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Analog antenna combining in transmit correlated channels: Transceiver design and performance evaluation ☆

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ABSTRACT

In this paper we study space–time coding schemes for a novel OFDM-based MIMO system which performs adaptive signal combining in radio-frequency (RF). Assuming perfect channel knowledge at the receiver and statistical channel state information at the transmitter, we consider the problem of selecting the transmit and receive RF weights (beamformers), as well as the time and frequency linear precoders, under the assumption of Rayleigh channels. The transmission scheme is based on orthogonal beam division multiplexing (OBDM) and minimum mean-square error (MMSE) receive beamforming, i.e., the data is transmitted by means of several transmit beamformers matched to the spatial correlation matrix, whereas the receive beamformers are selected to minimize the MSE of the linear MMSE receiver. Finally, the performance of the proposed scheme is evaluated by means of Monte Carlo simulations.

1. Introduction

In order to extract the benefits of multiple-input multiple-output (MIMO) wireless communication systems, all antenna paths must be independently acquired and processed at the baseband [1]. This has an extra cost, size, and power consumption in comparison with the single-input single-output (SISO) systems, which is in part responsible for the delay in the deployment of MIMO wireless transceivers.

To mitigate these drawbacks, and propelled by recent advances in SiGe-BiCMOS technology [8],[1] a novel RF-MIMO transceiver architecture has been proposed, which is shown in Fig. 1. With this architecture, the spatial processing is done at the radio-frequency (RF) front-end, which significatively reduces the hardware cost and power consumption [7]. Thus, a single stream of data is transmitted and received through an equivalent SISO channel, which is optimized with respect to the transmit and receive analog beamformers (RF weights).

From a signal processing point of view, the new architecture poses several challenging design problems. Firstly, since only one data stream is acquired and processed, the multiplexing gain of the system is always limited to one [29]. Nevertheless, it can be proven that some other important benefits of the MIMO channel, such as diversity or array gain, are kept by the new architecture [20]. On the other hand, the problem of designing the transmit and receive beamformers for an OFDM-based

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[1] This is a novel semiconductor silicon–germanium BiCMOS technology for the development of integrated circuits.

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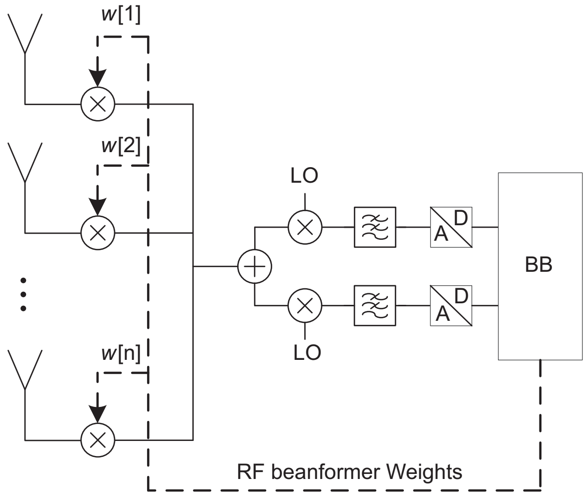

Fig. 1. Analog antenna combining in the RF path for MIMO communications systems. Exemplarily shown for a direct-conversion receiver.

Table 1
Complexity comparison between conventional and novel MIMO schemes.

Components	Conventional MIMO scheme	Novel MIMO scheme
Receive antennas	n_R	n_R
LNAs	n_R	n_R
Down-conversion chains	n_R	1
ADCs	n_R	1
FFTs	n_R	1

system is a challenging task [25] due to the fact that, unlike conventional MIMO-OFDM systems, the same pair of beamformers is applied to all the subcarriers. Moreover, in practice, the beamforming is performed by two vector modulators in each RF branch, both for the real and the imaginary parts. In a real transceiver, the applied weights must be quantified with a finite precision, which, jointly with some other RF impairments, is translated into a small performance degradation that will be evaluated in the results section by means of Monte Carlo simulations. Nevertheless, these limitations are justified by the significant reduction of the hardware and power consumption both in the baseband and the RF branches. We must note that, in conventional SISO systems, most of the power is usually consumed in the baseband branch and in the transmit part of the RF front-end [3]. Moreover, at 60 GHz communications, the FFT block takes about the 20% of digital baseband complexity [21,6]. Therefore, we can claim that the hardware and power consumption of this novel architecture is close to that of a conventional SISO system. In particular, this reduction of complexity of the RF part is summarized in Table 1.

In this paper, we focus on OFDM-based systems and address the problem of designing the transmit and receive beamformers in the case of perfect channel state information (CSI) at the receiver side, and statistical CSI at the transmitter side. The design criterion consists in the minimization of the bit error rate (BER) of the linear MMSE receiver. Thus, we firstly show that the optimal time and frequency precoders are given by the discrete Fourier transform (DFT) or the Walsh–Hadamard matrices, and reduce the problem to the design of the beamformers. The design of the receive beamformer follows the lines in [25], which considers the problem of point-to-point channels with perfect channel knowledge at the transmitter and the receiver side (CSIT+CSIR). Therefore, the main novelty of this work in comparison with [25] consists in the optimal selection of the beamformers when the CSI at the transmitter is only statistical. On the other hand, the design of the transmit beamformers resembles the conventional problem of optimal precoding for correlated MIMO channels and, in fact, the solution is rather similar: power waterfilling and transmission along the strongest modes of the transmit correlation matrix. However, a direct application of this solution to the analog combining architecture would require significative changes in the power of the RF transmit beamformers through time and, therefore, is not adequate for practical implementation. To avoid this problem, an additional set of constraints enforcing transmit beamformers of constant energy must be introduced in the optimization problem. These new constraints are satisfied by mixing the strongest modes of the transmit correlation matrix, which is again achieved by the DFT or Walsh–Hadamard matrices.

Finally, several simulation examples illustrate the great advantage of the proposed architecture over a conventional SISO system (its natural competitor), whereas the performance degradation with respect to conventional MIMO systems is justified by the reduction in the system cost and power consumption.

2. Preliminaries

2.1. Notation

Throughout this paper we will use bold-faced upper case letters to denote matrices, e.g., $\mathbf{X}$, bold-faced lower case letters for column vector, e.g., $\mathbf{x}$, and light-faced lower case letters for scalar quantities. Superscripts $(\cdot)^T$, $(\cdot)^H$, and $(\cdot)^*$ denote transpose, Hermitian, and complex conjugate, respectively. $\|\mathbf{A}\|$, $\mathrm{Tr}(\mathbf{A})$, $\mathrm{rank}(\mathbf{A})$, and $\mathrm{vec}(\mathbf{A})$ will denote, respectively, the Frobenius norm, trace, rank, and column-wise vectorized version of matrix $\mathbf{A}$. Finally, $E[\cdot]$ denotes the expectation operator.

