the ergodic and the outage capacities do not depend on the number of OFDM tones. Equation (15) suggests that high spatial correlation at the individual channel taps does not always produce low ergodic capacity because $\mathbf{R}_S$ is the coherent sum of the spatial correlation matrices at the individual taps. On the contrary, according to (15)-(17), higher spatial correlation always leads to lower outage capacity.

A. Channel with a common correlation matrix

Assuming a spatially balanced channel with the same correlation matrix for all the taps: $\mathbf{R}_n = p_n\mathbf{R}$, where $\mathbf{R}$ is the common spatial correlation matrix with unit entries in its main diagonal, and $\{p_n\}$ is the common PDP. This situation tipically arises when the antennas are very close at the transmit and/or receive array, and when the transmitter and/or receiver are surrounded by local scatterers, so the angular spectrums are omnidirectional for any tap. In this case,

$$\|\mathbf{R}_S\|_F^2 = \|\mathbf{R}\|_F^2\ ,\quad \sum_{n=0}^{L-1}\|\mathbf{R}_n\|_F^2 = \|\mathbf{R}\|_F^2\sum_{n=0}^{L-1}p_n^2.$$

Note that the spatially uncorrelated channel can be viewed as a particular case where $\mathbf{R}$ is a diagonal matrix. In this case $\|\mathbf{R}\|_F^2 = n_R n_T$ because of the channel normalization. In spatially correlated channels, $\|\mathbf{R}\|_F^2 \geq n_R n_T$, therefore μ_I will be always lower than in the corresponding uncorrelated channels. The variance σ_I^2 will be higher in spatially-correlated channel than in the corresponding uncorrelated channel. Therefore, as it is expected, the spatial selectivity also improves the outage capacity.

In the particular case of uniform PDP's, the variance reduces to

$$\sigma_I^2 \approx \left(\frac{R\rho\log_2 e}{n_T(R+\rho n_R)}\right)^2\ \|\mathbf{R}\|_F^2\ \frac{1}{L}. \quad (18)$$

This is the lower variance for all the possible PDP's of length L. The variance for a two-rays PDP is obtained by setting $L=2$, regardless the delay between the two taps. By setting $L=1$ we obtain the variance for a one-ray PDP which corresponds to a channel with frequency flat response. In this case, the σ_I^2 coincides with the variance of the mutual information in narrowband spatially correlated MIMO-STBC channels with Rayleigh fading [10].

B. One-side spatially correlated channels

We first focus on channels spatially correlated in reception and uncorrelated in transmission. This situation usually arises in the uplink of a typical NLOS urban outdoor channel when the transmitter is surrounded by local scatterers and the receiver is not obstructed by local scatterers. Assuming that the correlation at the receiver array does not depend on the transmit antenna, the correlation matrices at the channel taps can be expressed as follows

$$\mathbf{R}_n = \mathbf{R}_{\mathbf{T}n}^T \otimes \mathbf{R}_{\mathbf{R}n} = \mathbf{I}\otimes\mathbf{R}_{\mathbf{R}n},$$

where $\otimes$ denotes the Kronecker product, the superscript $(\cdot)^T$ denotes the matrix transpose operator, $\mathbf{R}_{\mathbf{R}n}$ is the $n_R \times n_R$ receive correlation matrix for the n-th tap and $\mathbf{R}_{\mathbf{T}n}$ is the $n_T \times n_T$ transmit correlation matrix for the n-th tap which, in this case, equals the identity matrix $\mathbf{I}$. Now,

$$\|\mathbf{R}_S\|_F^2 = n_T\,\|\mathbf{R}_{\mathbf{R}S}\|_F^2\ ,\quad \sum_{n=0}^{L-1}\|\mathbf{R}_n\|_F^2 = n_T\sum_{n=0}^{L-1}\|\mathbf{R}_{\mathbf{R}n}\|_F^2\,, \quad (19)$$

where $\mathbf{R}_{\mathbf{R}S} = \sum_{n=0}^{L-1}\mathbf{R}_{\mathbf{R}n}$ is the sum of the correlation matrices in reception. Substituting (19) in (15) and (16) we obtain the corresponding expressions for the mean and variance of the mutual information. Analogous expression are obtained when the channel is spatially uncorrelated in reception and correlated in transmission.

