An Interference Alignment Algorithm for Structured Channels

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***Abstract*—In this paper we propose a new interference alignment (IA) algorithm specifically designed to work with structured channels (e.g., diagonal or block-diagonal). Multiple-input multiple-output (MIMO) structured channels arise when symbol extensions -either in time or frequency- are employed jointly with the spatial dimension in the design of the precoders. In this case, the rank constraint in the direct channels must explicitly be taken into account into the optimization problem to ensure that there is no degrees-of-freedom (DoF) loss. To this end, we propose an algorithm that minimizes the interference leakage while ensuring that the direct links are full rank and the transmitters satisfy a power constraint. The algorithm is based upon an alternating optimization procedure, which solves a generalized eigenvalue problem at each step. We show through simulations the advantages of the proposed algorithm in several scenarios that use symbols extensions or improper (a.k.a. asymmetric) signalling.**

***Keywords*—Interference alignment, interference channel, degrees-of-freedom, generalized eigenvalue problem.**

I. Introduction

Interference alignment (IA) is a key technique to achieve the degrees-of-freedom (DoF) of interference networks [1], [2]. With IA, the available space-time-frequency dimensions are utilized in such a way that the interference at every receiver is confined into a reduced-dimensional subspace, leaving the other dimensions free of interference. Since the majority of interference channels do not admit closed-form alignment solutions, it is usually necessary to resort to iterative algorithms such as those proposed in [3]–[5]. These algorithms minimize, in slightly different forms, the interference leakage[1], and are specially suited when the alignment is performed over the spatial dimension only. The multiple-input multiple-output (MIMO) channels for spatial IA techniques are typically assumed to be generic with entries drawn from a continuous distribution. Under this assumption, constraining the precoders and decoders to be full column rank matrices (e.g., unitary) ensures that the equivalent direct channels after applying the precoders and decoders will also be full rank almost surely. Minimizing the interference leakage without paying attention to the direct links is then sufficient to achieve perfect alignment with generic MIMO channels. However, when symbol extensions are applied, the block-diagonal structure induced in the channel matrices makes this property no longer true, and therefore these algorithms may converge to rank-deficient solutions.

[1]Algorithms that minimize other cost functions (e.g., mean-squared error) have also been proposed in the literature [6]–[9], but they do not attain in general perfect alignment solutions and hence are not useful to elucidate whether an alignment problem is feasible or not.

To overcome this problem, other IA algorithms have recently been proposed that consider the rank of the direct channels into the optimization problem. In [10] the nuclear norm of the interference subspace is minimized subject to a constraint in the minimum eigenvalue of the direct channels, in an attempt to preserve the dimension of the desired signal space while reducing the dimensionality of the interference subspace. However, probably due to the convex relaxation and other heuristics in the method, it fails to provide all available DoF for many feasible scenarios. Alternatively, the algorithm proposed in [11] minimizes the total interference leakage while constraining the direct channels to be the identity matrix. The whole procedure consists of an alternating optimization procedure that has closed-form solution at each step. However, the proposed solution depends on the inverse of the interference covariance matrices, which become rank-deficient as the algorithm converges. Therefore, it may experience severe numerical errors when perfect alignment solutions are computed.

As an alternative to these methods, in this paper we propose an algorithm that minimizes the interference leakage subject to a rank constraint on the direct channel (expressed as a minimum eigenvalue constraint), and a constraint on the transmitted power. We use an alternating optimization procedure which, at each step, finds the optimal precoders and decoders as the solution of a generalized eigenvalue problem. The computational cost is therefore similar to the original method for generic channels [3]. Likewise to other alternating minimization or coordinate descent procedures, there is no guarantee of global convergence. However, simulation results indicate that the proposed method provides better results than [4], [10]. The remainder of the paper is organized as follows: Section II describes the system model. The proposed algorithm is derived in Section III. In Section IV we provide some numerical examples to illustrate its performance. Finally, Section V concludes the paper.

II. System Model and Previous Work

Let us consider a multiple-input multiple-output (MIMO) interference channel (IC) comprised of K transmitter-receiver pairs that uses symbol extensions for the alignment. The ith transmitter and receiver are equipped with M_i and N_i antennas, respectively, each user wishes to send d_i data streams and the number of symbols extensions is set to L. Symbol extensions over time or frequency dimensions are necessary to achieve IA when no spatial dimensions are available (single-input single-output -SISO- systems) and may increase the achievable DoF of MIMO-ICs [1]. We denote this general IC as $[\prod_{k=1}^{K}(M_i \times N_i, d_i), L]$, modifying the notation in [12] to include the symbol extensions. The channel matrix from transmitter j to receiver i at the symbol extension n is denoted as $\mathcal{H}_{ij}[n]$, and the overall $LN_i \times LM_i$ channel matrix has the following block-diagonal structure

$$\mathbf{H}_{ij} = \begin{pmatrix} \mathcal{H}_{ij}[1] & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathcal{H}_{ij}[2] & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathcal{H}_{ij}[L] \end{pmatrix}, \tag{1}$$

where $\mathbf{0}$ denotes a zero matrix of the appropriate dimensions. Notice that in the case of constant channels, the blocks along the main diagonal are identical.