2.2. Problem statement

Conventional MIMO-OFDM baseband schemes have access to the signals at each one of the transmitting/receiving antennas and, consequently, can apply a different pair of beamformers in each subcarrier. However, as it can be seen in Fig. 1, with the novel analog RF combining architecture a per-carrier beamforming design is not possible since all the subcarriers are affected by the same pair of beamformers. Notice that with the RF combining architecture a single FFT must be computed after the analog beamforming (at the receiver side), which notably simplifies the hardware and the system computational complexity, but also complicates the beamforming design problem due to the coupling among subcarriers. This coupling imposes some tradeoffs and represents the main challenge for the design of the beamformers.

2.3. General system model

Let us start by defining a transmission block as a set of M OFDM symbols with N subcarriers. These symbols will be sequentially transmitted and received using M different pairs of beamformers, which are assumed to change synchronously, i.e., each OFDM symbol is transmitted with a different pair of beamformers. Due to technological reasons, the beamformer weights could have to remain fixed during the transmission of several OFDM symbols. However, we must note that in that case the transmission blocks could be distributed among several beamformers as shown in Fig. 2.

Now, considering a data matrix $\mathbf{D} \in \mathbb{C}^{N\times M}$ containing NM information symbols, we can define the transmission matrix

$$\mathbf{S} = \mathbf{G}_\mathrm{F}\mathbf{D}\mathbf{G}_\mathrm{T}, \tag{1}$$

where $\mathbf{G}_\mathrm{F} \in \mathbb{C}^{N\times N}$ and $\mathbf{G}_\mathrm{T} \in \mathbb{C}^{M\times M}$ are, respectively, the frequency and time precoding matrices. Here, we must note that the total energy associated to a transmission block is

$$\|\mathbf{S}\|^2 = \|\mathbf{s}\|^2 = \|\mathbf{G}\mathbf{d}\|^2, \tag{2}$$

where $\mathbf{s} = \mathrm{vec}(\mathbf{S})$, $\mathbf{d} = \mathrm{vec}(\mathbf{D})$ and $\mathbf{G} = \mathbf{G}_\mathrm{T} \otimes \mathbf{G}_\mathrm{F}$. Therefore, in order to preserve the transmission energy, the precoding matrices should have a unitary Kronecker product, i.e., $\mathbf{G}^H\mathbf{G} = \mathbf{I}$.

After linear precoding, each row of $\mathbf{S}$ is associated to a subcarrier, whereas each column represents the linearly precoded data in one OFDM symbol. Thus, the n-th column of $\mathbf{S}$ will be transmitted using the transmit and receive beamformers $\mathbf{w}_\mathrm{T}[n] \in \mathbb{C}^{n_\mathrm{T}\times 1}$ and $\mathbf{w}_\mathrm{R}[n] \in \mathbb{C}^{n_\mathrm{R}\times 1}$, where n_T and n_R are the number of transmit and receive antennas, respectively. The elements of these beamformers are given by the RF weights shown in Fig. 1. Furthermore, we will assume, without loss of generality, unit energy beamformers, i.e., $\|\mathbf{w}_\mathrm{T}[n]\| = \|\mathbf{w}_\mathrm{R}[n]\| = 1$. Fig. 3 shows this OFDM-based scheme at the transmitter side. In this figure, it is shown that the data symbols are linearly precoded in frequency and time. Each OFDM symbol contains N precoded data symbols. After the insertion of the guard and pilot symbols, the N_c symbols are modulated by the IFFT block. Then, the cyclic prefix is inserted and the OFDM symbol, applying $\mathbf{w}_\mathrm{T}$, is transmitted through the n_T antennas.

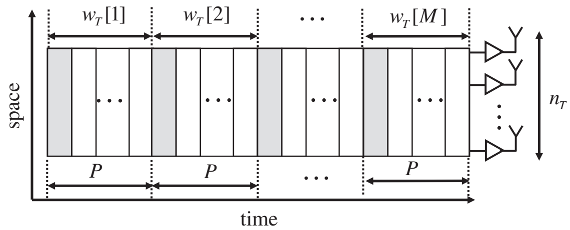

Fig. 2. Distribution of a transmission block. The gray columns denote the M time and frequency precoded OFDM symbols of one block, which are transmitted using M different beamformers. In this example the beamformers remain fixed during the transmission of P OFDM symbols. The minimum achievable P depends on the limitations of the RF implementation. On the other hand, the maximum allowable P is imposed by the temporal coherence of the channel.

With the above definitions, and assuming a cyclic prefix longer than the impulse response of the MIMO channel, the equivalent system model in the frequency domain for the n-th pair of beamformers ($n=1,\ldots,M$) can be written as

$$y_k[n] = h_k[n]s_k[n] + n_k[n], \quad k = 1,\ldots,N, \tag{3}$$

where $y_k[n]$ denotes the observed signal at the k-th subcarrier, $n_k[n]$ denotes the i.i.d. Gaussian noise with variance σ^2, $s_k[n]$ is the element in the k-th row and n-th column of $\mathbf{S}$, and the equivalent channel after TX and RX beamforming is

$$h_k[n] = \mathbf{w}_\mathrm{R}^H[n]\mathbf{H}_k\mathbf{w}_\mathrm{T}[n], \quad k = 1,\ldots,N, \tag{4}$$

where $\mathbf{H}_k \in \mathbb{C}^{n_\mathrm{R}\times n_\mathrm{T}}$ represents the response of the MIMO channel at the k-th subcarrier.

3. General analog beamforming criterion with CSIT and CSIR

In this section we review the problem of designing the optimal pair of beamformers under perfect knowledge of the frequency-selective MIMO channel $\mathbf{H}_k$, and the noise variance. This problem was addressed in [25], where a general beamforming criterion for the novel MIMO transceiver was proposed. In the case of perfect CSI, it is easy to prove that the optimal TX/RX strategy amounts to using a single pair of beamformers. That is, in this case we do not need to spread the information among several beamformers, and the time precoding matrix $\mathbf{G}_\mathrm{T}$ is not necessary.

