

## Beamforming Design for Simplified Analog Antenna Combining Architectures

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**Abstract**—Analog antenna combining architectures have recently received increased interest due to their reduced size and power consumption, in comparison with the conventional multiple-input–multiple-output (MIMO) approach that operates in the baseband. In this paper, we design algorithms for three new radio-frequency (RF) architectures that are based on different combinations of phase shifters and variable gain amplifiers. From a baseband point of view, each architecture poses a different beamforming design problem in which the transmit/receive beamformers are constrained to have constant-modulus complex, real, or nonnegative real weights, respectively. Assuming perfect channel knowledge, we consider the problem of designing these constrained RF beamformers under orthogonal frequency-division-multiplexing (OFDM) transmissions. For the general MIMO case, the resulting optimization problems are not convex and are, therefore, difficult to solve. However, the single-input–multiple-output (SIMO) and multiple-input–single-output cases either have a closed-form solution or can be reformulated as convex problems. Exploiting this fact, an alternating optimization procedure is used to find a suboptimal solution for the MIMO case. The performance of the different RF-MIMO architectures is compared by means of Monte Carlo simulations, which allows us to conclude that the architecture based solely on phase shifters (RF equal-gain beamforming) provides the best results.

**Index Terms**—Analog antenna combining, equal-gain beamforming (EGB), equal-phase beamforming (EPB), multiple-input–multiple-output (MIMO) beamforming, orthogonal frequency-division multiplexing (OFDM).

### I. INTRODUCTION

Conventional multiple-input–multiple-output (MIMO) systems require all the antenna paths to be independently acquired and jointly processed at the baseband. This increases the cost of the transceiver, which is approximately proportional to the number of analog-to-digital converters (ADCs) [1], [2]. For this reason, the implementation of conventional MIMO transceivers becomes a major problem in low-cost wireless terminals, where the hardware complexity is strictly limited.

To increase the energy efficiency of such systems without excessively increasing the size and hardware cost, several schemes based on shifting part of the spatial signal processing from the baseband to the radio-frequency (RF) front-end have been proposed in [1]–[3]. Although these analog combining architectures are restricted to

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process a single stream of data, they can still extract the spatial diversity and array gain of the MIMO channel.

In particular, the RF-MIMO architecture proposed in [2] and [4] uses, at each branch, a phase splitter and two variable gain amplifiers (VGAs), which essentially implement a multiplication by a complex weight directly in the RF domain. Subsequently, the weighted RF signals are added before acquisition. In the case of flat-fading channels, this RF-MIMO scheme, which we refer here to as maximum-ratio beamforming (RF-MRB), can implement the optimal MIMO beamforming solution by properly choosing the gains of the in-phase and quadrature signals at each branch. The problem is more involved in the case of frequency-selective channels and orthogonal frequency-division-multiplexing (OFDM) transmissions, because the same analog beamformer is applied to all subcarriers, and thus, the problem is inherently coupled [3]. The RF-MRB architecture and its beamforming design algorithms have been thoroughly studied in several publications [2]–[5]. These works suggest that the performance gap of RF-MRB with respect to conventional architectures is justified by the reduction in hardware cost and power consumption. Following this line of research, the motivation of this paper is to investigate other alternative analog antenna combining architectures that could further reduce the system complexity without having a high impact on performance. Thus, we consider three alternative analog antenna combining architectures. The first scheme, which we refer to as RF real-weight beamforming (RF-RWB), applies at each branch a sign switch (i.e., a controllable  $0^\circ/180^\circ$  phase shifter) followed by a VGA, which jointly permit a change in the amplitude and sign of each incoming RF signal before adding them up. The second scheme, which we refer to as RF equal-phase beamforming (RF-EPB), uses only a VGA per RF branch. Finally, the third scheme, which we refer to as RF equal-gain beamforming (RF-EGB), changes only the phase of the RF signals by means of analog phase shifters. From a baseband point of view, these three RF-MIMO architectures pose different constraints on the beamforming problem. That is, instead of having complex RF weights (like those of the RF-MRB or the conventional MIMO schemes), we have real (RF-RWB), nonnegative real (RF-EPB), and constant-modulus complex (RF-EGB) RF weights.

Assuming perfect channel knowledge, our design criterion consists of maximizing the signal-to-noise ratio (SNR) at the output of the receive beamformer. We will first show that the beamforming design problems for MIMO channels are highly nonconvex, and therefore, we will have to resort to an iterative algorithm based on the successive solution of the equivalent single-input–multiple-output (SIMO) and multiple-input–single-output (MISO) problems. This alternating optimization algorithm is guaranteed to converge (although not necessarily to the global optimum), and with a proper initialization method, it provides very satisfactory results, which is illustrated by means of several simulation examples.

