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Maximum Sum-Rate Interference Alignment Algorithms for MIMO Channels

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***Abstract*—Alternating minimization algorithms are typically used to find interference alignment (IA) solutions for multiple-input multiple-output (MIMO) interference channels with more than $K = 3$ users. For these scenarios many IA solutions exit, and the initial point determines which one is obtained upon convergence. In this paper, we propose a new iterative algorithm that aims at finding the IA solution that maximizes the average sum-rate. At each step of the alternating minimization algorithm, either the precoders or the decoders are moved along the direction given by the gradient of the sum-rate. Since IA solutions are defined by a set of subspaces, the gradient optimization is performed on the Grassmann manifold. The step size of the gradient ascent algorithm is annealed to zero over the iterations in such a way that during the last iterations only the interference leakage is being minimized and a perfect alignment solution is finally reached. Simulation examples are provided showing that the proposed algorithm obtains IA solutions with significant higher throughputs than the conventional IA algorithms.**

I. Introduction

Interference alignment (IA) is a recently proposed technique to achieve the maximum degrees of freedom (DoF) for the K-user interference channel [1]. IA jointly designs the signals transmitted by all users in such a way that the interfering signals at each receiver fall into a reduced-dimensional subspace. The receivers can then extract the projection of the desired signal that lies in the interference-free subspace.

In this paper we propose an algorithm for IA in the K-user multiple-input multiple-output (MIMO) interference channel, where every transmitter and receiver has N antennas and each user wishes to send d streams of data. Unfortunately, closed-form solutions for such systems are known so far only for certain cases, namely for the 3-user channel where each user achieves $d = N/2$ DoF[1] [1], and also for $K = N + 1$ users when only $d = 1$ stream is transmitted by each user [2], [3][2]. For other IA configurations, iterative algorithms can be employed to alternately optimize the precoders at the transmitters and the decoders (or, equivalently, the interference subspaces) at the receivers [5], [6]. Interestingly, for many interference channels there seems to be a very large number of different IA solutions [4], [7], and the initial set of precoders determines which one is obtained after convergence. Although all these IA solutions perfectly cancel the interference and are, in consequence, asymptotically equivalent, their performance in terms of average sum-rate may differ significantly under noise.

In this paper we propose an iterative alternating minimization algorithm that aims to find the best IA solution in terms of sum-rate, regardless of the initialization point. The basic idea is to move the precoders obtained at each step of the alternating minimization procedure along the direction given by the gradient of the sum-rate. Since any set of precoders and decoders achieving interference alignment is invariant to a change of basis (i.e., an IA solution is just defined by a set of subspaces), this gradient optimization is performed on the Grassmann manifold [8]. To achieve a solution that perfectly cancels the aggregated interference from non-intended users, the step size of the gradient ascent approach is annealed to zero over the iterations. As an alternative to the sum-rate criterion, the maximization of the average sum power in the interference free subspaces is also considered for comparison.

II. Iterative algorithms for IA

We consider a K-user ($K > 3$) interference channel where each user wishes to achieve d degrees of freedom and is equipped with N antennas at both sides of the link[3]. We denote this symmetric interference channel as $(N \times N, d)^K$. Let $\mathbf{V}_k \in \mathbb{C}^{N \times d}$ be the unitary precoder for user k, then, the received signal at receiver k is given as

$$\mathbf{y}_k = \mathbf{H}_{k,k}\mathbf{V}_k\mathbf{s}_k + \sum_{l \neq k} \mathbf{H}_{k,l}\mathbf{V}_l\mathbf{s}_l + \mathbf{n}_k, \qquad (1)$$

where $\mathbf{H}_{k,l} \in \mathbb{C}^{N \times N}$ is the flat fading MIMO channel from transmitter l to receiver k, $\mathbf{s}_l \in \mathbb{C}^{d \times 1}$ is the desired signal

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[1] If N is odd, a two time-slot symbol extension is required.

[2] Closed-form IA solutions have been recently found in [4] for other MIMO interference channels, e.g., for the 4-user channel where each transmit-receive link has 2 antennas at the transmitter side, 3 antennas at the receiver side and sends $d = 1$ stream.