The perfect CSI at the receiver can be obtained with the transmission of $n_\mathrm{T}n_\mathrm{R}$ OFDM training symbols from the transmitter to the receiver. These symbols are transmitted and received using different combinations of orthogonal transmit–receive beamformers in order to estimate the $n_\mathrm{T}n_\mathrm{R}$ equivalent SISO channels. The number of required

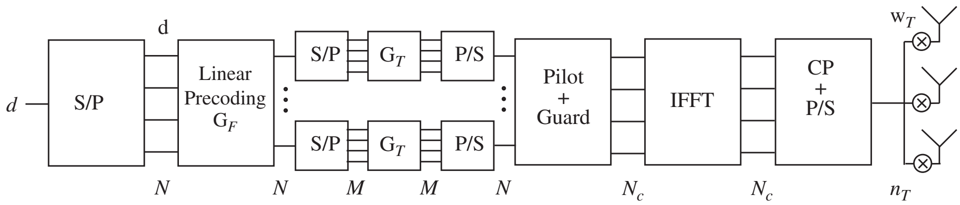

Fig. 3. Block diagram of the OFDM-based transmitter with analog antenna combining. The receiver diagram is equivalent.

pilot subcarriers to estimate each equivalent SISO channel corresponds to the length of the channel impulse response. Afterwards, the receiver feedbacks the channel estimates to the transmitter, which obviously incurs in a slight reduction of the system throughput. However, under slow fading scenarios, this procedure is repeated less frequently, and the decrease of the system throughput becomes negligible.

The proposed algorithm consists of minimizing the following cost function:

$$f_\alpha(\mathbf{w}_\mathrm{T},\mathbf{w}_\mathrm{R}) = \frac{1}{\alpha-1}\log\left(\frac{1}{N}\sum_{k=1}^{N}\mathrm{MSE}_k^{\alpha-1}\right), \qquad (5)$$

where N is the number of subcarriers and α is a real parameter which controls the overall system performance [25]. Thus, our optimization problem can be written as

$$\begin{aligned}&\underset{\mathbf{w}_\mathrm{T},\mathbf{w}_\mathrm{R}}{\text{minimize}} \quad f_\alpha(\mathbf{w}_\mathrm{T},\mathbf{w}_\mathrm{R})\\ &\text{subject to} \quad \|\mathbf{w}_\mathrm{T}\|=1,\\ &\|\mathbf{w}_\mathrm{R}\|=1,\end{aligned} \qquad (6)$$

which is (in general) a highly non-convex problem. Thus, in order to obtain a local solution, in [25] the authors proposed a gradient search algorithm. Although the proposed algorithm is in general suboptimal, we saw that with a proper initialization it provides very accurate results. The details of this algorithm are beyond the scope of this paper but a deeper description can be found in [25].

Regarding α in (5) there are some interesting choices of the parameter. If the parameter α is set to zero, the optimization problem in (7) can be rewritten as the maximization of the received SNR. This problem has been previously addressed by other authors in the contexts of analog combining [14] and pre-FFT schemes [19,10,17,12], and it is also closely related to the statistical eigen beamforming transmission mode defined in the WiMAX standard [26,27]. When α approaches 1 the proposed criterion reduces to the maximization of the capacity of the equivalent SISO channel after beamforming. For $\alpha=2$ the proposed criterion amounts to minimizing the overall MSE of the optimal linear receiver. Moreover, it can be proved that, in the important case of quadrature amplitude modulation (QAM) constellations, and under optimal linear precoding of the information symbols, the minimization of the MSE is equivalent to the minimization of the bit error rate (BER) of the optimal linear receiver [1,15]. Based on this observation, in this work we focus on minimization of the MSE ($\alpha=2$).

4. General analog beamforming criterion with statistical CSIT and perfect CSIR

In this section, as the main contribution of this paper, we analyze in detail the case with perfect CSI at the receiver (CSIR) and only correlation CSI at the transmitter. We start by introducing the general problem under frequency selective channels with transmit antenna correlation and afterwards we analyze some interesting particular cases. Our goal consists in designing the system parameters $\mathbf{G}_\mathrm{F}$, $\mathbf{G}_\mathrm{T}$, $\mathbf{w}_\mathrm{T}[n]$ and $\mathbf{w}_\mathrm{R}[n]$ to minimize the averaged bit error rate (BER) of the system associated to the linear minimum mean square (MMSE) receiver, i.e., the minimization of the function $\mathrm{BER}(\mathbf{G}_\mathrm{T},\mathbf{G}_\mathrm{F},\mathbf{w}_\mathrm{T}[n],\mathbf{w}_\mathrm{R}[n])$. Therefore, our optimization problem is

$$\begin{aligned}&\underset{\mathbf{G},\mathbf{G}_\mathrm{T},\mathbf{G}_\mathrm{F},\mathbf{w}_\mathrm{T}[n],\mathbf{w}_\mathrm{R}[n]}{\text{minimize}} \quad \mathrm{BER}(\mathbf{G}_\mathrm{T},\mathbf{G}_\mathrm{F},\mathbf{w}_\mathrm{T}[n],\mathbf{w}_\mathrm{R}[n])\\ &\text{subject to} \quad \|\mathbf{w}_\mathrm{T}[n]\|=\|\mathbf{w}_\mathrm{R}[n]\|=1, \quad n=1,\ldots,M,\\ &\mathbf{G}^H\mathbf{G}=\mathbf{I},\\ &\mathbf{G}=\mathbf{G}_\mathrm{T}\otimes\mathbf{G}_\mathrm{F}.\end{aligned} \qquad (7)$$

4.1. Design of the frequency and time precoders

The design of linear precoding schemes for OFDM systems has been addressed, under different criteria, in [1,13,28]. Here, we follow the same principles applied on the matrix $\mathbf{G}$. In the case of linear receivers and QAM constellations, the basic idea consists in writing the averaged BER as a function of the MSE associated to the information symbols

$$\mathrm{BER} = \frac{1}{NM}\sum_{k=1}^{N}\sum_{n=1}^{M}\mathrm{BER}_k[n] = \frac{1}{NM}\sum_{k=1}^{N}\sum_{n=1}^{M} g(\mathrm{MSE}_k[n]), \qquad (8)$$

where $\mathrm{BER}_k[n]$ and $\mathrm{MSE}_k[n]$ represent, respectively, the BER and MSE associated to the information symbol in the k-th row and n-th column of $\mathbf{D}$, and $g(x)$ is the function

$$g(x) = \frac{\alpha}{\log_2 M}\mathcal{Q}(\sqrt{\beta(x^{-1}-1)}) \qquad (9)$$

where α, β, and M are parameters which depend on the QAM constellation. It can be easily proven that this function is convex for practical values of x [15].

