Spatial Shaping and Precoding Design for Underlay MIMO Interference Channels

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***Abstract*—In this paper, we study coexistence issues between an underlay single-beam interference channel (IC) and a primary point-to-point link (PPL) that has a rate constraint. We derive spatial shaping constraints at the secondary transmitters and show that they generalize the so-called interference temperature (IT) when the PPL transmits multiple streams. We propose a successive convex approximation algorithm to compute the spatial shaping matrices that maximize the allowed transmit power of the IC while ensuring the rate constraint of the PPL. Then, we provide general interference leakage minimization (MinIL) and maximum signal-to-interference-plus-noise ratio (MaxSINR) algorithms that can incorporate both types of constraint. An additional power control step is included in the optimization procedure to further improve the sum-rate of the IC. Different numerical examples are provided to illustrate the effectiveness of the proposed techniques and to compare the performance improvement when the IC is subject to spatial shaping constraints in comparison to IT.**

***Keywords*—Cognitive radio, interference channel, interference shaping, interference temperature, precoding design**

I. Introduction

The deployment of heterogeneous networks (HetNets) is a promising solution for increasing capacity, flexibility and reducing the costs for future cellular networks. In these scenarios, the use of proper interference management techniques is crucial for an efficient utilization of the scarce radio resources. Following the cognitive radio (CR) paradigm [1], [2], underlaying networks are allowed to coexist with the primary or legacy network as long as a given performance at the latter is ensured. Thus, the so-called interference temperature (IT) has been proposed as a way to measure and constrain the interference level generated at the primary by the underlaying networks.

In this paper, we focus on coexistence issues between an interference channel (IC) and a legacy point-to-point link (PPL) that has a given rate constraint. This model can represent, for instance, a device-to-device (D2D) communication network underlaying a cellular network (see, e.g., [3] and references therein). The IC, as an independent network, has received much attention in the last few years. In this scenario, a novel approach called interference alignment (IA) [4]– [6] has recently been proposed to manage interference and achieve the maximum degrees-of-freedom (DoF), which asymptotically approach the sum-rate capacity. Following these lines, many interesting results about performance limits have been provided [7]– [10], and many different algorithms have been proposed for designing linear precoders and decoders [11]– [18]

On the other hand, the IC in the context of CR has been also widely studied in the literature [19]– [24]. The achievable DoF of a cognitive IC that coexists with a PPL are studied in [19]. If the legacy user performs the optimal strategy, namely, singular value decomposition (SVD) and waterfilling power allocation, some eigenmodes may be left unused, in which the IC opportunistically confines the transmitted signals. On the other hand, it is shown in [20] and [21] that primary cooperation by means of interference suppression decoding, can lead to a significant improvement of the achievable sum-rate of the IC with negligible primary rate reduction. A non-iterative IA scheme is proposed in [22], in which the IC is also constrained to cause zero interference to the primary. An IC constrained by IT is studied in [23], where an IA algorithm that minimizes the interference leakage (IL) subject to the IT constraint is proposed. The algorithm proposed in [23] follows the alternating optimization approach, and a semidefinite relaxation programm (SDP) is solved at each step. The authors also provide an extension in [24] to deal with channel uncertainties and to take the desired channel of the secondary users also into account. Coexistence issues have been also considered for other secondary networks, such as single-input single-output (SISO) or multiple access channels (MAC). See, e.g., [25] and [26] and references therein.

In this paper, we consider an underlay IC that coexists with a PPL that has a rate constraint.[1] We first show that, when the primary link transmits multiple streams, constraining the spatial structure of interference [28] is crucial for the IC to achieve good sum-rate performance. To this end, we derive transmit shaping constraints at the secondary transmitters and show that they generalize the IT constraints for single-beam secondary networks. A successive convex approximation algorithm is then proposed to obtain suitable shaping matrices and the minimum interference leakage (MinIL) and maximum signal-to-interference-plus-noise ratio (MaxSINR) algorithms [11] are extended to incorporate such constraints. An additional power control step based on gradient descent is introduced to enhance the sum-rate of the IC.

The rest of the paper is organized as follows. Section II

[1] We have performed a preliminary study in [27].

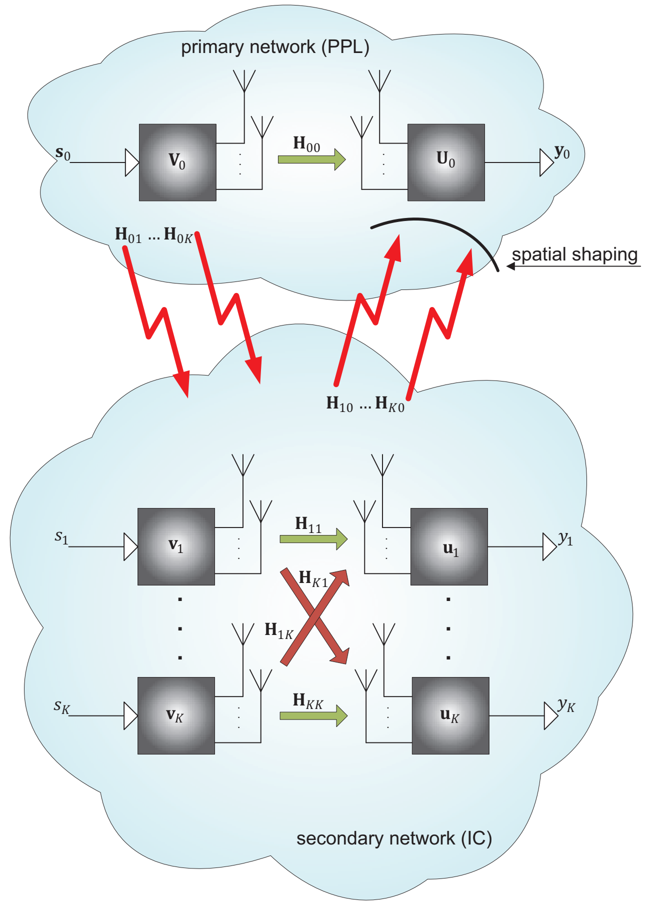


Fig. 1. Considered scenario comprised of a secondary interference channel (IC) and a primary point-to-point link (PPL). Spatial shaping is used to control the interference generated at the PPL by the secondary network.

describes the system model. The spatial shaping constraints are discussed in Section III and the proposed algorithm is presented. In Section IV, the MinIL and MaxSINR algorithms are extended to include shaping constraints, and simulation examples are provided in Section V. Finally, Section VI summarizes the concluding remarks.