Throughout this paper, boldfaced uppercase letters denote matrices, boldfaced lowercase letters column vectors, and light-faced lowercase letters scalar quantities. Superscripts  $(\cdot)^T$  and  $(\cdot)^H$  denote transpose and Hermitian, respectively.  $\|\mathbf{A}\|$ ,  $\text{Tr}(\mathbf{A})$ ,  $\text{rank}(\mathbf{A})$ , and  $\text{diag}(\mathbf{A})$  denote, respectively, Frobenius norm, trace, rank, and the diagonal version of matrix  $\mathbf{A}$ .  $\text{vec}(\mathbf{A})$  is the column-wise vectorized version of matrix  $\mathbf{A}$ , and  $\text{unvec}(\mathbf{a})$  is the inverse of the  $\text{vec}(\mathbf{A})$  operation, i.e.,  $\text{unvec}(\text{vec}(\mathbf{A})) = \mathbf{A}$ .  $\mathbf{A} \succeq \mathbf{0}$  means that  $\mathbf{A}$  is Hermitian and positive semidefinite, whereas  $\mathbf{A} \geq \mathbf{0}$  means that the elements of  $\mathbf{A}$  are nonnegative.  $\mathbb{C}$  and  $\mathbb{R}$  are the complex and real fields, respectively, and  $\Re(\mathbf{A})$  denotes the real part of the complex matrix  $\mathbf{A}$ . Finally,  $\mathbf{I}$  and  $\mathbf{0}$  are the identity and zero matrices of the required dimensions.

## II. CONVENTIONAL AND RADIO-FREQUENCY MULTIPLE-INPUT–MULTIPLE-OUTPUT ARCHITECTURES

This section revisits the conventional MIMO architecture that processes the signals in the baseband domain, and introduces the original analog antenna combining system (RF-MRB), as well as the newly proposed analog beamforming architectures (RF-RWB, RF-EPB, and RF-EGB).

### A. Conventional MIMO

In conventional MIMO systems, beamforming is performed at the baseband. At the receiver side (the transmitter operates analogously), the signal at each antenna must be downconverted and analog-to-digital converted (ADC) before combining. Therefore, the cost of the conventional MIMO transceiver is approximately proportional to the number of ADCs used, which are the most power-hungry components. While this architecture establishes an upper bound on the performance of any multiantenna system, it remains expensive for practical implementation, especially at mobile handheld terminals. Considering a MIMO system with  $n_T$  transmit (Tx) and  $n_R$  receive (Rx) antennas and assuming a transmission scheme based on OFDM with  $N_c$  data carriers and using a cyclic prefix longer than the channel impulse response, the communication system after Tx–Rx baseband beamforming may be decomposed into the following set of parallel and noninterfering single-input single-output (SISO) equivalent channels:

$$y_k = \mathbf{w}_{R,k}^H \mathbf{H}_k \mathbf{w}_{T,k} s_k + \mathbf{w}_{R,k}^H \mathbf{n}_k, \quad k = 1, \dots, N_c \quad (1)$$

where  $y_k \in \mathbb{C}$  is the observation associated with the  $k$ th data carrier,  $s_k \in \mathbb{C}$  is the transmitted symbol,  $\mathbf{n}_k \in \mathbb{C}^{n_R \times 1}$  represents the complex circular independent identically distributed (i.i.d.) Gaussian noise with zero mean and variance  $\sigma^2$ ,  $\mathbf{w}_{T,k} \in \mathbb{C}^{n_T \times 1}$  and  $\mathbf{w}_{R,k} \in \mathbb{C}^{n_R \times 1}$  are the transmit and receive beamformers in digital baseband (a pair of beamformers for each subcarrier  $k$ ), and  $\mathbf{H}_k \in \mathbb{C}^{n_R \times n_T}$  represents the MIMO channel for the  $k$ th data carrier.

### B. Maximum Ratio Beamforming Architecture (RF-MRB)

The original RF-MIMO architecture considered in [2]–[4] changes the amplitude and phase of the transmitted/received RF signals by means of vector modulators (VMs), as illustrated in Fig. 1. Essentially, each VM is implemented through a phase splitter and two VGAs. The weighted RF signals are finally summed up and then downconverted. In this case, the frequency-domain observations at baseband are

$$y_k = \mathbf{w}_R^H \mathbf{H}_k \mathbf{w}_T s_k + \mathbf{w}_R^H \mathbf{n}_k, \quad k = 1, \dots, N_c \quad (2)$$

where  $\mathbf{w}_T \in \mathbb{C}^{n_T \times 1}$  and  $\mathbf{w}_R \in \mathbb{C}^{n_R \times 1}$  are the transmit and receive beamformers in RF. In this paper, we assume that the analog circuitry is ideal, and therefore, each beamformer weight in (2) can take any value within the field of complex numbers. Therefore, for flat-fading MIMO channels, this architecture can implement the optimal MRB at both sides of the link, which justifies the nomenclature RF-MRB. The key differences with the conventional MIMO architecture are twofold. On one hand, the Tx and Rx beamformers are the same for all the subcarriers, and in consequence, the beamforming design problem under OFDM transmissions is coupled [3]. On the other hand, the RF-MRB only requires one downconversion RF chain, and therefore, it significantly reduces the complexity of the conventional MIMO transceiver. However, the RF-MRB architecture still requires a considerable amount of RF components, which motivates us to present three alternative analog combining architectures with less hardware circuitry.