Interestingly, due to the unitarity of the precoding matrix $\mathbf{G}$ and assuming unit power transmissions, without loss of generality (i.e., $E[|s_k[n]|^2]=1$), the total MSE can be written as

$$\overline{\mathrm{MSE}} = \frac{1}{NM}\sum_{k=1}^{N}\sum_{n=1}^{M}\mathrm{MSE}_k[n] = \frac{1}{NM}\sum_{k=1}^{N}\sum_{n=1}^{M}\mathrm{MSE}_{s_k}[n], \qquad (10)$$

where

$$\mathrm{MSE}_{s_k}[n] = \frac{1}{1+\gamma|h_k[n]|^2}, \qquad (11)$$

denotes the MSE in the estimate of $s_k[n]$, and $\gamma=1/\sigma^2$ is the SNR. Thus, noting that $\overline{\mathrm{MSE}}$ does not depend on the specific unitary precoding matrix $\mathbf{G}$, and taking into account that the averaged BER is a Schur-convex function [15], we have

$$\mathrm{BER} = \frac{1}{NM}\sum_{k=1}^{N}\sum_{n=1}^{M} g(\mathrm{MSE}_k[n]) \geq g(\overline{\mathrm{MSE}}) \qquad (12)$$

and the lower bound is achieved when all the $\mathrm{MSE}_k[n]$ are equal [1,15].

Finally, in order to ensure a uniform distribution of the total MSE among the information symbols, the optimal precoding matrix $\mathbf{G}$ must be unitary with constant modulus entries, such as the DFT or Walsh–Hadamard matrices [1,15]. However, we must note that, in our particular problem, $\mathbf{G}$ must also satisfy the Kronecker structure $\mathbf{G}=\mathbf{G}_\mathrm{T}\otimes\mathbf{G}_\mathrm{F}$. Fortunately, the Kronecker product preserves the unitarity and constant modulus properties, i.e., given two unitary matrices $\mathbf{G}_\mathrm{T}$ and $\mathbf{G}_\mathrm{F}$ with constant modulus entries, the product $\mathbf{G}_\mathrm{T}\otimes\mathbf{G}_\mathrm{F}$ is unitary with constant modulus elements. Thanks to this property, we can conclude that the separated time and frequency precoding structure proposed in this paper is optimal. To summarize, we propose to independently chose the precoding matrices $\mathbf{G}_\mathrm{T}$ and $\mathbf{G}_\mathrm{F}$ as any DFT or Walsh–Hadamard matrices, which reduces the averaged BER to

$$\mathrm{BER} = g(\overline{\mathrm{MSE}}) = g\left(\frac{1}{NM}\sum_{k=1}^{N}\sum_{n=1}^{M}\frac{1}{1+\gamma|h_k[n]|^2}\right). \tag{13}$$

4.2. Design of the beamformers

In this subsection, the transmit and receive beamformers are designed in order to minimize the BER of the analog combining system. The proposed scheme assumes spatially correlated Rayleigh channels, and it is based on a set of M unit-energy transmit beamformers. At the receiver side the channel and transmit beamformers are known, which reduces the problem to the minimization of the MSE, i.e., we can use the algorithm in [25]. At the transmitter side the beamformers would distribute the transmit power isotropically in the case without transmit correlation. Nevertheless, when the channels are spatially correlated at the transmitter side, the design of the optimal beamformers is more involved.

4.2.1. Receive beamformers

As we have previously shown, under the optimal precoding matrices $\mathbf{G}_\mathrm{T}$ and $\mathbf{G}_\mathrm{F}$, the problem of minimizing the BER reduces to the minimization of the total MSE. Thus, taking (13) into account, the criterion for the design of the receive beamformers can be rewritten as the following uncoupled optimization problems:

$$\underset{\mathbf{w}_\mathrm{R}[n]}{\text{minimize}} \quad \sum_{k=1}^{N}\frac{1}{1+\gamma|h_k[n]|^2} \quad \text{such that } \|\mathbf{w}_\mathrm{R}[n]\|=1, \tag{14}$$

for $n=1,\ldots,M$. Since the receiver knows the MIMO channel and the transmit beamformers, the above problems are equivalent to that of designing the minimum MSE (MinMSE) receive beamformer in an analog antenna combining SIMO system under OFDM transmissions. This problem, which in the flat-fading case reduces to the well known maximum ratio combining (MRC) receiver, has been studied in [24,25]. Thus, we can directly apply the algorithm proposed in [25], which in practice rapidly converges to a satisfactory solution. Specifically, the updating rule is

$$\mathbf{w}_\mathrm{R}[n] \leftarrow \mathbf{w}_\mathrm{R}[n] + \mu\tilde{\mathbf{H}}[n]\mathbf{w}_\mathrm{R}[n], \quad \begin{cases} n=1,\ldots,M, \\ k=1,\ldots,N, \end{cases} \tag{15}$$

where

$$\tilde{\mathbf{H}}[n] = \sum_{k=1}^{N}\mathrm{MSE}_{s_k}^2[n]\mathbf{h}_k[n]\mathbf{h}_k^H[n], \quad \begin{cases} n=1,\ldots,M, \\ k=1,\ldots,N \end{cases} \tag{16}$$

can be seen as a weighted correlation matrix, and

$$\mathbf{h}_k[n] = \mathbf{H}_k\mathbf{w}_\mathrm{T}[n], \quad \begin{cases} n=1,\ldots,M, \\ k=1,\ldots,N \end{cases} \tag{17}$$

defines the equivalent frequency selective SIMO channels after fixing the transmit beamformers.

4.2.2. Transmit beamformers

The design of the transmit beamformers is more involved due to the fact that only correlation CSI is available at the transmitter side. A simple alternative to design the transmit beamformers consists in the minimization of the pairwise error probability (PEP). However, this is a very difficult problem in the analog antenna combining case, because the receive beamformer depends in a far from trivial way on the TX beamformer. Here, in order to simplify our analysis, we focus on the case of having full access to the signals at all the receive antennas. Obviously, this can be seen as an upper bound for analog antenna combining schemes, and the bound is tight in the cases of flat-fading channels or only one receive antenna. However, this approximation allows us to obtain a neat formulation for the optimal transmit beamformers and, as we will see in the simulations section, the bound is reasonably close to the performance of the proposed scheme.