II. System Model

We consider a K-user single-beam multiple-input multiple-output (MIMO) IC that coexists with a MIMO PPL, as depicted in Fig. 1. Under this setting, and denoting by N_i and M_j the number of antennas at receiver i and transmitter j, respectively $(i,j = 0,\ldots,K$, where $i,j = 0$ denotes the primary user), the signal received by each user can be expressed as

$$y_i = \mathbf{u}_i^H \left(\sum_{j=1}^{K} \mathbf{H}_{ij}\mathbf{v}_j s_j + \mathbf{H}_{00}\mathbf{V}_0\mathbf{s}_0 + \mathbf{n}_i \right) \ , \ i = 1,\ldots,K \tag{1}$$

$$\mathbf{y}_0 = \mathbf{U}_0^H \left(\mathbf{H}_{00}\mathbf{V}_0\mathbf{s}_0 + \sum_{j=1}^{K} \mathbf{H}_{0j}\mathbf{v}_j s_j + \mathbf{n}_0 \right) \ , \tag{2}$$

where $\mathbf{H}_{ij} \in \mathcal{C}^{N_i \times M_j}$ is the MIMO channel between transmitter j and receiver i, $\mathbf{U}_0 \in \mathcal{C}^{N_0 \times d_0}$ and $\mathbf{V}_0 \in \mathcal{C}^{M_0 \times d_0}$ are the decoding and precoding matrices of the primary user, respectively; $\mathbf{u}_i \in \mathcal{C}^{N_i \times 1}$ and $\mathbf{v}_i \in \mathcal{C}^{M_i \times 1}$ are the decoding and precoding vectors of the ith secondary user, respectively; $\mathbf{n}_i \in \mathcal{C}^{N_i \times 1}$ is the noise at receiver i which is assumed to be distributed as $\mathcal{CN}(0, \sigma^2\mathbf{I})$ and $\mathbf{s}_0 \in \mathcal{C}^{d_0 \times 1}$, $s_j \in \mathcal{C}$ are the symbols transmitted by the primary and the jth secondary user, respectively. We follow the notation in [29] and refer to this secondary network as $\prod_{k=1}^{K}(M_k \times N_k, 1)$.

Let us assume that the primary user is oblivious of the actual interference covariance matrix and performs the optimal transmission strategy in the absence of interference, namely, SVD and waterfilling power allocation [30][2]. Thus, its achievable rate as a function of the aggregate interference covariance matrix, $\mathbf{Q}$, is given by

$$R_{\text{PPL}}\left(\mathbf{Q}\right) = \log\det\left(\mathbf{I} + \left(\sigma^2\mathbf{I} + \mathbf{Q}\right)^{-1}\boldsymbol{\Sigma}_S\right) \ , \tag{3}$$

where $\boldsymbol{\Sigma}_S$ is a diagonal matrix. Without loss of generality, the minimum rate constraint can be therefore expressed as $R_{\text{PPL}}\left(\mathbf{Q}\right) \geq (1-\alpha)\, R_{\text{PPL}}\left(\mathbf{0}\right)$, with $\alpha \in [0,1]$.

An interesting issue that comes up at this point is related to the knowledge that each network has about the other one and how they cooperate with each other. In this work we consider that they have limited knowledge about each other and hence the required overhead is reduced as much as possible. More specifically, each secondary user only needs to know its transmit covariance constraint in order to optimize its transmit direction, whereas the primary user requires the local channels, $\mathbf{H}_{0i}$, in order to select suitable constraints.

III. Spatial Shaping Constraints

In this section, we derive shaping constraints [28] for the secondary network to ensure that the rate requirement at the primary user is satisfied. We assume that the constraints are obtained by the PPL receiver and are sent to the IC through a feedback link. Typically, IT constraints are used to control the total interference power that the secondary users generate at the primary receiver. When the PPL uses a single-beam scheme there is not much left to do. However, in the multi-stream case, how the interference is distributed among the different streams strongly affects the achievable rate of the PPL. To this end, we will constraint the transmit covariance of each secondary user as

$$\mathbf{v}_j\mathbf{v}_j^H \preceq \mathbf{S}_j \ , \ j = 1,\ldots,K \tag{4}$$

where $\preceq$ stands for the Löwner ordering, i.e., $\mathbf{A} \preceq \mathbf{B}$ means that $\mathbf{B} - \mathbf{A}$ is positive semidefinite; $\mathbf{S}_j \in \mathcal{S}_+^{M_j}$ is the spatial shaping constraint of the jth secondary transmitter and $\mathcal{S}_+^N$ is the set of $N \times N$ positive semidefinite matrices. By imposing constraints as given by (4), the transmitted signal of each secondary users is spatially restricted by limiting or even forbidding (if $\mathbf{S}_j$ is rank deficient) the signal power in some directions. The spatial shaping matrix, $\mathbf{S}_j$, must be selected such that the rate requirement at the primary user is ensured for all transmit directions satisfying (4). The following lemma

[2]Note that some streams may remain unused due to the waterfilling power allocation.

establishes the connection between spatial shaping and IT constraints.

Lemma 1: Let $\mathbf{S}_j \in \mathcal{S}_+^{M_j}$ and $\mathbf{v}_j \in \mathcal{C}^{M_j \times 1}$. Then $\mathbf{v}_j\mathbf{v}_j^H \preceq \mathbf{S}_j$ holds if and only if $\mathbf{v}_j^H\mathbf{S}_j^{-1}\mathbf{v}_j \leq 1$.

Proof: It is easy to see that $\mathbf{v}_j\mathbf{v}_j^H \preceq \mathbf{S}_j$ is equivalent to $\lambda_{\max}(\mathbf{S}_j^{-1}\mathbf{v}_j\mathbf{v}_j^H) \leq 1$, where $\lambda_{\max}(\cdot)$ denotes maximum eigenvalue. Since this matrix is rank-one, its maximum eigenvalue is given by $\mathbf{v}_j^H\mathbf{S}_j^{-1}\mathbf{v}_j$, which concludes the proof[3]. ■

Corollary 1: The IT constraint is equivalent to the spatial shaping constraint $\mathbf{v}_j\mathbf{v}_j^H \preceq \rho_j(\mathbf{H}_{0j}^H\mathbf{U}_0\mathbf{U}_0^H\mathbf{H}_{0j})^{-1}$, where ρ_j is the IT for user j.