[3] We make this model assumption mainly for notational convenience. The algorithms discussed in this paper can be used without modification when the transmitters and receivers have different number of antennas.

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for the l-th user and $\mathbf{n}_k$ is the additive and spatially white Gaussian noise at the k-th receiver[4].

Let $\mathbf{U}_k \in \mathbb{C}^{N\times d}$ be an orthonormal basis of the linear receiver for user k whose columns span the sought interference-free subspace. Previously proposed algorithms for interference alignment find a set of precoders $\mathbf{V}_k$ and decoders $\mathbf{U}_k$ that minimize the interference leakage cost function [5], [6], which is defined as

$$IL = \sum_{k=1}^{K}\sum_{l\neq k}^{K} \left\|\mathbf{U}_k^H\mathbf{H}_{kl}\mathbf{V}_l\mathbf{V}_l^H\mathbf{H}_{kl}^H\mathbf{U}_k\right\|_F^2, \qquad (2)$$
$$\text{subject to} \quad \mathbf{V}_k^H\mathbf{V}_k = \mathbf{I}_d \;\text{ and }\; \mathbf{U}_k^H\mathbf{U}_k = \mathbf{I}_d,$$

where $\mathbf{I}_d$ denotes the $d \times d$ identity matrix.

The cost function in (2) is unitarily invariant in both $\mathbf{V}_k$ and $\mathbf{U}_k$. For example, the same solution is achieved for any unitary $\mathbf{Q}$ by replacing $\mathbf{V}_k$ by $\mathbf{V}_k\mathbf{Q}$. Thus, the true domain of the optimization problem is over the Grassmann manifold [8], which is the space of d-dimensional subspaces of an N-dimensional space. Recognizing the subspaces represent an equivalence class and provide the same IA solution, in (2) we choose arbitrary orthogonal basis, $\mathbf{V}_k^H\mathbf{V}_k = \mathbf{I}_d$ and $\mathbf{U}_k^H\mathbf{U}_k = \mathbf{I}_d$, as class representatives.

In [5] it has been shown that if $\mathbf{V}_k$ and $\mathbf{U}_k$ are respectively the optimal IA precoders and decoders for the channels $\mathbf{H}_{k,l}$; then, if we reverse the direction of communication, $\mathbf{U}_k$ are now the IA precoders and $\mathbf{V}_k$ are the optimal decoders for the reciprocal channels $\mathbf{H}_{k,l}^H$. Although channel reciprocity is not strictly required to apply iterative IA algorithms [6], it provides a useful interpretation for this kind of alternating minimization algorithms. Basically, the minimization of (2) is carried out by fixing a subset of variables (either $\mathbf{V}_k$ or $\mathbf{U}_k$), and then optimizing for the remaining variables. Once a subset is fixed, the optimum values for the other subset can be obtained in closed form.

It has been observed experimentally that for $K > 3$ users this iterative algorithm converges to a perfectly aligned solution when the total number of transmitted streams is below the theoretically achievable DoF. Moreover, depending on the initial point, IA solutions with different performance can be obtained.

III. Maximum sum-rate IA algorithm

A. Problem statement

The sum-rate of the MIMO interference channel is given by

$$R = \sum_{k=1}^{K} \log\left|\mathbf{I}_N + \left(\sigma^2\mathbf{I}_N + \sum_{l\neq k}\mathbf{Q}_{kl}\right)^{-1}\mathbf{Q}_{kk}\right|, \qquad (3)$$

where $\mathbf{Q}_{kl}$ denotes the $N \times N$ covariance matrix of the signal from the l-th transmitter to the k-the receiver, and σ^2 is the variance of the additive white Gaussian noise. Since the algorithm proposed in this section tries to maximize the sum-rate by alternatively optimizing the precoders and the decoders, it will be useful to special (3) for a given set of precoders

$$\mathbf{Q}_{kl} = \mathbf{H}_{kl}\mathbf{V}_l\mathbf{V}_l^H\mathbf{H}_{kl}^H, \qquad (4)$$

and also for the reciprocal channel where the roles of precoders and decoders are interchanged, i.e.,

$$\mathbf{Q}_{lk} = \mathbf{H}_{kl}^H\mathbf{U}_l\mathbf{U}_l^H\mathbf{H}_{kl}. \qquad (5)$$