The signal received by user i is given by

$$\mathbf{y}_i = \sum_{j=1}^{K} \mathbf{H}_{ij}\mathbf{V}_j\mathbf{s}_j + \mathbf{r}_i \ , \tag{2}$$

where $\mathbf{V}_j \in \mathbb{C}^{M_j L \times d_j}$ is the precoding matrix of transmitter j, $\mathbf{s}_j \in \mathbb{C}^{d_j}$ is the vector of symbols transmitted by user j and $\mathbf{r}_i \in \mathbb{C}^{N_i L}$ is the additive white Gaussian noise with zero mean and variance σ^2. At the receiver side, we apply a decoding matrix, $\mathbf{U}_i \in \mathbb{C}^{N_i L \times d_i}$, to the received signal (2) yielding

$$\mathbf{z}_i = \underbrace{\mathbf{U}_i^H \mathbf{H}_{ii}\mathbf{V}_i\mathbf{s}_i}_{\text{desired signal}} + \underbrace{\sum_{j \neq i} \mathbf{U}_i^H \mathbf{H}_{ij}\mathbf{V}_j\mathbf{s}_j + \mathbf{U}_i^H \mathbf{r}_i}_{\text{interference+noise}} \ . \tag{3}$$

The interference alignment problem is to design the decoders and precoders in such a way that the interfering signals at each receiver fall into a reduced-dimensional subspace. The receivers can then extract the projection of the desired signal that lies in the interference-free subspace. To this end, it is required that the polynomial equations

$$\mathbf{U}_i^H \mathbf{H}_{ij}\mathbf{V}_j = \mathbf{0}, \quad i \neq j \tag{4}$$

are satisfied, while the signal subspace for each user must be linearly independent of the interference subspace and must have dimension d_i, that is

$$\text{rank}\left(\mathbf{U}_i^H \mathbf{H}_{ii}\mathbf{V}_i\right) = d_i, \quad \forall i \ . \tag{5}$$

When no symbol extensions are applied and the MIMO channels $\mathcal{H}_{ij}$ are generic (which happens for instance when their entries are independently drawn from a continuous distribution), condition (5) is satisfied with probability one if the precoders and decoders are full rank. In other words, with generic channels there is no need to explicitly introduce the rank constraint into the optimization problem. That is why, if the problem is feasible, the well-known alternating minimization algorithm [3], [4], is able to find an alignment solution by simply minimizing the interference leakage function:

$$\underset{\{\mathbf{U}_i\}\{\mathbf{V}_j\}}{\text{minimize}} \ \sum_i \sum_{i \neq j} \left\| \mathbf{U}_i^H \mathbf{H}_{ij}\mathbf{V}_j \right\|_F \ . \tag{6}$$

where $\|\cdot\|_F$ denotes Frobenius norm. However, when IA is also performed over symbol extensions in time or frequency, the resulting block-diagonal structure in the channel matrices, see Eq. (1), makes this property no longer true. In such scenarios, algorithms that minimize interference leakage without explicitly taking (5) into account, may yield rank-deficient direct channels.

Several worthy attempts have recently been made to avoid this problem and hence develop a fully general IA algorithm. In [11], the authors propose to constrain the direct links to be the identity matrix, i.e., $\mathbf{U}_i^H \mathbf{H}_{ii}\mathbf{V}_i = \mathbf{I}$, using again the interference leakage as the cost function to be minimized. The problem is also solved by alternating the optimization of the precoders and decoders. However, the solutions at each step depend on the inverse of the interference plus noise covariance matrix. When IA solutions are wished to be computed, i.e., the noise term is set to zero, the interference covariance matrix becomes rank deficient as the algorithm proceeds, which may cause severe numerical issues. Further, the norms of the precoders and decoders are not constrained in the optimization problem. It might happen that after a normalization step, which is necessary in practice to satisfy the maximum transmitted power budget, the minimum eigenvalues of the direct channels would end up with a too low value, thus yielding a DoF loss in practice. Another method relevant for the present discussion is described in [10]. This work minimizes the nuclear norm (a convex relaxation for the rank) of the interference matrices subject to the constraint that the minimum eigenvalue of the direct channels is equal to or greater than a given parameter. This method works satisfactorily in some simple scenarios, but in others a DoF loss is observed as pointed out in [10]. Moreover, no norm constraints are considered in the optimization, and therefore the rank of the direct channels is again not guaranteed after the normalization step.