The above corollary shows that the IT constraint is equivalent to a fixed spatial shaping, which suggests that (4) is in fact a generalization of the traditional IT constraint. Therefore, the algorithms that we propose in this paper can be applied to IT constraints by replacing $\mathbf{S}_j$ with $\rho_j(\mathbf{H}_{0j}^H\mathbf{U}_0\mathbf{U}_0^H\mathbf{H}_{0j})^{-1}$.

To obtain suitable spatial shaping matrices, $\mathbf{S}_j$, we can consider the following optimization problem

$$\begin{aligned}
\mathcal{P}_1: \quad & \underset{\{\mathbf{S}_j\}_1^K, \mathbf{Q}}{\text{maximize}} && \sum_{j=1}^{K} \mathrm{Tr}\left(\mathbf{S}_j\right) , \\
& \text{subject to} && R_{\text{PPL}}\left(\mathbf{Q}\right) \geq (1-\alpha)\, R_{\text{PPL}}\left(\mathbf{0}\right) , \qquad (5)\\
& && \sum_{j=1}^{K} \mathbf{U}_0^H\mathbf{H}_{0j}\mathbf{S}_j\mathbf{H}_{0j}^H\mathbf{U}_0 \preceq \mathbf{Q} , \\
& && \mathbf{0} \preceq \mathbf{S}_j \preceq P_j\mathbf{I} , \; j=1,\ldots,K ,
\end{aligned}$$

where P_j is the power budget of the jth transmitter. In $\mathcal{P}_1$, the total allowed transmit power of the IC is maximized subject to the rate constraint of the PPL. This problem, however, is non-convex due to (5), which makes the problem difficult to solve. In [31], an optimization framework for finding local optima of non-convex problems was proposed, based on convex approximations of the non-convex constraints. The key idea is to replace the non-convex constraints by a convex approximation at a given point, and solve the resulting convex problem. Doing this iteratively the method is shown to converge to a local optimum of the original non-convex problem. This approach, although being effective, may suffer from slow convergence depending on the complexity of the function to be approximated. In order to alleviate the computational demands of such successive convex approximation algorithms, we make a first approximation on the rate constraint (5) to reduce the number of variables and to simplify the rate function, by constraining the interference covariance matrix to be diagonal, i.e.,

$$\mathbf{Q} = \mathbf{C} \in \mathcal{D}_+^{d_0} , \qquad (6)$$

where $\mathcal{D}_+^N$ denotes the set of $N \times N$ diagonal matrices with positive elements. Therefore, the rate achieved by the primary user as a function of $\mathbf{C}$ can be expressed as a sum of logarithms, simplifying the rate constraint (5) and, consequently, easing the application of a successive convex approximation algorithm. On the other hand, when we constrain the interference covariance matrix, $\mathbf{Q}$, to be diagonal, the Löwner ordering in $\mathcal{P}_1$ can be replaced by a more convenient matrix partial ordering that expands the feasible set as much as possible. To this end, we consider the following partial ordering

$$\sum_{j=1}^{K} \mathbf{U}_0^H\mathbf{H}_{0j}\mathbf{S}_j\mathbf{H}_{0j}^H\mathbf{U}_0 \preceq_{\mathcal{D}} \mathbf{C} , \qquad (7)$$

where $\mathbf{A} \preceq_{\mathcal{D}} \mathbf{B}$ means $(\mathbf{A})_{ii} \leq (\mathbf{B})_{ii}$, for all i, with $(\mathbf{A})_{ii}$ denoting the ith element of the diagonal of $\mathbf{A}$. Note that $\preceq_{\mathcal{D}}$ establishes a partial order on $\mathcal{S}_+^N$ since it is reflexive, antisymmetric and transitive [32]. As we show in the following lemmas, the partial ordering (7) increases the size of the feasible set without affecting the rate of the primary user.

Lemma 2: Let $\mathcal{L} = \{\mathbf{Q} \in \mathcal{S}_+^{d_0} : \mathbf{Q} \preceq \mathbf{C}\}$ and $\mathcal{G} = \{\mathbf{Q} \in \mathcal{S}_+^{N_0} : \mathbf{Q} \preceq_{\mathcal{D}} \mathbf{C}\}$ for a given $\mathbf{C} \in \mathcal{D}_+^{d_0}$. Then $\mathcal{L} \subset \mathcal{G}$.

Proof: Suppose that $\mathbf{Q} \preceq \mathbf{C}$ for a given $\mathbf{Q}$. Therefore, $\mathbf{a}^H\mathbf{Q}\mathbf{a} \leq \mathbf{a}^H\mathbf{C}\mathbf{a}$, for all $\mathbf{a}$. Setting $\mathbf{a}$ an all-zero vector with a one in the ith entry, it is clear that $\mathbf{Q} \preceq_{\mathcal{D}} \mathbf{C}$ also holds, which proves that $\mathcal{L} \subseteq \mathcal{G}$. To prove that $\mathcal{L}$ is strictly a subset of $\mathcal{G}$, suppose that $\mathbf{Q} =_{\mathcal{D}} \mathbf{C}$ for a given $\mathbf{Q}$, i.e., $(\mathbf{Q})_{ii} = (\mathbf{C})_{ii}$ for all i. As the eigenvalues of any Hermitian matrix majorize its diagonal [33], it turns out that $\mathbf{Q} \not\preceq \mathbf{C}$, which concludes the proof. ■

The foregoing lemma shows that the partial ordering (7) is less strict than the Löwner ordering. Furthermore, as shown by the following lemma, it can also be used to ensure the rate requirement of the primary user.

Lemma 3: Let $\mathbf{Q} \in \mathcal{S}_+^{d_0}$ be any matrix such that $\mathbf{Q} \preceq_{\mathcal{D}} \mathbf{C}$, for a given $\mathbf{C} \in \mathcal{D}_+^{d_0}$. Then $R_{\text{PPL}}(\mathbf{Q}) \geq R_{\text{PPL}}(\mathbf{C})$, where $R_{\text{PPL}}(\cdot)$ is given by (3).