Our goal in this paper is to find the unitary linear precoders and decoders that maximize the sum-rate R among those achieving zero interference leakage. More formally, our problem is

$$\max_{\mathbf{V}_k,\mathbf{U}_k} R, \qquad (6)$$

subject to the unitary constraints $\mathbf{V}_k^H\mathbf{V}_k = \mathbf{I}_d$, $\mathbf{U}_k^H\mathbf{U}_k = \mathbf{I}_d$, and also to the zero-interference-leakage constraint

$$IL = \sum_{k=1}^{K}\sum_{l\neq k}^{K} \left\|\mathbf{U}_k^H\mathbf{H}_{kl}\mathbf{V}_l\mathbf{V}_l^H\mathbf{H}_{kl}^H\mathbf{U}_k\right\|_F^2 = 0. \qquad (7)$$

Notice that when the interference is perfectly aligned, the interference channel is decoupled into a set of parallel Gaussian MIMO channels and the sum-rate in (3) reduces to

$$R = \sum_{k=1}^{K} \log\left|\mathbf{I}_N + \frac{1}{\sigma^2}\mathbf{U}_k^H\mathbf{H}_{kk}\mathbf{V}_k\mathbf{V}_k^H\mathbf{H}_{kk}^H\mathbf{U}_k\right|, \qquad (8)$$

which simply adds up the achievable rates in each interference-free $d \times d$ MIMO channel given by $\overline{\mathbf{H}}_k = \mathbf{U}_k^H\mathbf{H}_{kk}\mathbf{V}_k$, for $k = 1,\ldots,K$. It is then clear that better throughputs can be obtained by using non-unitary precoders and decoders, or by applying power waterfilling among the eigenmodes of the equivalent non-interfering $d\times d$ MIMO channels. Nevertheless, this additional increase in throughput can be obtained in a second step after the interference MIMO channel has been block-diagonalized by the maximum sum-rate IA solution. Here, we will concentrate on the first step and restrict ourselves to unitary precoders and decoders.

We stress that the solutions we are seeking in this paper minimize the interference leakage cost function, and within a class of equivalent solutions, find the one that provides the best sum rate performance. This is an important difference with other criteria recently proposed for the interference channel such as the minimum mean-square error (Min MSE) [9] or the maximum signal-to-interference noise ratio (Max SINR) [5], which directly optimize the chosen cost function but make no attempt to create an interference-free subspace. Although we acknowledge that these solutions (Max SINR or Min MSE) could provide better overall throughputs than the solutions proposed in this paper, at least for low and moderate SNRs, it must be clear that goal of this paper is to propose new IA iterative algorithms providing close-to-optimal performance (in terms of sum-rate) regardless of the initialization point.

[4]This assumption is also made for simplicity, but the algorithm proposed in this paper can also be applicable under non-white noise or in the presence of external interferers.

B. Sum-rate gradient on the Grassmann manifold

To find the maximum sum-rate IA solution we will use a gradient descent approach combined with the alternating minimization algorithm in [5], [6]. The basic idea is to move the solution obtained at each step of the alternating minimization algorithm in the direction of the gradient of (3) with respect to either the precoders or the decoders. A large step size is used for the gradient algorithm during the first iterations (exploration phase); then, it is progressively annealed to zero (convergence phase) in such a way that during the final stage of the algorithm only the interference leakage in (2) is being minimized and a perfect alignment solution satisfying the constraint $IL = 0$ is obtained upon convergence.