IV. Simulation results

To show the tightness of our approximation, we compare the analytical predictions of (17) with Monte Carlo simulations for a variety of channel conditions and system parameters. In all cases the analytical predictions are represented by solid lines and the Monte Carlo values are represented by markers. In every simulation, 20000 independent Monte Carlo runs have been performed.

Figure 1 shows the 10%-outage capacity for different MIMO configurations and code rates, comparing the analytical predictions of (17) with Monte Carlo simulations. The channel has a truncated exponential PDP with length $L=3$ and decay factor 2/3. The channel is spatially balanced, uncorrelated in transmission and correlated in reception with spatial correlation matrices given by the Jakes correlation model [14]. This model assumes uniform angular spectrum at both the transmitter and the receiver for all the channel taps. Also, it is assumed that the antennas are identical and single-polarized, at each array. According to this model the entries of $\mathbf{R}$ are given by

$$\rho^{ij,ks} = J_0\left(2\pi s_{ik}\right)J_0\left(2\pi s_{js}\right), \quad (20)$$

where $J_0\left(x\right)$ is the zero-order Bessel function of the first kind and s_{ik} and s_{js} are the distances (in wavelengths) between

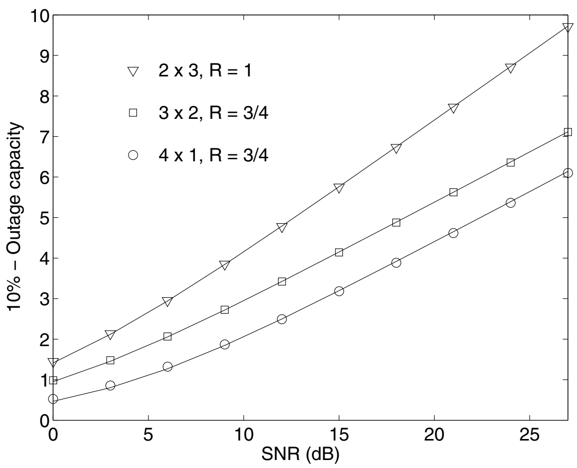

Fig. 1. 10%-Outage capacity of OSFBC for different MIMO configurations as a function of the average SNR at the receiver antennas. The curves compare the analytical predictions of (17) (solid lines) with Monte Carlo simulations (markers).

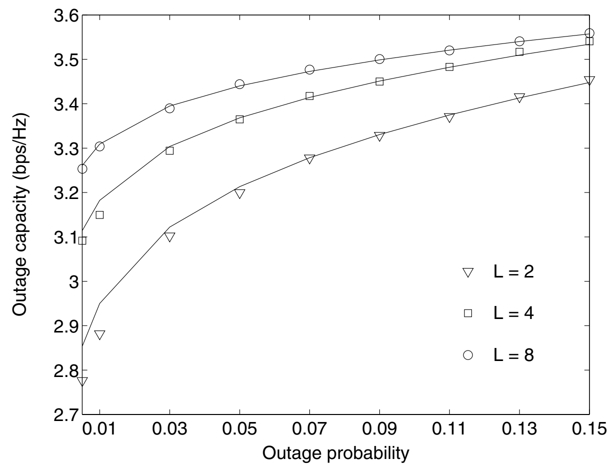

Fig. 2. Outage capacity of 3×3 OSFBC, as a function of the outage probability, for different channel lengths.

the corresponding antennas in the receive and transmit arrays, respectively. In the simulations we assume linear arrays with uniform antenna separations equal to $\lambda/6$ in both arrays. We also assume the same correlation matrix for all the channel taps. The number of OFDM tones was $K = 256$. The approximation error of (17)is lower than 0.06 bps/Hz in all cases.