Proof: To prove the lemma, we must show that the off-diagonal elements of $\mathbf{Q}$ do not reduce the achievable rate when its diagonal is fixed. To this end, let us consider that $\mathbf{Q} =_{\mathcal{D}} \mathbf{C}$, i.e., $\mathbf{Q} = \mathbf{C} + \mathbf{\Theta}$, where $\mathbf{\Theta}$ is any off-diagonal Hermitian matrix such that $\mathbf{C} + \mathbf{\Theta} \succeq \mathbf{0}$. Notice that, if $R_{\text{PPL}}(\mathbf{Q}) \geq R_{\text{PPL}}(\mathbf{C})$ holds for all $\mathbf{\Theta}$, then $R_{\text{PPL}}(\mathbf{Q}') \geq R_{\text{PPL}}(\mathbf{C})$ for any $\mathbf{Q}' \preceq_{\mathcal{D}} \mathbf{Q}$. The lemma is therefore proved if the following holds

$$\det\left(\mathbf{I} + \left(\bar{\mathbf{C}} + \mathbf{\Theta}\right)^{-1}\mathbf{\Sigma}_S\right) \geq \det\left(\mathbf{I} + \bar{\mathbf{C}}^{-1}\mathbf{\Sigma}_S\right) . \qquad (8)$$

Applying the determinant identities $\det(\mathbf{AB}) = \det(\mathbf{A})\det(\mathbf{B})$, for any squared matrices $\mathbf{A}$ and $\mathbf{B}$, and $\det(\mathbf{A}^{-1}) = 1/\det(\mathbf{A})$; the foregoing expression can be equivalently given by

$$\frac{\det\left(\bar{\mathbf{C}} + \mathbf{\Sigma}_S + \mathbf{\Theta}\right)}{\det\left(\bar{\mathbf{C}} + \mathbf{\Sigma}_S\right)} \geq \frac{\det\left(\bar{\mathbf{C}} + \mathbf{\Theta}\right)}{\det\left(\bar{\mathbf{C}}\right)} . \qquad (9)$$

As $\det(\mathbf{A}) \leq \prod_i (\mathbf{A})_{ii}$, with equality only when $\mathbf{A}$ is diagonal, and $\mathbf{\Sigma}_S$ is a diagonal matrix with positive entries, (9) holds for any $\mathbf{\Theta}$, which concludes the proof. ■

Finally, using Lemma 2, it is easy to see that the transmit covariances satisfying (4) also satisfy (7) for all $\mathbf{S}_j$ such that $\sum_{j=1}^{K} \mathbf{U}_0^H\mathbf{H}_{0j}\mathbf{S}_j\mathbf{H}_{0j}^H\mathbf{U}_0 \preceq_{\mathcal{D}} \mathbf{C}$.

[3]Notice that if $\mathbf{S}_j$ is rank deficient, there are some transmit direction in which user j is not allowed to transmit. In this case we have $\mathbf{g}_j^H\mathbf{\Sigma}_j^{-1}\mathbf{g}_j \leq 1$, where $\mathbf{\Sigma}_j$ is a diagonal matrix containing the non-null eigenvalues of $\mathbf{S}_j$ and $\mathbf{v}_j = \mathbf{F}_j\mathbf{g}_j$, with $\mathbf{F}_j$ being the eigenvectors of $\mathbf{S}_j$ associated to the non-null eigenvalues.

Set $f^{\star}_{\tilde{\mathcal{P}}_1^0} = 0$, $k = 0$ and a tolerance, ϵ; where $f^{\star}_{\tilde{\mathcal{P}}_1^k}$ denotes the optimal value of $\tilde{\mathcal{P}}_1^k$.
Choose an initial point $\mathbf{C}^0 = \mathbf{C}_{\text{init}}$.
repeat
1) $k = k + 1$.
2) Update the power allocation at the PPL by water-filling, taking $\mathbf{C}^{k-1}$ into account.
3) Replace (10) by its first order approximation at $\mathbf{C}^{k-1}$ to obtain $\tilde{\mathcal{P}}_1^k$, and solve this convex problem.

until $f^{\star}_{\tilde{\mathcal{P}}_1^k} - f^{\star}_{\tilde{\mathcal{P}}_1^{k-1}} \leq \epsilon$.

Algorithm 1: Successive convex approximation algorithm for finding local optima of $\tilde{\mathcal{P}}_1$.

Finally, using (6) and (7), $\mathcal{P}_1$ can be approximated by

$$\begin{aligned} \tilde{\mathcal{P}}_1: \quad & \underset{\{\mathbf{S}_j\}_1^K, \mathbf{C}}{\text{maximize}} && \sum_{j=1}^{K} \text{Tr}\left(\mathbf{S}_j\right), \\ & \text{subject to} && R_{\text{PPL}}\left(\mathbf{C}\right) \geq (1-\alpha) R_{\text{PPL}}\left(\mathbf{0}\right), \qquad (10) \\ & && \sum_{j=1}^{K} \mathbf{U}_0^H \mathbf{H}_{0j} \mathbf{S}_j \mathbf{H}_{0j}^H \mathbf{U}_0 \preceq_{\mathcal{D}} \mathbf{C}, \\ & && \mathbf{0} \preceq \mathbf{S}_j \preceq P_j \mathbf{I}, \; j = 1, \ldots, K. \end{aligned}$$

Although $\tilde{\mathcal{P}}_1$ is still a non-convex optimization problem, the first approximation of the rate constraint (10) facilitates the application of successive convex approximation methods. This is due to the fact that, as we have already pointed out, the rate function in (10) can be expressed as a sum of logarithms and depends on less variables than (5) (recall that $\mathbf{C}$ is a diagonal matrix), leading to less computational complexity and a less number of local optima. Following the optimization framework in [31], the rate constraint (10) is replaced by its first-order approximation at a given point, resulting in a convex problem. Doing this iteratively yields a sequence of convex approximations of $\tilde{\mathcal{P}}_1$, $\{\tilde{\mathcal{P}}_1^k\}$, that can be solved efficiently using standard numerical methods. The proposed successive convex approximation algorithm is described in Algorithm 1. Note that we have included a waterfilling step in Algorithm 1 to optimize the worst-case rate of the PPL, thus increasing the total allowed transmit power of the IC.

To obtain a good initial point for Algorithm 1, $\mathbf{C}_{\text{init}}$, we propose the following optimization problem

$$\begin{aligned} \mathcal{P}_2: \quad & \underset{\mathbf{C}_{\text{init}}}{\text{maximize}} && \text{Tr}\left(\mathbf{C}_{\text{init}}\right), \\ & \text{subject to} && R_{\text{PPL}}\left(\mathbf{C}_{\text{init}}\right) \geq (1-\alpha) R_{\text{PPL}}\left(\mathbf{0}\right), \\ & && \mathbf{0} \preceq \mathbf{C}_{\text{init}} \preceq \mathbf{C}_{\max}, \end{aligned}$$

where $(\mathbf{C}_{\max})_{ii} = (\sum_{j=1}^{K} P_j \mathbf{U}_0^H \mathbf{H}_{0j} \mathbf{H}_{0j}^H \mathbf{U}_0)_{ii}$ and zeros elsewhere. In $\mathcal{P}_2$, the allowed interference power at the PPL is maximized subject to the minimum rate constraint and an additional constraint that bounds the maximum allowed interference level at each stream to the worst case, which is represented by each entry of $\mathbf{C}_{\max}$. This may occur if the transmit directions are aligned to the channel eigenmodes from the secondary transmitters to the primary receiver. To solve this non-convex problem, we use the ensuing lemma.