The derivation of the PEP is based on the Chernoff bound [22]. Thus, the probability of decoding the codeword $\hat{\mathbf{s}}$ when $\mathbf{s}$ was transmitted is bounded by

$$P(\mathbf{s}\to\hat{\mathbf{s}}\,|\,\mathbf{H}) \le \exp\left(-\frac{\gamma}{4}\sum_{k=1}^{N}\|\mathbf{H}_k\mathbf{W}_\mathrm{T}\mathrm{diag}(\mathbf{s}_k-\hat{\mathbf{s}}_k)\|^2\right), \tag{18}$$

where $\mathbf{s}_k$ is the k-th row of $\mathbf{S}$, $\hat{\mathbf{s}}_k$ is the k-th row of the matrix $\hat{\mathbf{S}}$, γ is the signal to noise ratio and

$\mathbf{W}_\mathrm{T} = [\mathbf{w}_\mathrm{T}[1] \ \cdots \ \mathbf{w}_\mathrm{T}[M]]$.

Now, the average of (18) over the channel fading statistics yields [11]

$$P(\mathbf{s}\to\hat{\mathbf{s}}) \le \left|\mathbf{I}+\frac{\gamma}{4}\sum_{k=1}^{N}\mathbf{R}_k^{1/2}\mathbf{E}_k(\mathbf{s}\to\hat{\mathbf{s}})\mathbf{R}_k^{1/2}\right|^{-n_\mathrm{R}}, \tag{19}$$

where $\mathbf{R}_k$ is the $n_\mathrm{T}\times n_\mathrm{T}$ correlation matrix at the k-th subcarrier and $\mathbf{E}_k(\mathbf{s}\to\hat{\mathbf{s}})$ is the codeword distance product matrix at the k-th subcarrier,

$$\mathbf{E}_k(\mathbf{s}\to\hat{\mathbf{s}}) = \mathbf{W}_\mathrm{T}\ \mathrm{diag}(\mathbf{s}_k-\hat{\mathbf{s}}_k)\mathrm{diag}(\mathbf{S}_k-\hat{\mathbf{S}}_k)^H\mathbf{W}_\mathrm{T}^H. \tag{20}$$

Assuming that the TX correlation is the same in all the subcarriers, which is true when the channel impulse response is uncorrelated in time domain [5], (19) can be rewritten as

$$P(\mathbf{s}\to\hat{\mathbf{s}}) \le \left|\mathbf{I}+\frac{\gamma}{4}\mathbf{R}^{1/2}\sum_{k=1}^{N}\mathbf{E}_k(\mathbf{s}\to\hat{\mathbf{s}})\mathbf{R}^{1/2}\right|^{-n_\mathrm{R}}, \tag{21}$$

where $\mathbf{R}$ is the transmit correlation matrix of the system.

Obviously, the averaged PEP (and therefore the choice of $\mathbf{W}_\mathrm{T}$) depends on the specific pair of information vectors $(\mathbf{s},\hat{\mathbf{s}})$ considered. However, it seems reasonable to minimize the averaged PEP between the true information vector $\mathbf{s}$ and its closest neighbor $\hat{\mathbf{s}}$, i.e., those vectors which only differ from $\mathbf{s}$ in one element. With this choice, and taking into account that the optimal $\mathbf{G}_\mathrm{F}$ and $\mathbf{G}_\mathrm{T}$ have constant modulus entries, the codeword distance product becomes independent of the subcarrier and its expression is reduced to

$$\mathbf{E}(\mathbf{s}\rightarrow\hat{\mathbf{s}}) = \frac{Nd^2}{n_\mathrm{T}}\mathbf{W}_\mathrm{T}\mathbf{W}_\mathrm{T}^H, \quad (22)$$

where d is the minimum Euclidean distance in the particular constellation. Thus, the optimization problem in (7) reduces to

$$\underset{\mathbf{W}_\mathrm{T}}{\text{maximize}} \quad \left|\mathbf{I}+\frac{\gamma Nd^2}{4n_\mathrm{T}}\mathbf{W}_\mathrm{T}^H\mathbf{R}\mathbf{W}_\mathrm{T}\right|$$
$$\text{subject to} \quad \|\mathbf{w}_\mathrm{T}[n]\| = 1, \quad n=1,\ldots,n_\mathrm{T}, \quad (23)$$

which resembles the precoder design problem in conventional MIMO systems [2], with the additional constraint in the energy of the columns of $\mathbf{W}_\mathrm{T}$.

Fortunately, the individual energy constraints can be easily satisfied. In particular, writing the singular value decomposition (SVD)

$$\mathbf{W}_\mathrm{T} = \mathbf{U}_\mathrm{T}\boldsymbol{\Lambda}\mathbf{V}_\mathrm{T}^H, \quad (24)$$

where $\mathbf{U}_\mathrm{T}$, $\mathbf{V}_\mathrm{T}$ are $M\times M$ unitary matrices and

$$\boldsymbol{\Lambda} = \mathrm{diag}([\lambda_1,\ldots,\lambda_M]) \quad (25)$$

contains the singular values, it is easy to see that the determinant in (23) does not depend on the singular vectors $\mathbf{V}_\mathrm{T}$. Therefore, we can choose $\mathbf{V}_\mathrm{T}$ as any unitary matrix with constant modulus elements, such as the DFT or the Walsh–Hadamard matrix, which ensures [15]

$$\|\mathbf{w}_\mathrm{T}[n]\|^2 = \frac{\mathrm{Tr}(\boldsymbol{\Lambda}^2)}{M} \quad n=1,\ldots,M. \quad (26)$$

With this choice of $\mathbf{V}_\mathrm{T}$, (23) can be rewritten as

$$\underset{\mathbf{U}_\mathrm{T},\boldsymbol{\Lambda}}{\text{maximize}}\left|\mathbf{I}+\frac{\gamma Nd^2}{4n_\mathrm{T}}\boldsymbol{\Lambda}^2\mathbf{U}_\mathrm{T}^H\mathbf{R}\mathbf{U}_\mathrm{T}\right| \quad \text{such that } \mathrm{Tr}(\boldsymbol{\Lambda}^2)=M \quad (27)$$

and its solution is obtained from standard majorization results [15]. Specifically, writing the eigenvalue (EV) decomposition of $\mathbf{R}$ as

$$\mathbf{R} = \mathbf{U}_\mathbf{R}\boldsymbol{\Sigma}^2\mathbf{U}_\mathbf{R}^H \quad (28)$$

the optimal beam directions are directly given by $\mathbf{U}_\mathrm{T} = \mathbf{U}_\mathbf{R}$, whereas the optimal power allocation is obtained from a standard water-filling technique [18]

$$\lambda_n^2 = \left(\kappa - \frac{4n_\mathrm{T}}{\gamma Nd^2\sigma_n^2}\right)_+, \quad n=1,\ldots,M, \quad (29)$$

where σ_n^2 are the eigenvalues of $\mathbf{R}$,

$$(x)_+ = \begin{cases} 0, & x\le 0,\\ x, & x\ge 0\end{cases} \quad (30)$$

and κ is the water level, which is chosen to satisfy

$$\mathrm{Tr}(\boldsymbol{\Lambda}^2) = \sum_{n=1}^{n_\mathrm{T}}\lambda_n^2 = M. \quad (31)$$