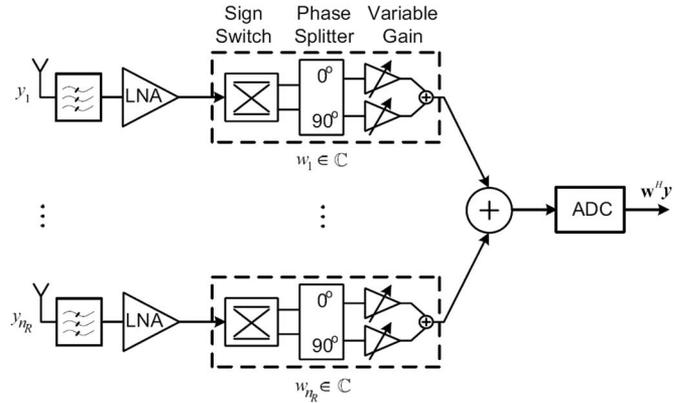


Fig. 1. RF-MRB.

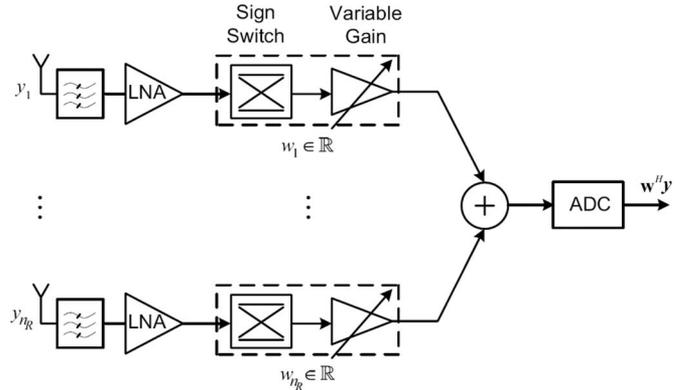


Fig. 2. RF-RWB.

### C. Real Weight Beamforming Architecture (RF-RWB)

A block diagram of the RF-RWB architecture is shown in Fig. 2, where we can see that each branch contains a VGA and a sign switch (a controllable  $0^\circ/180^\circ$  phase shifter). In comparison with the RF-MRB, we have removed the phase splitter and one VGA, thus simplifying the hardware and further reducing the power consumption. From a baseband point of view, the implications of this first simplified model consist in having Tx and Rx beamformers with real weights. That is, the RF-RWB baseband model is that in (2) with

$$\mathbf{w}_T \in \mathbb{R}^{n_T \times 1}, \quad \mathbf{w}_R \in \mathbb{R}^{n_R \times 1}$$

whereas the rest of the terms in (2) (MIMO channel, noise, etc.) are still complex.

### D. Equal Phase Beamforming Architecture (RF-EPB)

A further simplification of the foregoing architecture consists of eliminating the sign-switch block in Fig. 2, which results in the RF-EPB architecture shown in Fig. 3. With this topology, only the amplitudes of the RF signals are changed, whereas their relative phase differences are kept fixed. For this architecture, the baseband model in (2) remains valid, with the additional constraint of having beamformers with nonnegative entries, i.e.,

$$\mathbf{w}_T \in \mathbb{R}_+^{n_T \times 1}, \quad \mathbf{w}_R \in \mathbb{R}_+^{n_R \times 1}$$

where  $\mathbb{R}_+$  denotes the nonnegative orthant.

### E. Equal Gain Beamforming Architecture (RF-EGB)

The RF-EGB architecture, which is schematically shown in Fig. 4, replaces the VMs in Fig. 1 by phase shifters. This approach is well

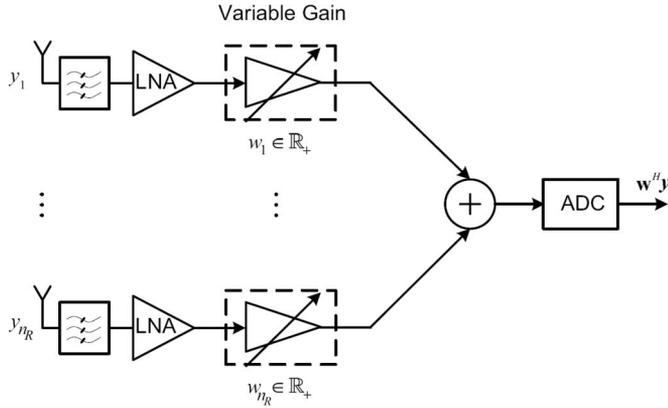


Fig. 3. RF-EPB.