Figure 2 shows results of outage capacity versus outage probability for a 3×3 MIMO channel with $K = 64$ OFDM tones. The code rate is $R = 3/4$. The different curves corresponds to uniform PDP's with different lengths (L), assuming that the average SNR is $\rho = 10$ dB in all cases. There is common correlation matrix $(\mathbf{R})$ for all taps, which is obtained from the Jakes correlation model [14] (see (20)) assuming linear arrays with uniform antenna separations equal to $\lambda/5$ in both arrays. The figure shows that expression (17) is quite tight for any channel length and outage probability. The relative maximum approximation errors were 2.8%, 1.1% and 0.24% for the the cases $L = 2$, $L = 4$ and $L = 8$, respectively. The curves also show the dependency of the outage capacity with the channel length. Note that the variance of the mutual information is inversely proportional to L, as (18) shows.

Now, we consider channels spatially correlated in reception and uncorrelated in transmission, where the spatial correlation matrix is different at the channel taps. We consider the channel model used in [6] and [13]. This model is suitable for the uplink of a typical cellular suburban channel where the transmitter is surrounded by local scatterers and the receiver is not obstructed by local scatterers. The model assumes that each channel tap is due to the waves arriving from a scatterer cluster, where the waves from a given cluster experience the same delay. The model also assumes a linear array at the receiver with identical single-polarized antennas. For each cluster/tap the angle of arrival of the incoming waves (with respect the array axis) are Gaussian distributed around a mean value $(\bar{\theta}_n)$ with standard deviation σ_n^θ. In practice, this standard deviation depends on the scattering radius of the cluster and its distance to the receiver. Under these assumptions and for small angular spreads, the entries of the receive correlation matrices $\mathbf{R}_{\mathbf{R}n}$ can be expressed as follows [6], [13]

$$\rho_{R\ n}^{i,k} \approx p_n \exp\left[-j2\pi s_{ik} \cos\bar{\theta}_n - 2\left(\pi s_{ik}\,\sigma_n^\theta \sin\bar{\theta}_n\right)^2\right]. \tag{21}$$

Note that, unlike in previous results, there are different correlation matrices for each channel path. Figure 3 shows results of 1% - outage capacity as function of the spacing between the receive antennas for different MIMO configurations with two transmit antennas and variable number of receive antennas. The code rate is $R = 1$, the number of OFDM tones is $K = 128$ and the average SNR at the receiver branches is ρ = 15 dB, in all cases. We consider $L = 6$ taps /clusters with mean angles of arrival given by $(n+6)\,\pi/16,\ n = 0, \ldots,\ L-1$. That is the clusters are uniformly distributed around an arc of $5\pi/16$ radians. We also assume uniform PDP and identical angular standard deviation for all the clusters: $\sigma_n^\theta = (\pi/36)$. The mean and variance of the outage capacity are obtained considering (21) and (19). Once again, the analytical approximation of (17) closely matches the outage capacity (markers), with an relative approximation error lower than 2.8% in all cases.

V. Conclusions

In this work we have derived tigth closed-form approximations for the ergodic and outage capacity of OSFBC. The derived expressions only depend on the spatial correlation matrices of the MIMO channel at the channel taps and on the system parameters. The accuracy of the expressions reveals that these covariance matrices are the only channel statistics needed for a tight estimation of the capacity. The expressions are quite accurate for any spatial correlation conditions, power delay profiles, SNR and system parameters. They clearly show the dependency of the ergodic and outage capacity on the system and channel parameters.

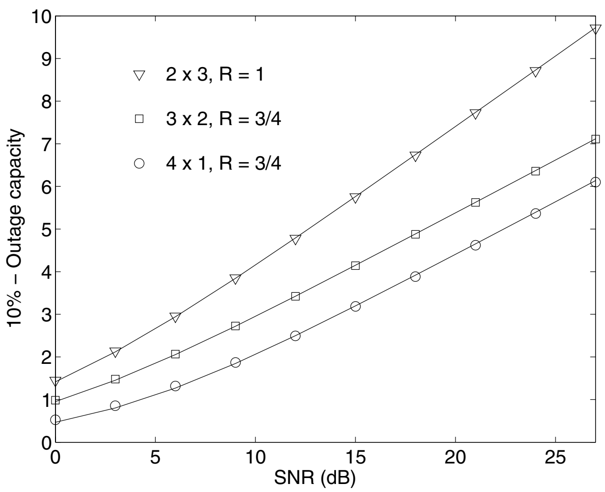

Fig. 3. 1%-Outage capacity of different OFDM-STBC systems as a function of the spacing between the receive antennas.

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