III. Proposed Algorithm

In this section, we propose an algorithm that avoids the main drawbacks of the methods in [10] and [11]. Our proposal minimizes the interference leakage as in [3], but introduces two additional constraints. First, the minimum singular value of $\mathbf{U}_i^H \mathbf{H}_{ii}\mathbf{V}_i$ for all direct links must be greater than or equal to a given parameter, $\sqrt{\epsilon}$. This ensures the desired rank for the signal subspaces. Second, the Frobenius norm of the precoders and decoders must be smaller than or equal to the available power budget (fixed to one without loss of generality). This avoids the need of any final normalization step that might result in a violation of the minimum eigenvalue constraint. With these considerations, our optimization problem can be written as follows

$$\begin{aligned} \underset{\{\mathbf{U}_i\}\{\mathbf{V}_j\}}{\text{minimize}} \quad & \sum_i \sum_{j \neq i} \left\| \mathbf{U}_i^H \mathbf{H}_{ij}\mathbf{V}_j \right\|_F \ , \\ \text{subject to} \quad & \mathbf{U}_i^H \mathbf{H}_{ii}\mathbf{V}_i\mathbf{V}_i^H \mathbf{H}_{ii}^H \mathbf{U}_i \succeq \epsilon \mathbf{I} \ , \\ & \text{Tr}\left(\mathbf{U}_i^H \mathbf{U}_i\right) \leq 1 \ , \\ & \text{Tr}\left(\mathbf{V}_j^H \mathbf{V}_j\right) \leq 1 \ , \end{aligned} \tag{7}$$

where $i, j = 1, \ldots, K$ and $\text{Tr}(\cdot)$ denotes the trace of a matrix and $\mathbf{A} \succeq \mathbf{B}$ implies that $\mathbf{A} - \mathbf{B}$ is positive semidefinite.

Problem (7) can be solved using an alternating optimization procedure similar to [3]. First, consider that $\mathbf{V}_j$ is kept fixed for all j. Then, (7) can be decomposed into K independent problems, $\forall i \in \{1,\ldots,K\}$, as

$$\begin{aligned} \mathcal{P}_1: \quad & \underset{\mathbf{U}_i}{\text{minimize}} && \operatorname{Tr}\left(\mathbf{U}_i^H \mathbf{Q}_{I_i} \mathbf{U}_i\right), \\ & \text{subject to} && \mathbf{U}_i^H \mathbf{Q}_{D_i} \mathbf{U}_i \succeq \epsilon \mathbf{I}, \\ & && \operatorname{Tr}\left(\mathbf{U}_i^H \mathbf{U}_i\right) \leq 1, \end{aligned} \tag{8}$$

where $\mathbf{Q}_{I_i} = \sum_{j\neq i} \mathbf{H}_{ij}\mathbf{V}_j\mathbf{V}_j^H\mathbf{H}_{ij}^H$ is the interference covariance matrix at the i-th receiver, and $\mathbf{Q}_{D_i} = \mathbf{H}_{ii}\mathbf{V}_i\mathbf{V}_i^H\mathbf{H}_{ii}^H$ is the covariance matrix of the transmitted signal. The optimal solution of $\mathcal{P}_1$ is formalized in the ensuing lemma.

Lemma 1: The optimal solution of $\mathcal{P}_1$ is given by

$$\mathbf{U}_i^\star = \sqrt{\epsilon}\breve{\mathbf{U}}_i \left(\breve{\mathbf{U}}_i^H \mathbf{Q}_{D_i} \breve{\mathbf{U}}_i\right)^{-\frac{1}{2}}, \tag{9}$$

where $\breve{\mathbf{U}}_i$ contains the d_i smallest generalized eigenvectors of the matrix pencil $(\mathbf{Q}_{I_i} + \mu_i\mathbf{I}, \mathbf{Q}_{D_i})$, with $\mu_i \geq 0$ being the Lagrange multiplier associated to the last constraint in $\mathcal{P}_1$.