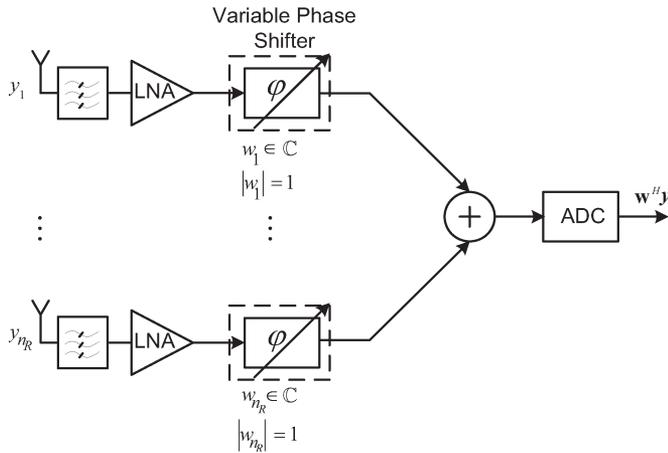


Fig. 4. RF-EGB.

known in the beamforming literature [6], [7] since it avoids the wide power variations that can happen in MRB schemes. Unlike the RWB and EPB schemes, which only make sense when applied in the RF domain, the EGB can be applied either in baseband [6], [7] or in RF. For this scheme, the baseband model of (2) is again valid but now with the additional constraint of having Tx and Rx beamformers with constant-modulus complex elements.

### III. BEAMFORMING DESIGN

In this section, we consider the problem of designing the RF weights or beamformers for the proposed architectures. Due to the lack of space, we assume perfect channel knowledge and only point out that the channel estimation can be performed by means of a training phase based on time-division multiple access and the estimation of the  $n_T n_R$  frequency-selective SISO channels. Moreover, the robustness to channel estimation errors of an RF-MRB system has been illustrated in [3] by means of simulations. As previously pointed out, one of the main design challenges comes from the fact that the same pair of beamformers is applied to all the subcarriers. It is interesting to mention here that a similar problem appears in the so-called ‘‘pre-fast Fourier transform (pre-FFT)’’ schemes, which are MIMO-OFDM systems that perform beamforming before the fast Fourier transform (FFT) calculations with the goal of reducing the computational cost. These ‘‘pre-FFT’’ algorithms operate at the baseband and have been widely studied in the literature [8]–[12].

Although the beamforming algorithms proposed in this paper can easily be extended to other beamforming criteria [3], here we focus on the maximization of the received SNR averaged over all subcarriers.

Without loss of generality, we can assume unit energy transmissions  $E[|s_k|^2] = 1$  and unit-norm beamformers  $\|\mathbf{w}_T\|^2 = \|\mathbf{w}_R\|^2 = 1$ , which results in

$$\text{SNR} = \frac{\sum_{k=1}^{N_c} |\mathbf{w}_R^H \mathbf{H}_k \mathbf{w}_T|^2}{N_c \sigma^2}. \quad (3)$$

Thus, the optimization problem associated with the analog antenna combining architectures can be written as

$$\begin{aligned} & \underset{\mathbf{w}_T, \mathbf{w}_R}{\text{maximize}} && \sum_{k=1}^{N_c} |\mathbf{w}_R^H \mathbf{H}_k \mathbf{w}_T|^2 \\ & \text{subject to} && \|\mathbf{w}_T\| \leq 1, \quad \mathbf{w}_T \in \mathcal{S}^{n_T \times 1} \\ & && \|\mathbf{w}_R\| \leq 1, \quad \mathbf{w}_R \in \mathcal{S}^{n_R \times 1} \end{aligned} \quad (4)$$

where we have replaced the equality constraints ( $\|\mathbf{w}_T\|^2 = \|\mathbf{w}_R\|^2 = 1$ ) with inequalities because the objective function is monotonically increasing in  $\|\mathbf{w}_T\|$  and  $\|\mathbf{w}_R\|$ , and where  $\mathcal{S}^{n \times 1}$  represents a subset of the complex field (i.e.,  $\mathcal{S}^{n \times 1} \subseteq \mathbb{C}^{n \times 1}$ ) that reflects the constraints imposed by the particular RF-MIMO beamforming architecture.