To solve this problem in an iterative fashion we first compute the gradient of (3) with respect to either $\mathbf{V}_k$ or $\mathbf{U}_k$, using (4) or (5), respectively. These gradients can be obtained as shown in [10]. For example, the derivative of the sum-rate with respect to $\mathbf{V}_k^*$ can be calculated as

$$\begin{aligned}\nabla_k R = &\sum_{i=1}^{K} \left(\mathbf{H}_{ik}\mathbf{C}_i^{-1}\mathbf{H}_{ik} - \operatorname{tr}\left(\mathbf{V}_k^H\mathbf{H}_{ik}\mathbf{C}_i^{-1}\mathbf{H}_{ik}\mathbf{V}_k\right)\right)\mathbf{V}_k \\ &+ \sum_{i\neq k}\left(\operatorname{tr}\left(\mathbf{V}_k^H\mathbf{H}_{ik}\overline{\mathbf{C}}_i^{-1}\mathbf{H}_{ik}\mathbf{V}_k\right) - \mathbf{H}_{ik}\overline{\mathbf{C}}_i^{-1}\mathbf{H}_{ik}\right)\mathbf{V}_k\end{aligned} \tag{9}$$

where

$$\begin{aligned}\mathbf{C}_i &= \sum_{l=1}^{K}\mathbf{Q}_{il} + \sigma^2\mathbf{I}_N, \\ \overline{\mathbf{C}}_i &= \sum_{l\neq k}\mathbf{Q}_{il} + \sigma^2\mathbf{I}_N.\end{aligned}$$

The gradient expression with respect to $\mathbf{U}_k^*$ can be obtained similarly, just by substituting each MIMO channel by its reciprocal and the precoders by the corresponding decoders in (9). Also, when the decoders are being updated the covariance matrices are given by (5) instead of (4).

A gradient algorithm for maximizing the sum-rate based exclusively on (9) (i.e., without any alternating minimization step) has been recently proposed in [10], [11]. For complex interference networks such as the $(5\times5, 2)^4$ scenario, however, this gradient algorithm is extremely slow and often gets trapped in local minima. Our approach to the problem is different since we are explicitly seeking for an interference alignment solution that, additionally, maximizes the sum-rate. Another important difference is that we perform the optimization on the Grassmann manifold. Specifically, if the unconstrained gradient of the sum-rate with respect to the k-th precoder is $\nabla_k R$, then the tangent vector on the Grassmann manifold is given by

$$\nabla_k^G R = \left(\mathbf{I}_N - \mathbf{V}_k\mathbf{V}_k^H\right)\nabla_k R. \tag{10}$$

Using this tangent vector, the solution obtained at each step of the alternating minimization approach, which simply seeks to minimize the interference leakage function, is moved following the geodesic on the Grassmann manifold according to

$$\mathbf{V}_k = \left(\mathbf{V}_k\mathbf{F}(\cos\mathbf{\Sigma}t)\mathbf{F}^H\right) + \left(\mathbf{G}(\sin\mathbf{\Sigma}t)\mathbf{F}^H\right), \tag{11}$$

where $\nabla_k^G R = \mathbf{G}\mathbf{\Sigma}\mathbf{F}^H$ is the compact singular value decomposition of the $N \times d$ gradient matrix given in (10).

Alternative cost functions, which might have less complex derivatives than the sum-rate, can be optimized in a similar way. For instance, in this paper we will also consider the maximization of the power received in the interference-free subspaces, which is given by

$$P = \sum_{k=1}^{K}\operatorname{tr}\left(\mathbf{U}_k^H\mathbf{H}_{kk}\mathbf{V}_k\mathbf{V}_k^H\mathbf{H}_{kk}^H\mathbf{U}_k\right), \tag{12}$$

where $\operatorname{tr}(\mathbf{A})$ denotes the trace of matrix $\mathbf{A}$. This cost function can be used to some extent as a proxy for the sum-rate, with the advantage that the derivatives of (12) with respect to the precoders an decoders are much simpler than (9). Specifically, the derivative of (12) with respect to $\mathbf{V}_k^*$ is $\nabla_k P = \mathbf{Q}_{kk}\mathbf{V}_k$ with $\mathbf{Q}_{kk}$ given as in (4). The derivative with respect to $\mathbf{U}_k^*$ can be obtained analogously as $\nabla_k P = \mathbf{Q}_{kk}\mathbf{U}_k$ where $\mathbf{Q}_{kk}$ is now given as in (5).