Lemma 4: Let us denote by $\mathbf{C}^{\star}_{\text{init}} = \text{diag}(c_1^{\star}, \ldots, c_{N_0}^{\star})$ the optimal solution of $\mathcal{P}_2$, where $c_j^{\star}$ is associated to the jth weakest mode of $\mathbf{H}_{00}$. Then, the following holds

$$c_j^{\star} < \left(\mathbf{C}_{\max}\right)_{jj} \;\Rightarrow\; c_{j+1}^{\star} = 0, \; j = 1, \ldots, N_0 - 1. \qquad (11)$$

Proof: As $\mathbf{C}_{\text{init}}$ is diagonal, we have $R_{\text{PPL}}(\mathbf{C}_{\text{init}}) = \sum_j \log[1 + (\mathbf{\Sigma}_S)_{jj}/(\sigma^2 + c_j)]$. The derivative of $R_{\text{PPL}}(\mathbf{C}_{\text{init}})$ with respect to c_j, $\nabla_{c_j} R_{\text{PPL}}$, is monotone decreasing and $\nabla_{c_j} R_{\text{PPL}} \leq \nabla_{c_{j+1}} R_{\text{PPL}}$ for $c_j = c_{j+1}$, $j = 1, \ldots, N_0 - 1$, i.e., the weaker the mode, the more interference power it tolerates to meet a given data rate. As the interference level is limited by $\mathbf{C}_{\max}$, we obtain (11), which concludes the proof. ■

The above lemma allows us to find the optimal solution of $\mathcal{P}_2$ stream-wise and with a closed-form expression, as detailed in Algorithm 2.

Set $\mathbf{C} = \text{diag}(c_1, \ldots, c_{N_0}) = \mathbf{0}$ and $j = 1$.
while $j \leq N_0$ **and** $R_{\text{PPL}}(\mathbf{C}) > (1-\alpha) R_{\text{PPL}}(\mathbf{0})$ **do**
$\Delta R = R^j_{\text{PPL}}(c_j) - (R_{\text{PPL}}(\mathbf{C}) - (1-\alpha) R_{\text{PPL}}(\mathbf{0}))$.
if $\Delta R \leq 0$ **then**
$c_j = (\mathbf{C}_{\max})_{jj}$.
else
$c_j = \min\left(\frac{(\mathbf{\Sigma}_S)_{jj}}{2^{\Delta R} - 1} - \sigma^2, (\mathbf{C}_{\max})_{jj}\right)$.
end if
j=j+1.
end while
where $R^j_{\text{PPL}}(a)$ is the rate achieved by the PPL at mode j when it experiences an interference power of a.

Algorithm 2: Algorithm for solving $\mathcal{P}_2$ and find a good initial point for Algorithm 1.

IV. Precoding design under spatial shaping

In this section, we propose two different precoding designs that consider the spatial shaping constraint (4). More specifically, we extend the well-known MinIL and MaxSINR algorithms to obtain precoders satisfying (4), and therefore guaranteeing the rate of the primary user. Recall that, by Corollary 1, the IT constraint can also be considered as a particular case of (4) in the proposed algorithms. Notice that, in our model, the additional constraint affects only the design of the precoders, whereas the decoders are optimized exactly as in the original algorithm[4]. Hence, we consider in the following the design of the transmit directions, $\mathbf{v}_j$.

A. MinIL algorithm

At each step of the MinIL, the precoders (decoders) are optimize subject to norm constraints, while the decoders (precoders) are fixed, so that the IL is successively minimized [11]. Therefore, for fixed decoders, the optimal precoder of the jth transmitter is obtained by solving the following optimization problem

$$\begin{aligned} \mathcal{P}_3: \quad & \underset{\mathbf{v}_j}{\text{minimize}} && \mathbf{v}_j^H \mathbf{R}_j \mathbf{v}_j, \\ & \text{subject to} && \mathbf{v}_j^H \mathbf{v}_j = \lambda_{\max}(\mathbf{S}_j), \\ & && \mathbf{v}_j^H \mathbf{S}_j^{-1} \mathbf{v}_j \leq 1, \qquad (12) \end{aligned}$$

[4]The additional interference from the PPL transmitter can be included in the interference covariance matrix when computing the decoders.

where $\mathbf{R}_j = \sum_{i \neq j}^{K} \mathbf{H}_{ij}^H \mathbf{u}_i \mathbf{u}_i^H \mathbf{H}_{ij}$. Recall that (12) is equivalent to (4) due to Lemma 1. The optimal solution of $\mathcal{P}_3$ is formalized in the following lemma.

Lemma 5: The optimal solution of $\mathcal{P}_3$ is given by

$$\mathbf{v}_j^\star = \sqrt{\lambda_{\max}(\mathbf{S}_j)}\nu_{\min}\left[(1-\mu_j)\,\mathbf{R}_j + \mu_j \mathbf{S}_j^{-1}\right] , \quad (13)$$

where $\nu_{\min}(\cdot)$ denotes eigenvector with minimum eigenvalue and $\mu_j \in [0,1]$.

Proof: The Lagrangian of $\mathcal{P}_3$ is given by

$$\begin{aligned}\mathscr{L}(\mathbf{v}_j, \tilde{\mu}_j, \eta_j) =& \mathbf{v}_j^H \mathbf{R}_j \mathbf{v}_j + \tilde{\mu}_j \left(\mathbf{v}_j^H \mathbf{S}_j^{-1} \mathbf{v}_j - 1\right) + \\ & \eta_j \left[\lambda_{\max}(\mathbf{S}_j) - \mathbf{v}_j^H \mathbf{v}_j\right] , \end{aligned} \quad (14)$$

where $\eta_j, \tilde{\mu}_j \geq 0$ are the Lagrange multipliers of the first and second constraints of $\mathcal{P}_3$, respectively. Evaluating the derivative of the Lagrangian with respect to $\mathbf{v}_j^*$ we obtain

$$\nabla_{\mathbf{v}_j^*} \mathscr{L}(\mathbf{v}_j, \tilde{\mu}_j, \eta_j) = 0 \Rightarrow \left(\mathbf{R}_j + \tilde{\mu}_j \mathbf{S}_j^{-1}\right)\mathbf{v}_j = \eta_j \mathbf{v}_j . \quad (15)$$

The above expression is an eigenvalue problem, whose solution satisfying the power constraint is given by (13), where $\mu_j = \frac{\tilde{\mu}_j}{1+\tilde{\mu}_j}$. ■

Since $\mu_j \in [0,1]$ and $\mathbf{v}_j^H \mathbf{S}_j^{-1} \mathbf{v}_j$ decreases monotonically as μ_j increases, its value can be easily obtained using bisection, such that (12) is satisfied with equality (if active).