4.3. *Further discussion*

In the previous subsections we have studied the design of the time and frequency precoders and the transmit and receive beamforming matrices in the case of statistical CSI at the transmitter side and perfect CSI at the receiver. For the initial design we supposed a frequency selective channel with correlation at the transmitter side but by relaxing these assumptions the design of the precoders or the beamformers can be done in an easier way in some cases.

In the particular case of spatially uncorrelated channels, the design of the transmit beamformers becomes significatively easier. In particular, the optimal solution is given by [20,9]

$$\mathbf{W}_\mathrm{T}^H\mathbf{W}_\mathrm{T} = \mathbf{I} \quad \text{for } n_\mathrm{T}\ge M, \quad (32)$$

$$\mathbf{W}_\mathrm{T}\mathbf{W}_\mathrm{T}^H = \mathbf{I} \quad \text{for } n_\mathrm{T}\le M, \quad (33)$$

i.e., as one could expect, the available power has to be isotropically distributed, which is the idea after the orthogonal beam division multiplexing (OBDM) scheme proposed in [20]. Furthermore, it is easy to prove that for $M\ge n_\mathrm{T}$, the beamformers in $\mathbf{W}_\mathrm{T}$ extract the spatial diversity at the transmitter side (n_T) and maximize the coding gain. It is also easy to show that, at high SNRs, the OBDM scheme is optimal.

Note that when the channel is flat-fading the optimal receive beamformer is given by the maximal ratio combining (MRC) receiver

$$\mathbf{w}_\mathrm{R}[n] = \frac{\mathbf{H}\mathbf{w}_\mathrm{T}[n]}{\|\mathbf{H}\mathbf{w}_\mathrm{T}[n]\|}, \quad n=1,\ldots,M, \quad (34)$$

which maximizes the SNR and also minimizes the BER [16]. In this case the matrix $\mathbf{G}_\mathrm{F}$ is not necessary and the optimal matrix $\mathbf{G}_\mathrm{T}$ can be designed as the DFT matrix or the Walsh–Hadamard [20,23].

5. Results

In this section, the performance of the proposed technique is evaluated by means of Monte Carlo simulations. In all the experiments, we consider a block-fading model in which the channel response remains constant at least for a coherence interval of PM symbols (i.e. the frame duration seen in Fig. 2). In all the experiments, we consider a 4×4 MIMO system with 64 data subcarriers and QPSK information symbols which are linearly precoded in frequency and time with the matrices proposed in the previous section. An i.i.d. Rayleigh MIMO channel model with exponential power delay profile has been assumed. In particular, the total power associated to the l-th tap is

$$E[\|\mathbf{H}[l]\|^2] = (1-\rho)\rho^l n_\mathrm{T}n_\mathrm{R}, \quad l=0,\ldots,L_c-1, \quad (35)$$

where L_c is the length of the channel impulse response (L_c=20 in all the simulations). The exponential parameter ρ has been selected as $\rho=0.4$ and $\rho=0.7$ to compare the performance of the techniques depending on the frequency selectivity. The transmit correlation matrix has been obtained from the Jakes model [4] with antenna spacing of $d_\mathrm{A}=0.1\lambda$ and $d_\mathrm{A}=0.25\lambda$, where λ is the wavelength.

5.1. Studied schemes

The evaluated systems are summarized as follows:

- Full-MIMO: We consider a scheme performing MRT (maximum ratio transmission) and MRC in each subcarrier. Obviously, this implies perfect channel knowledge at both sides of the link, and therefore it can be seen as a non-tight upper bound for the performance of the proposed system and, in general, for the performance of any analog beamforming scheme. Here, we must note that this scheme only transmits one data stream, which ensures the fairness in the comparison with the proposed analog antenna combining architecture.
- SISO: This can be seen as the natural competitor of the proposed system, which provides better performance at the expense of a slight increase of complexity.
- MSE+MSE: MinMSE analog antenna beamforming at the transmitter and at the receiver. This is an upper bound for the performance the proposed OBDM+MSE scheme since it assumes perfect channel knowledge (or feedback of the optimal beamformer) at the transmitter.
- OBDM+MRC: OBDM scheme at the transmitter and MRC in each subcarrier at the receiver. This scheme requires a conventional multi-antenna receiver with one down-conversion chain for each antenna, and therefore it can be seen as an upper bound for the performance of the proposed architecture. Moreover, the PEP analysis in Section 4.2.2 provides a tight bound for this system.
- OBDM+MSE: The proposed analog antenna combining architecture with OBDM at the transmitter and Min-MSE beamforming at the receiver.

5.2. Experiments

The spatial power distribution at the transmitter side is provided by the water-filling technique. Fig. 4 illustrates this power distribution among the different modes under two different scenarios where the distance between antennas is $d_A = 0.1\lambda$ and $d_A = 0.25\lambda$. As can be seen, the transmission scheme varies from pure beamforming for very low SNRs, which means using n_T identical beamformers, to isotropic radiation for high SNRs, which is equivalent to the OBDM scheme ($\mathbf{W}_T$ unitary).