Unfortunately, in the general MIMO case, the problem in (4) is not convex and therefore difficult to solve. For that reason, following the procedure proposed in [7], we resort to an alternating optimization algorithm, which iterates between the equivalent SIMO (with the Tx beamformer fixed) and MISO (with the Rx beamformer fixed) problems. It is easy to prove that this algorithm converges (the cost function is bounded below, and in each iteration, it does not increase), and as we will show later, with proper initialization, it provides satisfactory solutions. Equipped with this alternating optimization approach, the rest of this section focuses on how to solve the SIMO/MISO cases for the different architectures. Specifically, taking for example the SIMO case (the MISO case can be solved in a similar manner), the optimization problem in (4) reduces to

$$\begin{aligned} & \underset{\mathbf{w}_R}{\text{maximize}} && \mathbf{w}_R^H \mathbf{R}_{\text{SIMO}} \mathbf{w}_R \\ & \text{subject to} && \|\mathbf{w}_R\| \leq 1, \quad \mathbf{w}_R \in \mathcal{S}^{n_R \times 1} \end{aligned} \quad (5)$$

where

$$\mathbf{R}_{\text{SIMO}} = \sum_{k=1}^{N_c} \mathbf{h}_{\text{SIMO}_k} \mathbf{h}_{\text{SIMO}_k}^H \quad (6)$$

and  $\mathbf{h}_{\text{SIMO}_k} \in \mathbb{C}^{n_R \times 1}$  is the SIMO channel vector for the  $k$ th data carrier.

#### A. Conventional MIMO and RF-MRB Architectures

Remember that in the conventional MIMO case, we can use a different pair of Tx–Rx beamformers for each subcarrier; therefore, the optimal beamformers can be obtained in a per-carrier basis. More specifically, the optimal beamformers for the  $k$ th subcarrier are given by the singular vectors associated with the largest singular value of  $\mathbf{H}_k$ .

On the other hand, the beamforming design problem for the RF-MRB architecture under OFDM transmission has been analyzed in [3], [8], and [12]. In particular, for SIMO channels, the optimal receive beamformer can be obtained in closed form as the principal eigenvector of the matrix  $\mathbf{R}_{\text{SIMO}}$  in (6).

#### B. RF-RWB Architecture

The RF-RWB architecture cannot implement the previous solution because it is constrained to work with beamformers whose weights are real ( $\mathcal{S}^{n_R \times 1} = \mathbb{R}^{n_R \times 1}$ ). However, for the SIMO case, it has been

proved in [13] that the optimal RWB can easily be obtained in closed form as the principal eigenvector of the real matrix

$$\mathbf{G}_{\text{SIMO}} = \Re(\mathbf{R}_{\text{SIMO}}). \quad (7)$$

### C. RF-EPB Architecture

The RF-EPB architecture introduces the additional constraint of having beamformers with nonnegative entries, that is,  $\mathcal{S}^{n_R \times 1} = \mathbb{R}_+^{n_R \times 1}$ . Defining the beamforming matrix as  $\mathbf{W}_R = \mathbf{w}_R \mathbf{w}_R^T$ , the problem in (5) can be rewritten as

$$\begin{aligned} & \underset{\mathbf{W}_R}{\text{maximize}} && \text{Tr}(\mathbf{G}_{\text{SIMO}} \mathbf{W}_R) \\ & \text{subject to} && \text{Tr}(\mathbf{W}_R) \leq 1 \\ & && \mathbf{W}_R \succeq 0 \\ & && \mathbf{W}_R \succeq 0 \\ & && \text{rank}(\mathbf{W}_R) = 1 \end{aligned} \quad (8)$$

where, excluding the rank-one constraint, we have a linear objective function and linear (and semidefinite positiveness) constraints. Thus, if we relax the nonconvex rank-one constraint, we can find the solution  $\mathbf{W}_R$  by means of standard convex optimization tools [14], [15]. Finally, we propose to obtain the receive beamformer  $\mathbf{w}_R$  as the principal eigenvector of the (generally not rank-one) matrix  $\mathbf{W}_R$ .

### D. RF-EGB Architecture

For this architecture, and excluding the particular case of flat-fading channels,<sup>1</sup> the beamformer design is more involved. Here, we write the EGB problem as

$$\begin{aligned} & \underset{\mathbf{W}_R}{\text{maximize}} && \text{Tr}(\mathbf{R}_{\text{SIMO}} \mathbf{W}_R) \\ & \text{subject to} && \text{diag}(\mathbf{W}_R) = 1/n_R \mathbf{I} \\ & && \mathbf{W}_R \succeq 0 \\ & && \text{rank}(\mathbf{W}_R) = 1 \end{aligned} \quad (9)$$

where  $\mathbf{W}_R = \mathbf{w}_R \mathbf{w}_R^H$  is the beamforming matrix. Again, the foregoing problem is not convex due to the rank-one constraint, but this can be circumvented by following a semidefinite relaxation approach, as previously discussed. Finally, the receive beamformer  $\mathbf{w}_R$  can be approximated using the phases of the principal eigenvector of the (in general not rank-one) beamforming matrix  $\mathbf{W}_R$ .