B. MaxSINR algorithm

The MaxSINR algorithm follows the same alternating optimization approach as the MinIL [11]. Therefore, for fixed decoders, the optimal precoder of the jth transmitter is obtained by solving the following optimization problem

$$\begin{aligned} \mathcal{P}_4: \quad & \underset{\mathbf{v}_j}{\text{maximize}} && \frac{\mathbf{v}_j^H \mathbf{H}_{jj}^H \mathbf{u}_j \mathbf{u}_j^H \mathbf{H}_{jj} \mathbf{v}_j}{\mathbf{v}_j^H \mathbf{R}_j \mathbf{v}_j + \bar{\sigma}^2} , \\ & \text{subject to} && \mathbf{v}_j^H \mathbf{S}_j^{-1} \mathbf{v}_j \leq 1 , \end{aligned} \quad (16)$$

where $\bar{\sigma}^2 = \sigma^2 + \|\mathbf{u}_j^H \mathbf{H}_{j0} \mathbf{V}_0\|^2$ and the power constraint is implicit in (16) (notice that, with the MaxSINR criterion, the power constraint can be relaxed to an inequality). The optimal solution of $\mathcal{P}_4$ is formalized in the following lemma.

Lemma 6: The optimal solution of $\mathcal{P}_4$ is given by

$$\bar{\mathbf{v}}_j = \nu_{\max}\left[\mathbf{S}_j \left(\mathbf{H}_{jj}^H \mathbf{u}_j \mathbf{u}_j^H \mathbf{H}_{jj} - \gamma^\star \mathbf{R}_j\right)\right] , \quad (17)$$

$$\mathbf{v}_j^\star = \left[\frac{\bar{\sigma}^2 \gamma^\star}{\bar{\mathbf{v}}_j^H \left(\mathbf{H}_{jj}^H \mathbf{u}_j \mathbf{u}_j^H \mathbf{H}_{jj} - \gamma^\star \mathbf{R}_j\right)\bar{\mathbf{v}}_j}\right] \bar{\mathbf{v}}_j , \quad (18)$$

where $\nu_{\max}(\cdot)$ denotes eigenvector with maximum eigenvalue and $\gamma^\star$ is the optimal SINR.

Proof: $\mathcal{P}_4$ can be equivalently expressed as

$$\begin{aligned} & \underset{\mathbf{v}_j, \gamma}{\text{maximize}} && \gamma , \\ & \text{subject to} && \frac{\mathbf{v}_j^H \mathbf{H}_{jj}^H \mathbf{u}_j \mathbf{u}_j^H \mathbf{H}_{jj} \mathbf{v}_j}{\mathbf{v}_j^H \mathbf{R}_j \mathbf{v}_j + \bar{\sigma}^2} \geq \gamma , \\ & && \mathbf{v}_j^H \mathbf{S}_j^{-1} \mathbf{v}_j \leq 1 . \end{aligned} \quad (19)$$

repeat
1) Perform one step of the MinIL or MaxSINR algorithm.
2) Perform the power control to obtain the scaling factors, ϕ_j, for $j = 1, \ldots, K$.
3) Update the maximum transmit power of each secondary user by setting $\mathbf{S}_j = \phi_j \mathbf{S}_j$, for $j = 1, \ldots, K$.

until Stopping criterion is met

Algorithm 3: MinIL and MaxSINR algorithms with power control.

For a fixed value of γ, (19) can be written as a minimization problem with a SINR constraint as follows

$$\begin{aligned} & \underset{\mathbf{v}_j}{\text{minimize}} && \mathbf{v}_j^H \mathbf{S}_j^{-1} \mathbf{v}_j , \\ & \text{subject to} && \frac{\mathbf{v}_j^H \mathbf{H}_{jj}^H \mathbf{u}_j \mathbf{u}_j^H \mathbf{H}_{jj} \mathbf{v}_j}{\mathbf{v}_j^H \mathbf{R}_j \mathbf{v}_j + \bar{\sigma}^2} \geq \gamma . \end{aligned} \quad (20)$$

After some manipulations in the last constraint of the foregoing problem, the Lagrangian of (20) can be written as

$$\begin{aligned}\mathscr{G}(\mathbf{v}_j, \beta_j) =& \mathbf{v}_j^H \mathbf{S}_j^{-1} \mathbf{v}_j + \\ & \beta_j \left[\bar{\sigma}^2 \gamma - \mathbf{v}_j^H \left(\mathbf{H}_{jj}^H \mathbf{u}_j \mathbf{u}_j^H \mathbf{H}_{jj} - \gamma \mathbf{R}_j\right) \mathbf{v}_j^H\right] , \end{aligned} \quad (21)$$

where $\beta_j \geq 0$ is the Lagrange multiplier associated to the last constraint of (20). Equating the derivative of the Lagrangian with respect to $\mathbf{v}_j^*$ to zero yields

$$\nabla_{\mathbf{v}_j^*} \mathscr{G}(\mathbf{v}_j, \beta_j) = 0 \Rightarrow \mathbf{S}_j \left(\mathbf{H}_{jj}^H \mathbf{u}_j \mathbf{u}_j^H \mathbf{H}_{jj} - \gamma \mathbf{R}_j\right) \mathbf{v}_j = \frac{1}{\beta_j} \mathbf{v}_j . \quad (22)$$