C. Proposed algorithm

After choosing a set of random unitary precoders, the proposed algorithm alternates between two steps: one for updating the decoders (with fixed precoders) and other for updating the precoders (with fixed decoders). In this subsection we describe the calculations made in both steps.

Step I: Given a set of precoders, $\mathbf{V}_k$, we update the decoders $\mathbf{U}_k$ for $k = 1, \ldots, K$; as follows:

1) Compute the interference covariance matrices at the receivers
$$\mathbf{Q}_k = \sum_{l\neq k}^{K}\mathbf{H}_{kl}\mathbf{V}_l\mathbf{V}_l^H\mathbf{H}_{kl}^H. \tag{13}$$
2) Compute the decoders minimizing the leakage interference (2) as
$$\mathbf{U}_k = \nu_{min}^d(\mathbf{Q}_k) \tag{14}$$
where $= \nu_{min}^d(\mathbf{Q}_k)$ denotes the unitary matrix whose columns are the eigenvectors corresponding to the d smallest eigenvalues of $\mathbf{Q}_k$. This is the basic step of the alternating minimization algorithm [5], [6].
3) Compute the gradient of the sum-rate with respect to $\mathbf{U}_k$ as in (9). Notice that for this step we are actually computing the direction of increase in the sum-rate for the reciprocal channel when we use the $\mathbf{U}_k$ as precoders.
4) Project the gradient in the Grassmann tangent space using (10) to obtain $\nabla_k^G R$.
5) Compute the compact SVD of the $N \times d$ matrix $\nabla_k^G = \mathbf{G}\mathbf{\Sigma}\mathbf{F}^H$.
6) The new decoder is finally obtained by moving $\mathbf{U}_k$ along the geodesic in the Grassmannian manifold in the direction given by $\nabla_k^G R$, i.e.,
$$\mathbf{U}_k = \left(\mathbf{U}_k\mathbf{F}(\cos\mathbf{\Sigma}t)\mathbf{F}^H\right) + \left(\mathbf{G}(\sin\mathbf{\Sigma}t)\mathbf{F}^H\right). \tag{15}$$

which is just (11) but particularized this time for the k-th decoder. The distance between the starting and the final point in the Grassmann manifold is controlled by the step size t in (15). We will describe later a procedure to update this parameter over the iterations.

Step II: In this step we proceed in a similar fashion, but exchange the roles of the precoders and decoders. The only difference with *Step I* is that for the reciprocal network the interference covariance matrices are now computed as

$$\mathbf{Q}_k = \sum_{l \neq k}^{K} \mathbf{H}_{kl}^H \mathbf{U}_l \mathbf{U}_l^H \mathbf{H}_{kl}. \quad (16)$$

The precoders minimizing the interference leakage are again obtained as $\mathbf{V}_k = \nu_{min}^d(\mathbf{Q}_k)$, and the derivative of the sum-rate with respect to $\mathbf{V}_k$ is calculated as in (9). Once we have obtained the direction through which each transmitter sees less interference and the direction on the tangent space in which the sum-rate is increased, we move the solution along the geodesic as in (15).

IV. Simulation Results

In this section we evaluate the performance of the proposed maximum sum-rate IA and maximum power IA algorithms in comparison to the algorithm in [5] by means of numerical simulations. We consider the $(5 \times 5, 2)^4$ interference channel. Each MIMO channel, $\mathbf{H}_{kl}$, is i.i.d. Rayleigh with unit-variance entries (i.e., $\mathbf{H}_{kl}[i, j] \sim C(0, 1)$).