## IV. INITIALIZATION AND FURTHER COMMENTS

As previously pointed out, the convergence of the proposed alternating optimization algorithm is guaranteed by the fact that the cost function is bounded below, and in each iteration, the cost cannot increase. However, this does not guarantee the convergence to the global optimum, and it would be advisable to come up with a sufficiently accurate initialization technique. To this end, we start by rewriting the optimization problem in (4) as

$$\begin{aligned} & \underset{\mathbf{w}_T, \mathbf{w}_R, \mathbf{w}}{\text{maximize}} && \sum_{k=1}^{N_c} |\mathbf{h}_k^H \mathbf{w}|^2 \\ & \text{subject to} && \|\mathbf{w}\| \leq 1, \quad \mathbf{w} \in \mathcal{S}^{n_T n_R \times 1} \\ & && \mathbf{w} = \mathbf{w}_R \otimes \mathbf{w}_T^* \end{aligned} \quad (10)$$

where  $\mathbf{h}_k = \text{vec}(\mathbf{H}_k)$  is the vectorized version of the MIMO channel for the  $k$ th data carrier, and we have exploited the particular

structure of the sets  $\mathcal{S}$ . Of course, (10) is a nonconvex problem due to the nonconvex Kronecker structure constraint. However, if we relax this constraint, the optimization problem becomes identical to those addressed in the previous section, i.e., we have a virtual SIMO channel  $\mathbf{h}_k$  and a virtual beamformer  $\mathbf{w}$  combining the Tx and Rx beamformers. Therefore, we only need to apply the previous methods followed by the Euclidean projection onto the set of vectors with the required Kronecker structure, i.e., the Tx–Rx beamformers are obtained from the singular vectors associated with the largest singular value of the  $n_R \times n_T$  matrix  $\mathbf{W} = \text{unvec}(\mathbf{w})$ , where  $\text{unvec}(\cdot)$  denotes the inverse operator of  $\text{vec}(\cdot)$ .

Finally, it is easy to see that the computational complexity (per iteration) of the proposed techniques is of order  $\mathcal{O}(N_c n^2 + n^3)$  (with  $n = \max(n_T, n_R)$ ) for the RF-MRB and RF-RWB and  $\mathcal{O}(N_c n^2 + n^7)$  for the RF-EPB and RF-EGB, which is a direct consequence of the costs  $\mathcal{O}(N_c n^2)$  to obtain the matrix  $\mathbf{R}_{\text{SIMO}}$ ,  $\mathcal{O}(n^3)$  due to the extraction of the principal eigenvector and  $\mathcal{O}(n^7)$  to solve a semidefinite programming problem with  $n \times n$  matrices and one linear constraint [16].

## V. SIMULATION RESULTS

The proposed simplified RF-MIMO architectures are evaluated in this section by means of some numerical examples. In particular, we have compared the following schemes:

- 1) conventional baseband MIMO-OFDM with maximum ratio transmission and maximum ratio combining per subcarrier (denoted as Full-MIMO);
- 2) RF-MRB architecture with complex beamformer weights (Fig. 1);
- 3) RF-RWB architecture with real beamformer weights (Fig. 2);
- 4) RF-EPB architecture with nonnegative real beamformer weights (Fig. 3);
- 5) RF-EGB architecture with complex constant-modulus weights (Fig. 4);
- 6) a conventional SISO system, which can be seen as the natural competitor of the analog antenna combining architectures.

In all the simulations, the transmitted signals belong to a quadrature phase-shift keying (QPSK) constellation, the performance is evaluated in terms of bit error rate (BER), and we consider a  $4 \times 4$  MIMO system with 64 subcarriers. The MIMO channel follows an i.i.d. Rayleigh distribution with exponential power delay profile, where the power associated with the  $l$ th tap is  $E[|\mathbf{H}[l]|^2] = (1 - \rho)\rho^l n_T n_R$ , and the exponential parameter  $\rho$  has been selected as  $\rho = 0$  for flat-fading channels and  $\rho = 0.4$  for frequency-selective channels.

Figs. 5 and 6 show the simulation results for flat-fading and frequency-selective channels with uncoded transmissions. In both cases, the analog combining architectures clearly outperform the SISO system, and the gap with respect to the Full-MIMO system clearly depends on the frequency selectivity of the channel, which is a direct consequence of the fact that the same pair of beamformers is applied to all the subcarriers. However, the gap with respect to the conventional MIMO architecture decreases in the case of coded transmissions. In particular, Fig. 7 shows the results for a frequency-selective channel and coded transmissions using the IEEE 802.11a standard [17], which uses  $N_c = 48$  out of the 64 subcarriers for data transmission. In this case, we have selected a transmission rate of 12 Mb/s, which uses QPSK signaling and a code rate 1/2. The data bits are encoded with a convolutional code and block interleaved, as specified in the IEEE 802.11a standard. The receiver is based on the soft Viterbi decoder. As can be seen, in this case, the convolutional encoder makes the slope of the BER curves very similar, and the proposed analog antenna

<sup>1</sup>It is easy to prove that, for SIMO flat fading channels, the optimal beamformer is directly given by the projection of the channel coefficients onto the complex unit circle.