Proof: The Lagrangian of (8) is given by

$$\begin{aligned} \mathcal{L}_i\left(\mathbf{U}_i, \mathbf{\Lambda}_i, \mu_i\right) =& \operatorname{Tr}\left(\mathbf{U}_i^H \mathbf{Q}_{I_i} \mathbf{U}_i\right) + \operatorname{Tr}\left(\mathbf{\Lambda}_i\left[\epsilon\mathbf{I} - \mathbf{U}_i^H \mathbf{Q}_{D_i} \mathbf{U}_i\right]\right) \\ &+ \mu_i\left[\operatorname{Tr}\left(\mathbf{U}_i^H\mathbf{U}_i\right) - 1\right], \end{aligned} \tag{10}$$

where $\mathbf{\Lambda}_i \succeq 0$ and μ_i are the Lagrange multipliers associated to the first and last constraint in $\mathcal{P}_1$, respectively. Equating the complex gradient of (10) to zero yields

$$\nabla_{\mathbf{U}_i^*}\mathcal{L}_i\left(\mathbf{U}_i, \mathbf{\Lambda}_i\right) = 0 \Rightarrow \left(\mathbf{Q}_{I_i} + \mu_i\mathbf{I}\right)\mathbf{U}_i = \mathbf{Q}_{D_i}\mathbf{U}_i\mathbf{\Lambda}_i. \tag{11}$$

Equation (11) is a generalized eigenvalue problem, whose solution satisfying the constraint in (8) is given by (9). ∎

Finally, μ_i is chosen such that the norm constraint is satisfied. If the constraint is active, the optimal value of μ_i can be obtained using a bisection method. Since μ_i is unbounded above, let us consider a new variable, $\tilde{\mu}_i$, such that $\tilde{\mu}_i = \mu_i/(1+\mu_i)$. Notice that $\tilde{\mu}_i \in [0,1]$. Taking this into account, it is easy to see that (11) is equivalent to $[(1-\tilde{\mu}_i)\mathbf{Q}_{I_i} + \tilde{\mu}_i\mathbf{I}]\mathbf{U}_i = \mathbf{Q}_{D_i}\mathbf{U}_i\tilde{\mathbf{\Lambda}}_i$, where $\tilde{\mathbf{\Lambda}}_i = 1/(1+\mu_i)\mathbf{\Lambda}_i$. Therefore, the bisection method can be performed over $\tilde{\mu}_i$, between 0 and 1. The optimal value of $\tilde{\mu}_i$ is thus successively bounded by checking the total power required for the optimal solution given by (9), at each step of the bisection method.

Analogously, with $\mathbf{U}_i$ fixed for all i, (7) can be decomposed into K independent problems, $\forall j \in \{1,\ldots,K\}$, as

$$\begin{aligned} \mathcal{P}_2: \quad & \underset{\mathbf{V}_j}{\text{minimize}} && \operatorname{Tr}\left(\mathbf{V}_j^H \mathbf{R}_{I_j} \mathbf{V}_j\right), \\ & \text{subject to} && \mathbf{V}_j^H \mathbf{R}_{D_j} \mathbf{V}_j \succeq \epsilon \mathbf{I}, \\ & && \operatorname{Tr}\left(\mathbf{V}_j^H \mathbf{V}_j\right) \leq 1, \end{aligned} \tag{12}$$

where $\mathbf{R}_{D_j} = \mathbf{H}_{jj}^H\mathbf{U}_j\mathbf{U}_j^H\mathbf{H}_{jj}$ and $\mathbf{R}_{I_j} = \sum_{i\neq j}\mathbf{H}_{ji}^H\mathbf{U}_i\mathbf{U}_i^H\mathbf{H}_{ji}$ are again the signal and interference covariance matrices, respectively. For completeness, the optimal solution of $\mathcal{P}_2$ is formalized in the following lemma, whose proof is analogous to that of Lemma 1.

Lemma 2: The optimal solution of $\mathcal{P}_2$ is given by

$$\mathbf{V}_j^\star = \sqrt{\epsilon}\breve{\mathbf{V}}_j \left(\breve{\mathbf{V}}_j^H \mathbf{R}_{D_j} \breve{\mathbf{V}}_j\right)^{-\frac{1}{2}}, \tag{13}$$

where $\breve{\mathbf{V}}_j$ contains the d_j smallest generalized eigenvectors of the matrix pencil $\left(\mathbf{R}_{I_j} + \nu_j\mathbf{I}, \mathbf{R}_{D_j}\right)$, with $\nu_j \geq 0$ being the Lagrange multiplier associated to the last constraint in $\mathcal{P}_2$.

A. Some final remarks

Remark 1: Let us notice that both the objective function, representing the total interference leakage, and the constraints, are in fact identical for problems $\mathcal{P}_1$ and $\mathcal{P}_2$. Therefore, at each iteration of the proposed alternating optimization procedure the objective function cannot increase and, as it is bounded below by zero, the convergence to a stationary point is guaranteed.