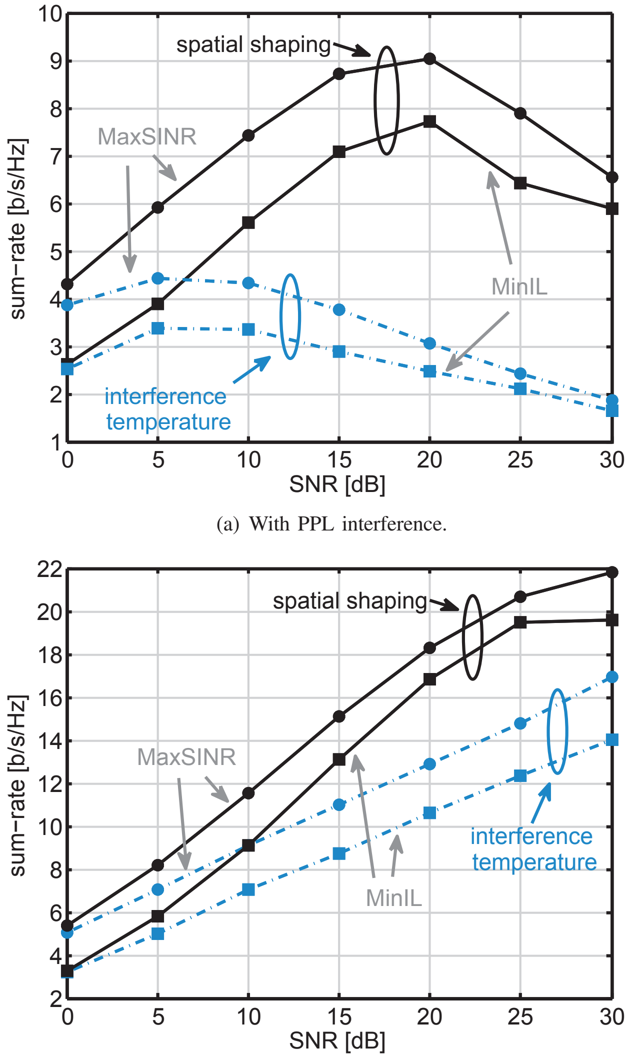
The above expression is an eigenvalue problem, whose solution achieving the optimal SINR is given by (17) and (18). ■

The optimal SINR, $\gamma^\star$, can be easily obtained using bisection as follows. First, it is easy to see that the optimal objective value of problem (20) increases monotonically with γ, which make it possible to apply a bisection method to obtain $\gamma^\star$. Second, γ is bounded above and below as $\gamma \in [0, \gamma^U]$, where $\gamma^U = \lambda_{\max}(\mathbf{H}_{jj}^H \mathbf{u}_j \mathbf{u}_j^H \mathbf{H}_{jj}, \mathbf{R}_j + \bar{\sigma}^2 \mathbf{I})$, with $\lambda_{\max}(\mathbf{A}, \mathbf{B})$ being the maximum generalized eigenvalue of the matrix pencil $(\mathbf{A}, \mathbf{B})$.

C. Power control step

Even when the IA problem is feasible, the IC may achieve a low sum-rate performance due to the shaping constraint. In these cases, optimizing the transmit power of each secondary user may play an important role to increase the sum-rate of the IC. To this end, we propose a simple centralized power control based on applying a gradient method at each step of the alternating optimization algorithm. This method optimizes the sum-rate of the secondary network as a function of the transmit powers by means of a gradient descent procedure. To this end, we express the sum-rate achieved by the IC as

$$\mathrm{SR}_{\mathrm{IC}}\left(\{\phi_j\}_1^K\right) = \sum_{i=1}^{K} \log\left(1 + \frac{\zeta_{ii}\phi_i}{\sigma^2 + \sum_{j \neq i} \zeta_{ij}\phi_j}\right) , \quad (23)$$



(a) With PPL interference.

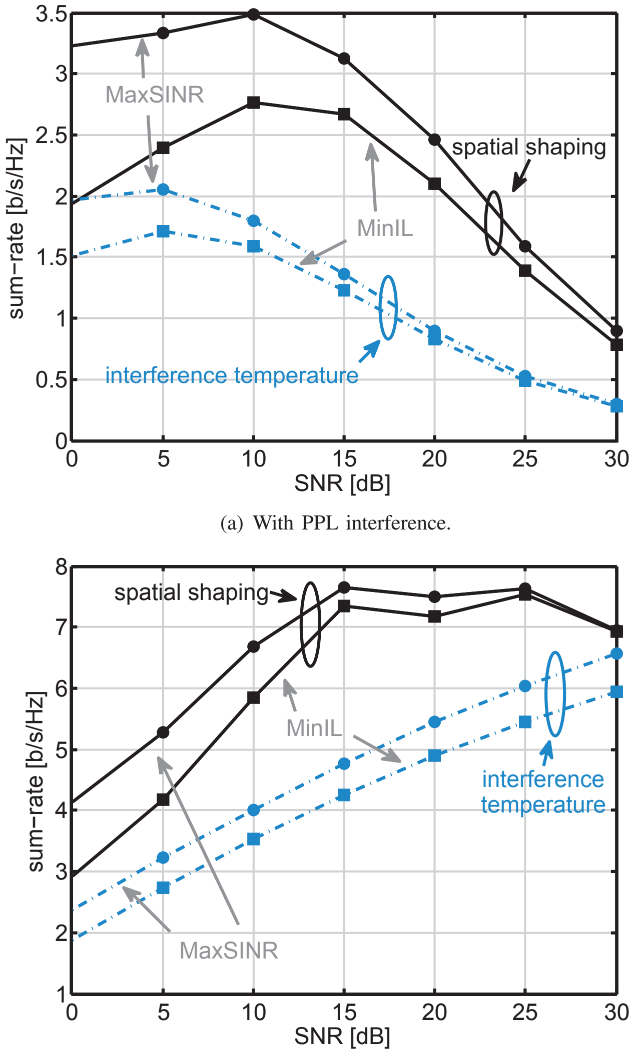
(b) Without PPL interference.

Fig. 2. Achievable sum-rate of the $(3\times 2,1)^3$ IC with a 3×3 PPL for spatial shaping and IT constraints, and $\alpha=0.5$. Situations with (a) and without (b) interference coming from the PPL are depicted.

where $\zeta_{ij} = |\mathbf{u}_i^H\mathbf{H}_{ij}\mathbf{v}_j|^2$ and $0 \leq \phi_j\|\mathbf{v}_j\|^2 \leq P_j$. The gradient of (23) with respect to the scaling factors of the transmit powers, ϕ_j, is given at the bottom of this page. The resulting MinIL and MaxSINR algorithms with power control are summarized in Algorithm 3.

V. Numerical Results

In this section we provide several numerical examples that illustrate the performance improvements that can be achieved by the proposed spatial shaping constraints in comparison to the traditional IT. In all the examples we consider a $(3\times 2,1)^3$ IC and a 3×3 PPL, and define the signal-to-noise ratio as $\mathrm{SNR} = 10\log_{10}(1/\sigma^2)$, i.e., we consider the same SNR for both primary and secondary networks. The entries of the channel matrices are i.i.d. complex Gaussian random variables with zero mean and unit variance. All results are averaged over 100 different channel realizations.