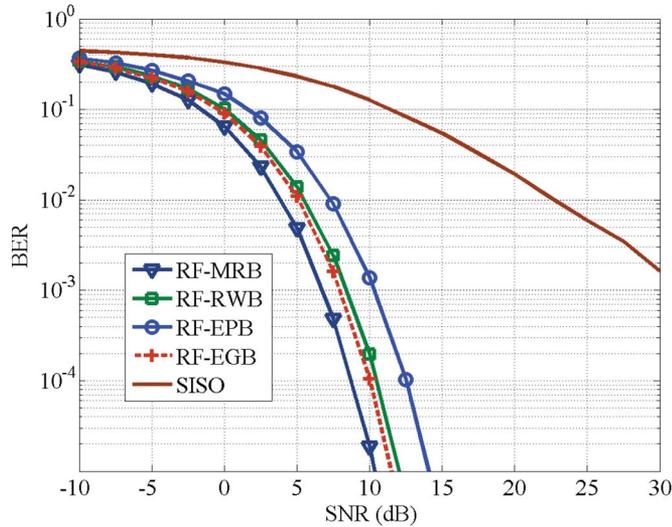


Fig. 5. Performance of the different RF architectures in a  $4 \times 4$  MIMO channel.

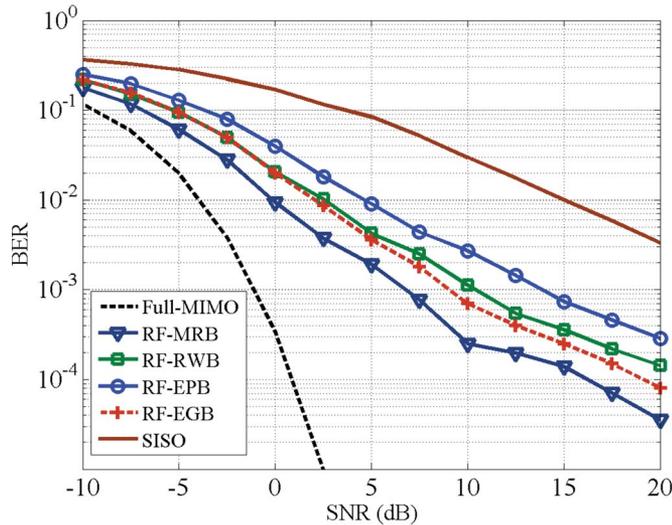


Fig. 6. BER versus SNR for the compared algorithms. Uncoded QPSK symbols for  $\rho = 0.4$ .

combining architectures offer an intermediate solution between the conventional SISO system and the complex Full-MIMO architecture.

Finally, we must note that the ordering among the alternative architectures is the same in the three figures, that is, RF-MRB, RF-EGB, RF-RWB, and RF-EPB from better to worse performance. This is easily explained as a direct consequence of the feasibility regions (and complexity) of the proposed schemes. Thus, the feasible beamformers of the RF-EPB are only a subset ( $\mathbb{R}_+$ ) of those for RF-RWB ( $\mathbb{R}$ ), which, at the same time, are contained in the feasible region associated with the RF-MRB ( $\mathbb{C}$ ). The explanation for the results of the RF-EGB architecture is a bit more complicated, but roughly speaking, we can say that it outperforms the RF-RWB architecture because, at least for Rayleigh channels, the phases of the beamformers are more informative than the amplitudes.

### VI. CONCLUSION

In this paper, we have presented three simplified analog antenna combining architectures (RWB, EPB, and EGB) that reduce the system cost and power consumption at the expense of a slight performance

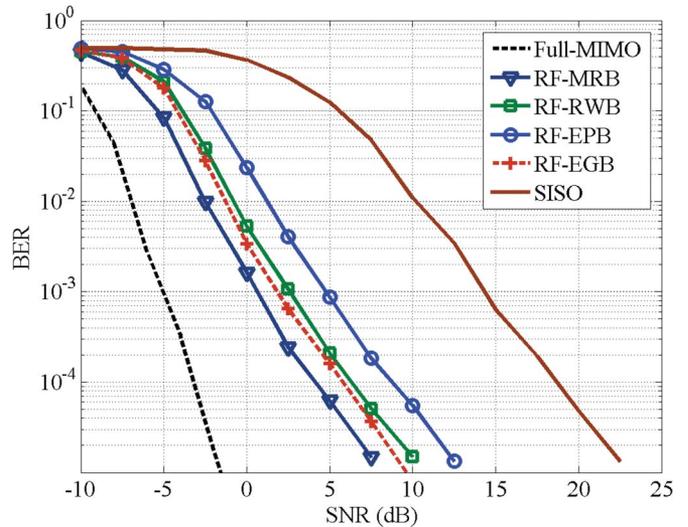


Fig. 7. BER for IEEE 802.11a-based system with transmission rate of 12 Mb/s, QPSK signaling, and convolutional encoder of rate 1/2 for  $\rho = 0.4$ .

degradation. From a baseband point of view, the three architectures result in new beamforming design problems, in which the Tx–Rx beamformers are constrained to have real weights (RWB), nonnegative real weights (EPB), or constant-modulus complex weights (EGB). For SIMO or MISO channels and OFDM-based transmissions, these beamforming problems can be solved using convex relaxation techniques, and the MIMO case is solved by means of an alternating optimization approach. The performance of the proposed architectures and algorithms has been illustrated by means of several simulation examples, which allows us to conclude that the proposed architectures represent an attractive low-cost alternative to conventional MIMO systems and other (more costly) analog antenna combining architectures. Finally, the best results are provided by the RF-EGB architecture, which reveals the importance of the phase of the RF signals in comparison with their amplitude in beamforming problems.