A. First example

In the first simulation example, we consider a *fixed* channel realization of the $(5 \times 5, 2)^4$ interference channel and a fixed signal-to-noise ratio (SNR) of 20 dB. For this scenario, we ran the different IA algorithms 200 times, each one starting from different random unitary precoders. The number of iterations was 3000, which ensures that the final interference leakage after convergence is always lower than $IL = 1e - 7$, thus indicating that all algorithms succeeded in aligning the interferences at the receivers. An important parameter of our proposed method is the step size t, which determines the distance moved along the geodesic at each iteration. In order to converge to a perfectly aligned solution, this parameter must be annealed to zero in a proper fashion. Specifically, we initialize the algorithm with a large step size ($t_0 = 0.1$) and after each iteration we decrease its value as $t_{n+1} = 0.995 t_n$. In this way, the final value of the step size is close to zero and we end up with a perfect IA solution that, additionally, optimizes the chosen criterion.

After each independent run has converged, the average sum-rate obtained by the three methods under comparison (Max Sum-Rate, Max Power and conventional IA) is computed as in (3); then, the probability distribution function (pdf) of the final sum-rate is estimated using Parzen windowing method and displayed in Fig. 1. As we can observe, even though we start each run from a different initial point, the final sum-rate provided by the proposed algorithm is more concentrated around higher values, which means that we are effectively guiding the convergence of the algorithm towards the same region of the cost function regardless of the initial point. A similar behavior is observed for Max Power criterion, for which the sum-rate pdf is also concentrated although its mean is significantly lower. Finally, the sum-rate pdf corresponding to the conventional IA algorithm is completely spread out, each run producing a solution with very different performance.

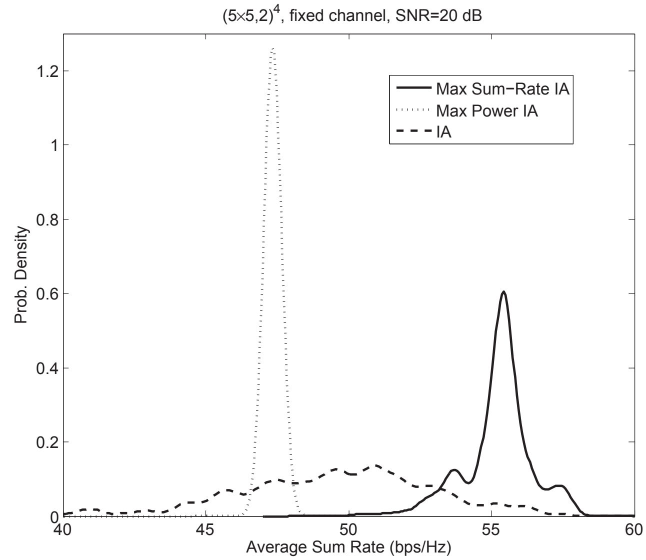

Fig. 1. Probability density function of the average sum-rate for the solutions obtained starting from different random points with the Max Sum-Rate, the Max Power and the conventional IA algorithms.

B. Second example

In the second simulation example, we consider 250 random realizations of the $(5 \times 5, 2)^4$ interference MIMO channel and evaluate the average sum-rate with respect to the SNR for the proposed IA algorithms (Max Sum-Rate and Max Power) and the conventional IA algorithm. We have also included in the comparison the results obtained by the Max SINR (signal-to-interference-plus-noise ratio) algorithm described in [5]. This algorithm also uses an alternating minimization procedure that, at each iteration, maximizes the SINR on a stream-by-stream basis[5].

For each method the precoders were initialized randomly and 3000 iterations were applied as before. Notice that the solution obtained for the conventional IA and the Max Power algorithm do not depend on the SNR; whereas the Max

[5]As pointed out in [5], orthogonal precoders and decoders are in general suboptimal for SINR maximization. However, for the $(5 \times 5, 2)^4$ interference network we have observed that at high SNRs a stream-by-stream optimization, without enforcing any constraint among the columns of the precoders, typically results in a rank-one precoder for at least one user (i.e., one of the streams is shut-off, thus losing one DoF and reducing the sum-rate). To avoid this problem we have included in the algorithm an additional step that orthogonalizes the columns of the precoders and the decoders after maximizing the SINR independently for each stream.

Sum-Rate and the Max SINR solutions depend on the noise variance. The results are depicted in Fig. 2.