In the second experiment we have evaluated the BER performance of the proposed scheme (OBDM+MSE) under different conditions. Figs. 5 and 6 show the BER performance when there is not transmit correlation ($d_A = \infty$). The noticeable decrease of BER in OBDM in comparison to the SISO system is due to the spreading of the information symbols along the n_T beamformers, as well as the optimal analog beamforming at the receiver side. The gap between OBDM and MSE+MSE is in both cases less than 5 dB in medium SNRs and represents the difference between having perfect CSIT or only statistical CSI at the transmitter side. Figs. 7 and 8 show the performance when the transmit correlation is $d_A = 0.25\lambda$. In this case all the curves remain similar except our proposed technique (OBDM+MSE)

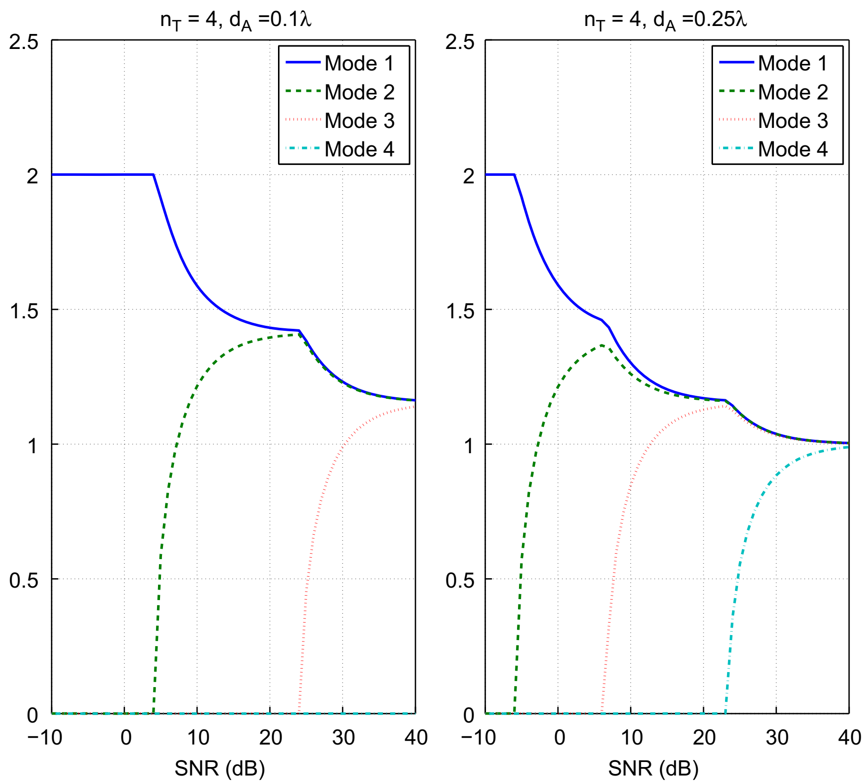

Fig. 4. Spatial power distribution under two scenarios with different antenna separation. (a) $n_T = 4$, $d_A = 0.1\lambda$. (b) $n_T = 4$, $d_A = 0.25\lambda$.

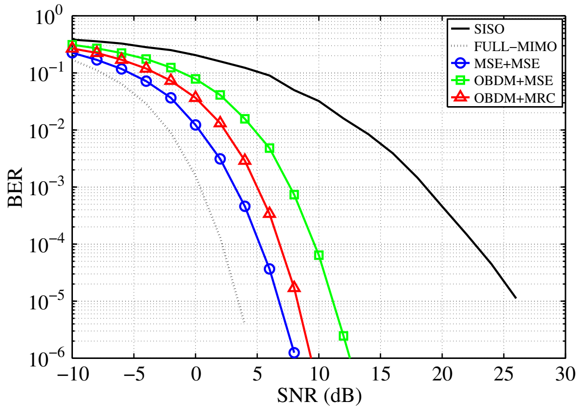

Fig. 5. Bit error rate for the evaluated schemes ($\rho=0.4$, $d_A=\infty$).

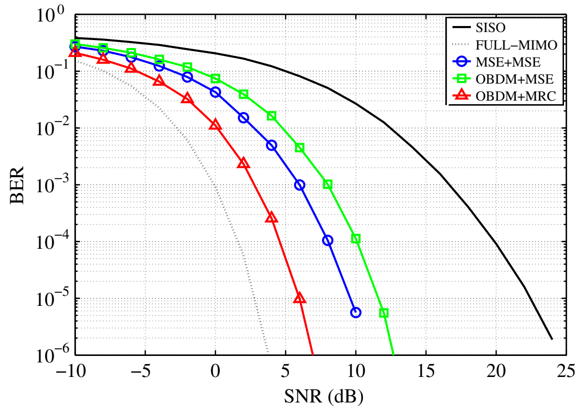

Fig. 8. Bit error rate for the evaluated schemes ($\rho=0.7$, $d_A=0.25\lambda$).

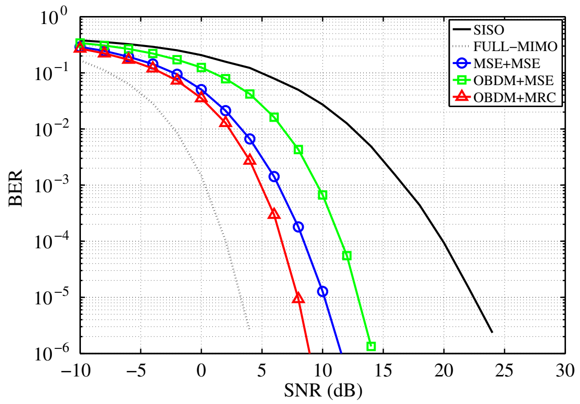

Fig. 6. Bit error rate for the evaluated schemes ($\rho=0.7$, $d_A=\infty$).

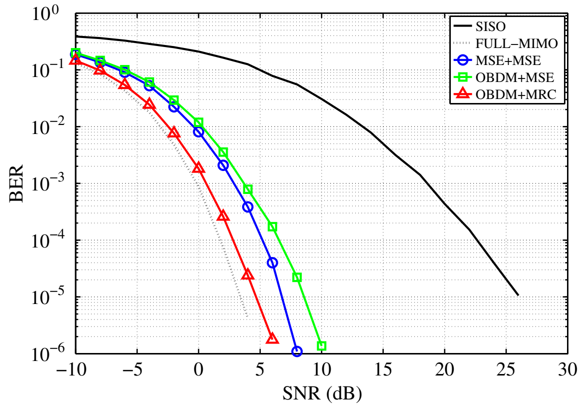

Fig. 9. Bit error rate for the evaluated schemes ($\rho=0.4$, $d_A=0.1\lambda$).

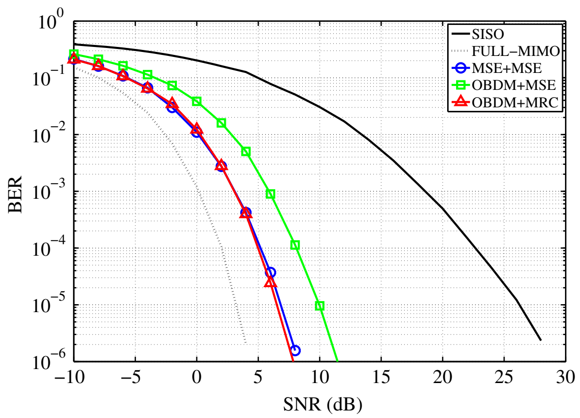

Fig. 7. Bit error rate for the evaluated schemes ($\rho=0.4$, $d_A=0.25\lambda$).