(a) With PPL interference.

(b) Without PPL interference.

Fig. 3. Achievable sum-rate of the $(3\times 2,1)^3$ IC with a 3×3 PPL for spatial shaping and IT constraints, and $\alpha=0.1$. Situations with (a) and without (b) interference coming from the PPL are depicted.

Fig. 2 and Fig. 3 show the achievable sum-rate of the IC for $\alpha=0.5$ and $\alpha=0.1$, respectively, and for both interference constraints (spatial shaping and IT). Two different cases are depicted in the figures: with interference from the PPL (a) and without interference from the PPL (b). The latter may

$$\nabla_{\phi_j}\mathrm{SR}_{\mathrm{IC}} = \frac{\zeta_{jj}}{\ln 2\left(\sigma^2+\sum\limits_{i\neq j}\zeta_{ji}\phi_i\right)\left[1+\left(\sigma^2+\sum\limits_{i\neq j}\zeta_{ji}\phi_i\right)^{-1}\zeta_{jj}\phi_j\right]} - \sum_{i\neq j}\frac{\zeta_{ij}\zeta_{ii}\phi_i}{\ln 2\left(\sigma^2+\sum\limits_{k\neq i}\zeta_{ik}\phi_k\right)^2\left[1+\left(\sigma^2+\sum\limits_{k\neq i}\zeta_{ik}\phi_k\right)^{-1}\zeta_{ii}\phi_i\right]}, \tag{24}$$

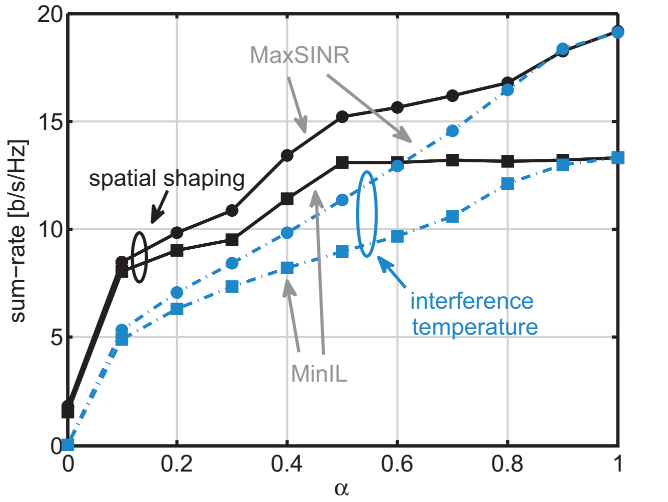


Fig. 4. Achievable sum-rate versus α for a $(3\times2,1)^3$ IC with a 3×3 PPL, for spatial shaping and IT constraints, and SNR $= 15\,$dB. In this example, the PPL does not interfere the IC.

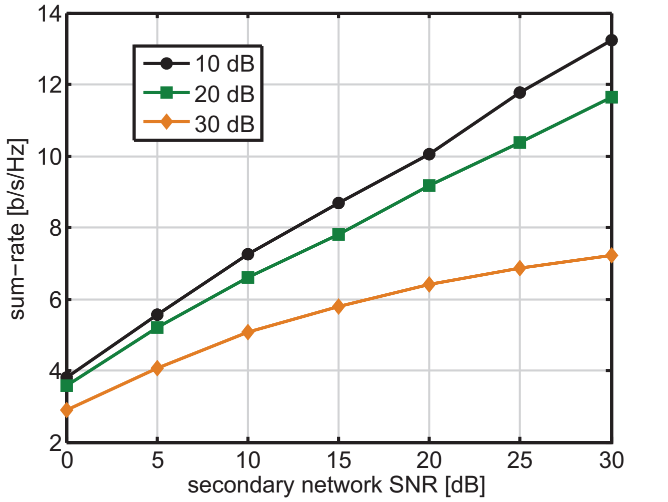


Fig. 5. Achievable sum-rate of the $(3\times2,1)^3$ IC for the MaxSINR algorithm as a function of its SNR, with a 3×3 PPL whose SNR is kept fixed at $10\,$dB, $20\,$dB and $30\,$dB, respectively; and $\alpha = 0.5$.

occur in scenarios where the primary transmitter is deployed far from the secondary network. While with both constraints the rate of the PPL is guaranteed, the spatial shaping constraint allows the IC to achieve much higher data rates than the IT thanks to controlling the spatial interference distribution at the primary receiver. We also observe that the MaxSINR algorithm provides significantly better results than MinIL in the whole SNR regime. When comparing situations with and without interference coming from the PPL, we observe that in the latter case the sum-rate performance is monotone decreasing from some SNR onwards. This is explained as follows: when the SNR increases, the effective noise tends to be dominated by the interference generated by the primary transmitter, thus avoiding the secondary network to improve any further. This effect, along with the fact that both interference constraints become more stringent as the SNR increases, explain the result observed in the Fig. 2(a) and 3(a).

Now we set SNR $= 15\,$dB and show the sum-rate of the IC as a function of α in Fig. 4. In this example, we assume that the PPL does not interfere the secondary network. The same relationship between the algorithms and both interference constraints as in the previous examples is observed. Also, IT and spatial shaping constraints attain practically the same performance as α approaches 1. This is reasonable since, when $\alpha = 1$, the IC is not constrained by the PPL. Alternatively, we consider now different SNR for the primary and secondary networks, and plot in Fig. 5 the achievable sum-rate of the IC, for the MaxSINR algorithm, as a function of its SNR when the SNR of the primary user is kept fixed, and $\alpha = 0.5$. The figure shows that, when the primary user experiences high SNR, the sum-rate of the secondary network decreases, which is owing to the fact that the waterfilling power allocation at the PPL tends to assign the power uniformly among the streams, thus resulting in a more stringent spatial shaping constraint.

VI. Conclusion

In this paper we have studied network coexistence between an IC and a PPL in the context of CR. We have shown that controlling the spatial structure of the interference is critical in order to provide high sum-rate to the IC, while ensuring the rate requirement at the PPL. We have observed that the spatial shaping constraint generalizes the IT in single-beam secondary networks, providing it with more flexibility to design the transmit directions. A successive convex approximation algorithm has been proposed to obtain the shaping matrices and we have then extended the MinIL and MaxSINR algorithms to incorporate such constraints. An additional power control step has been included to enhance the sum-rate of the IC. We have shown through different numerical examples the importance of controlling the spatial structure of the interference when the PPL transmits multiple streams.

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