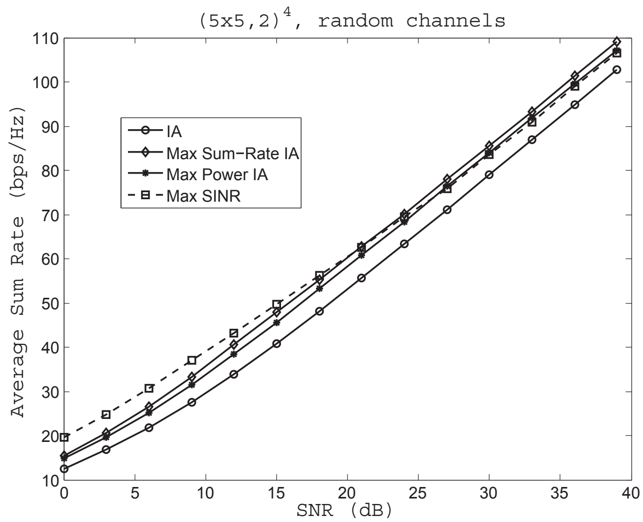

Fig. 2. Average sum-rate achieved by the Max Sum-Rate, the Max-Power, the Max SINR and the conventional IA solution.

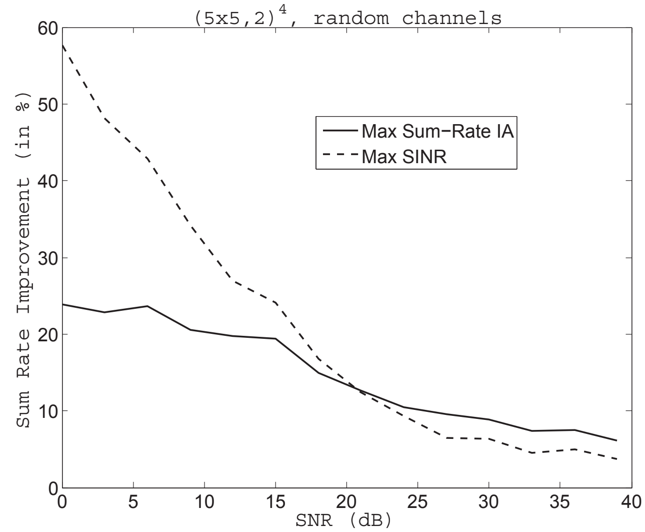

Fig. 3. Sum-rate improvement in percentage over the conventional IA solution.

As expected, for low SNRs the best results are provided by the Max SINR algorithm, but at high SNRs, where a perfectly aligned solution is desirable, the Max Sum-Rate provides the best results. It can also be observed that a large portion of the gain provided by the Max Sum-Rate algorithm is also provided by the Max Power criterion at a much reduced computational cost. These results suggest that the maximization of the received power in the interference-free subspace is a reasonable proxy of the sum-rate for IA solutions. Let us remark that the Max Power solution does not necessarily maximizes the average sum-rate. This is explained because for certain Max Power IA solutions the powers received by the 4 users can be very different (although obviously the total power is maximized). In these cases where the received powers are highly unbalanced, the user receiving less power limits the achievable sum-rate. The Max Sum-Rate solution, however, tends to provide more balanced powers among users, which is beneficial in terms of sum-rate.

To give an idea about the SNR range at which the Max Sum-Rate algorithm provides any advantage, Fig. 3 shows the sum-rate improvement in percentage of the Max SINR and the Max Sum-Rate solutions over the conventional IA. For SNRs larger than 20 dB the IA solution provided by the Max Sum-Rate algorithm achieves better throughput.

Finally, we study for this example the convergence of the proposed Max Sum-Rate and Max Power algorithms in comparison to the conventional IA. In Figs. 4 and 5 we plot the convergence curves (averaged over the 250 channel realizations) of the sum-rate (Eq. (3)) and the interference leakage (Eq. (2)), respectively.