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## Efficient Resource Allocation for OFDMA-Based Two-Hop Relay Systems

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**Abstract**—We propose an efficient resource allocation scheme for the downlink of orthogonal frequency-division multiple-access (OFDMA)-based two-hop relay systems. The proposed scheme reuses the subchannels within a single cell in the frequency-selective fading environment by allocating a subchannel to multiple links simultaneously. We first formulate a preliminary subchannel allocation problem in consideration of the quality-of-service (QoS) requirements for both real-time traffic and nonreal-time traffic. We also design an efficient postprocessing algorithm that modifies the solution obtained by solving the preliminary problem so that it can be used in practical OFDMA systems. Numerical results show that the proposed scheme can significantly improve system performance.

**Index Terms**—Orthogonal frequency-division multiple access (OFDMA), subchannel allocation, two-hop relay system.

### I. INTRODUCTION

Wireless cellular systems support a variety of multimedia services having different quality-of-service (QoS) requirements. In these systems, it is very challenging for a base station (BS) to guarantee the QoS

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of mobile stations (MSs) located at cell boundary or in deep shadowed area [1]. Recently, relay-assisted two-hop communication is emerging as an efficient method for alleviating this problem. The heavy path loss between the BS and the MSs at cell boundary as well as the severe signal attenuation between the BS and the MSs in deep shadowed area can be properly controlled by using the intermediate relay stations (RSs). Because of this practical advantage, many standardization groups, such as IEEE 802.16j [2], have paid attention to relay-assisted communication.

Many resource allocation schemes have been proposed for relay-assisted cellular systems (e.g., [3]–[6]). In [3], the authors investigated optimal resource allocation for orthogonal frequency-division multiplexing (OFDM)-based relay systems when only limited feedback information is available at the BS. Dang *et al.* proposed the scheme that improves throughput performance by accomplishing subcarrier allocation and relay selection at the same time [4]. On the other hand, by allowing multiple downlink transmitters (i.e., BS and RSs) in a cell to use the same resource simultaneously (*intracell reuse*) if the mutual interference between them is sufficiently low, the relay-deployed systems can utilize the radio resource more efficiently. Kaneko *et al.* proposed the resource allocation algorithm that exploits the intracell reuse of subchannels while supporting the QoS of users in orthogonal frequency-division multiple-access (OFDMA) relay systems [5]. This scheme has a limitation that the feasibility for intracell reuse of resource is checked based on only the path loss. The resource allocation problem in [6] takes account of the instantaneous channel fading in evaluating the channel states for allowing intracell reuse of the resource. However, it requires exhaustive search for finding the optimal solution, which is impractical for real-time (RT) operation. Furthermore, the scheme does not consider various traffic classes with different QoS requirements.

In this paper, we propose an efficient resource allocation scheme for the downlink of OFDMA-based relay systems, which simultaneously allocates the same subchannel to multiple links with little interference to one another. First, we formulate a preliminary resource allocation problem that aims at heightening the cell throughput while satisfying the QoS requirements of both RT and nonreal-time (NRT) traffic classes under the assumption of an infinite number of OFDM symbols in a frame. The solution for this resource allocation problem cannot be directly applicable to the practical systems because the number of OFDM symbols in a frame is finite in practice. Thus, we also design an effective postprocessing algorithm for adjusting the obtained solution for the system with a finite number of OFDM symbols in a frame, taking the intracell reuse of subchannels into account.

The remainder of this paper is organized as follows: In the next section, we describe the system model under consideration. Section III explains the proposed resource allocation scheme. In Section IV, the simulation model is presented, and the numerical results are discussed. This paper is concluded with Section V.

### II. SYSTEM MODEL

We consider the downlink of an OFDMA-based cellular system. Within each cell, there are one BS,  $R$  RSs, and  $N$  MSs. All subcarriers are organized into  $M$  disjointed subchannels, each of which consists of adjacent subcarriers within the channel coherence bandwidth. The transmission powers per subchannel of BS and RS are denoted by  $P_{BS}$  and  $P_{RS}$ , respectively. The system time is divided into frames. Frame length  $T_f$  corresponds to  $S$  OFDM symbol times and is approximately determined so that the channel gain remains constant in a frame.