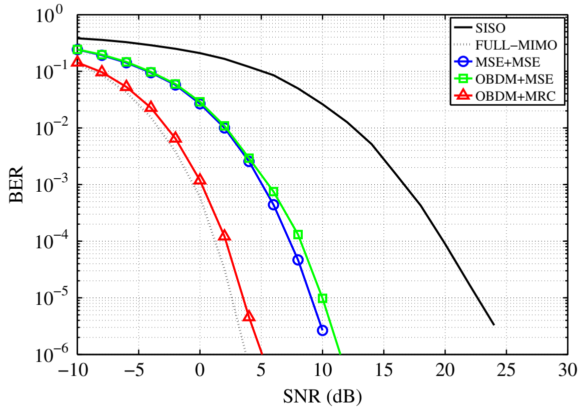

Fig. 10. Bit error rate for the evaluated schemes ($\rho=0.7$, $d_A=0.1\lambda$).

which improves substantially, reducing the gap with the MSE+MSE technique to 3 dB. Figs. 9 and 10 go deeply into this idea and illustrate that when the transmit correlation is higher ($d_A=0.1\lambda$), the difference between both techniques becomes lower. That is because when the correlation becomes higher the power is distributed in less modes. Furthermore it can be observed that when the correlation increases pure beamforming becomes closer to optimal beamforming. We can also see differences between the experiments with different frequency selectivity. It can be seen that when the selectivity increases ($\rho=0.7$) the gap between the proposed OBDM+MSE and the full-MIMO scheme also increases, which can be seen as a direct consequence of the fact that the proposed architecture applies the same pair of beamformers to all the subcarriers. Nevertheless, the increase of the selectivity has a bigger impact on the MSE+MSE technique, which allows us to conclude that, as the frequency diversity increases, the gap between the optimal analog beamforming schemes with and without CSIT decreases.

Finally, we have studied the impact of different realistic impairments in the BER performance of the proposed system. Particularly, three different sources of error have been introduced in order to test the performance of the proposed beamforming algorithm in a realistic RF combining system. Firstly, we have obtained least squares (LS) estimates of the $\mathbf{H}_k$ ($k=1,\ldots,N_c$). Secondly, we have simulated an error in the knowledge of the correlation matrix $\mathbf{R}$ at the transmitter. This error consists in the addition of i.i.d. noise to the matrix $\mathbf{Q}$, where $\mathbf{Q}$ is the Hermitian square root of $\mathbf{R}$. The power of this noise is the 10% of the averaged power of the $\mathbf{Q}$ entries. Finally, we have quantified the real and the imaginary part of the optimal transmit and receive beamformers with 5 and 2 bits, and we have established a dead region around zero where the weights cannot be in. Therefore, the real and imaginary components of the weights cannot take a value under a threshold, a 10% of the maximum value in our case. This means that, since the weights norm is 1, all the weight components with a value between -0.1 and 0.1 are set to 0.1.

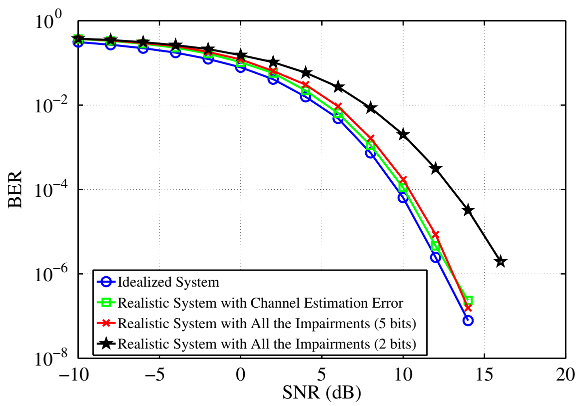

Fig. 11. Bit error rate for the proposed system in a realistic scenario under different impairments ($\rho=0.4$, $d_A=\infty$).

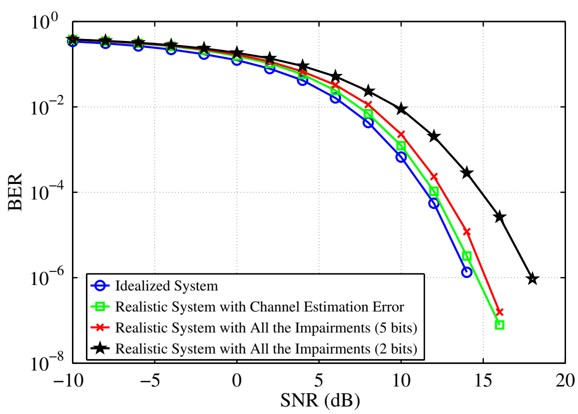

Fig. 12. Bit error rate for the proposed system in a realistic scenario under different impairments ($\rho=0.7$, $d_A=\infty$).

Figs. 11 and 12 show the BER performance degradation of the realistic system with different impairments in comparison with the idealized system. The curves show the slight degradation due to the channel estimator errors as well as due to all the impairments. It can be seen that the gap between the ideal system and that with all the impairments (including a quantification of the weights of 5 bits) is around 1 dB, which shows the robustness of the proposed system to the impairments. A curve of a realistic system where the beamformers are quantified with 2 bits has been also included, and it is seen that the degradation starts to be noticeable. Note that Figs. 11 and 12 have been obtained in a system without transmit correlation, but other simulations for different values of transmit correlation have shown similar results.

6. Conclusions

In this paper we have addressed the problem of designing the transceiver for a novel architecture based on analog antenna combining. One of the main challenges of the design problem consists in the fact that the same pair of beamformers (RF weights) has to be applied to all the subcarriers, which introduces a coupling that is not present in conventional OFDM-based MIMO systems. Considering the case of perfect channel knowledge at the receiver side, and statistical channel state information at the transmitter, we have shown that the optimal scheme is based on the minimization of the MSE at the receiver (MinMSE receive beamforming), whereas the transmitter distributes the information symbols among different spatial directions, matched to the channel spatial correlation matrix. Interestingly, several simulation examples have shown that the performance of the proposed architecture is not very far from that of alternative schemes such as the conventional MIMO systems, whereas the hardware and power consumption is closer to that of a conventional SISO system.

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