For each channel realization the three algorithms are initialized at the same point and a SNR of 30 dB was considered for this example. The conventional IA algorithm easily finds one of the many local minima of the interference leakage cost function, which explains the fast convergence during the first iterations. However, the final performance in terms of sum-rate is not necessarily optimal. The proposed Max Sum-Rate algorithm shows a slow, but steady, increase in sum-rate over iterations. From this figure we can conclude that the movement following the sum-rate gradient on the Grassmann manifold effectively guides the convergence of the algorithm towards an alignment solution with better overall throughput. The Max Power algorithm shows a similar convergence behavior, but the final sum-rate is lower. The convergence of the interference leakage cost function in Fig. 5 also reveals that, although

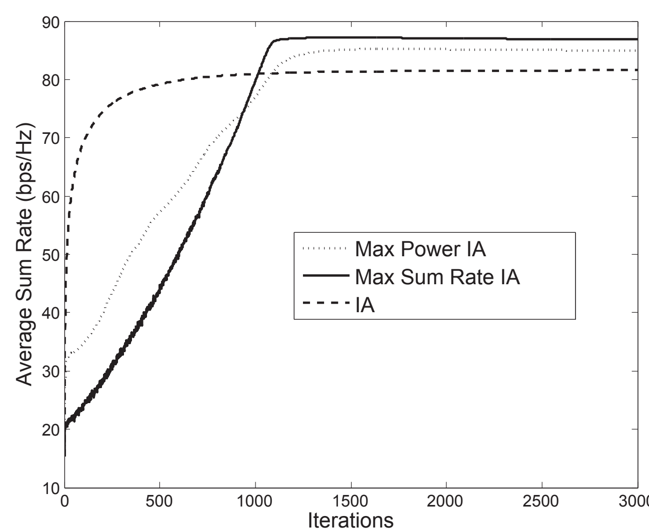

Fig. 4. Convergence of the average sum-rate for the Max Sum-Rate, the Max Power and conventional IA algorithms.

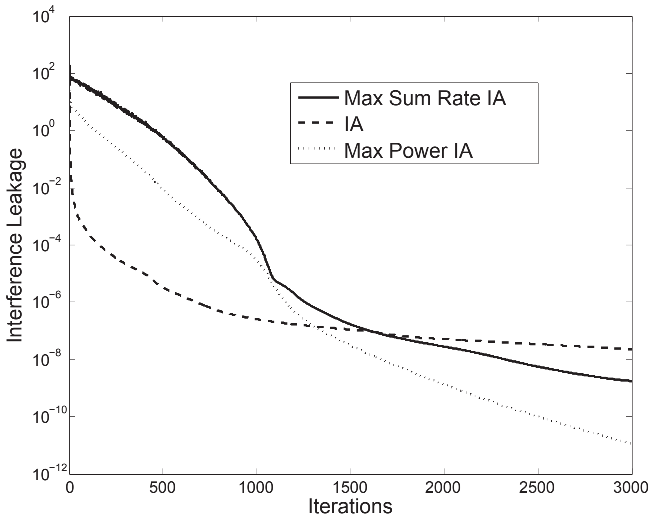

Fig. 5. Convergence of the interference leakage for the Max Sum-Rate, the Max Power and conventional IA algorithms.

the convergence of the Max Sum-Rate and the Max Power algorithms is slower during the first iterations, they finally reach a lower value on average. These results show the existing tradeoff between speed of convergence and final sum-rate. For realistic scenarios where some loss in sum-rate is acceptable, the faster convergence of conventional IA algorithms might be preferable.

V. Conclusion

In this paper we proposed a new iterative algorithm for interference alignment that jointly minimizes the interference leakage (through the conventional alternating minimization procedure) and maximizes the sum-rate (through a new gradient ascent procedure performed on the Grassmann manifold). The step size of the sum-rate gradient optimization is progressively decreased so that the minimization of the interference leakage takes the most prominent role during the last iterations. The result is an aligned solution that provides better throughput, irrespective of the initialization. The main drawback of the proposed method is that it may present slower convergence in comparison to the alternating minimization algorithm. The application of the homotopy method to solve the IA system of polynomial equations is currently being considered to speed up the overall procedure. Furthermore, a complete performance comparison with alternative criteria such as the Min MSE or the Max SINR is another step along this research line.

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