Remark 2: At each step of the algorithm the norm of the precoders and decoders is always equal to or smaller than 1. Upon convergence, if a norm turns out to be smaller than 1 we can always normalize it to satisfy $\operatorname{Tr}((\mathbf{V}_i^\star)^H\mathbf{V}_i^\star) = 1$ or $\operatorname{Tr}((\mathbf{U}_i^\star)^H\mathbf{U}_i^\star) = 1$. This normalization would cause no harm, since the minimum eigenvalue of all direct channels after normalizing would still be larger than $\sqrt{\epsilon}$.

Remark 3: Notice finally that the noise power can also be incorporated into the optimization problem. Specifically, the interference covariance matrices in $\mathcal{P}_1$ and $\mathcal{P}_2$ can be modified as $\mathbf{Q}_{I_i} + \sigma^2\mathbf{I}$ and $\mathbf{R}_{I_i} + \sigma^2\mathbf{I}$, respectively, for $i = 1,\ldots,K$. However, if we desire to compute a perfect alignment solution (for instance, to have evidence about whether a given IA problem can be feasible or not), the noise term must be omitted.

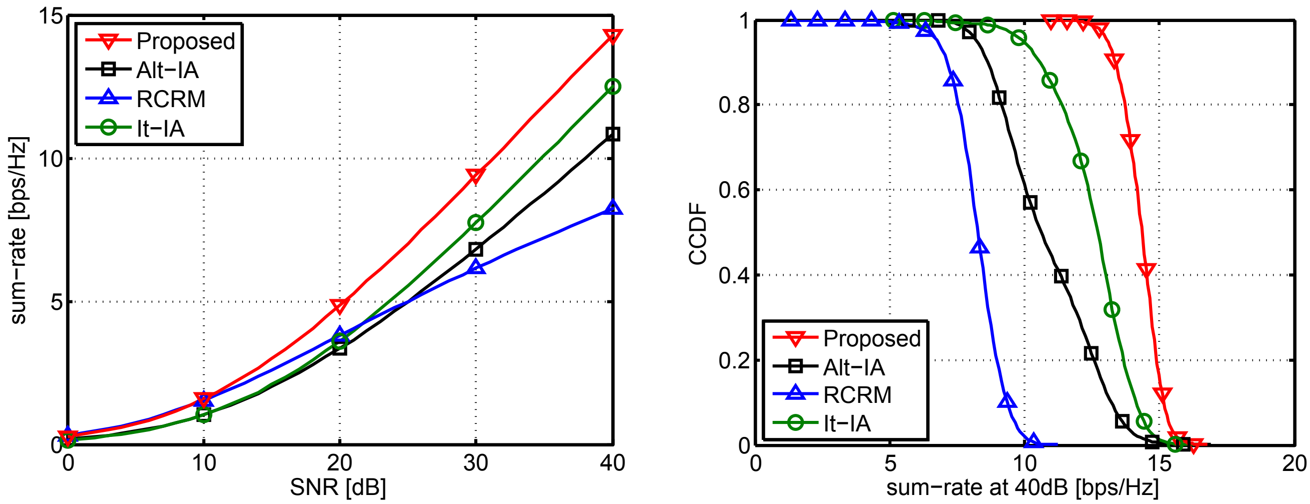
IV. Numerical Results

In this section we provide some numerical examples that illustrate the performance of the proposed method in different scenarios, and compare it with other existing algorithms, namely the alternating minimization algorithm (Alt-IA) [3], the rank-constrained rank-minimization algorithm (RCRM) [10] and the iterative algorithm (It-IA) [11]. In each simulation, the algorithms are evaluated in 500 different channel realizations. The entries of the channel matrices are i.i.d. zero-mean circular complex Gaussian random variables with unit variance. We set $\epsilon = 10^{-3}$, consider unit transmit power and define the signal-to-noise ratio as $\text{SNR} = 10\log_{10}\frac{1}{\sigma^2}$. Finally, as we are interested in computing perfect alignment solutions, we do not consider noise power in any of the algorithms.

In Fig. 1 we evaluate the aforementioned algorithms for the $[(1\times 1, 3)^4, 8]$ scenario with time-varying channels, which has $4\cdot 3/8 = 1.5$ DoF. In Fig. 1(a), we plot the average sum-rate versus the SNR, which is obtained as

$$\text{sum-rate}\left(\sigma^2\right) = \sum_{i=1}^{K}\log_2\left|\mathbf{I} + \left(\sigma^2\mathbf{U}_i^H\mathbf{U}_i + \mathbf{Q}_{I_i}\right)^{-1}\mathbf{Q}_{D_i}\right|. \tag{14}$$

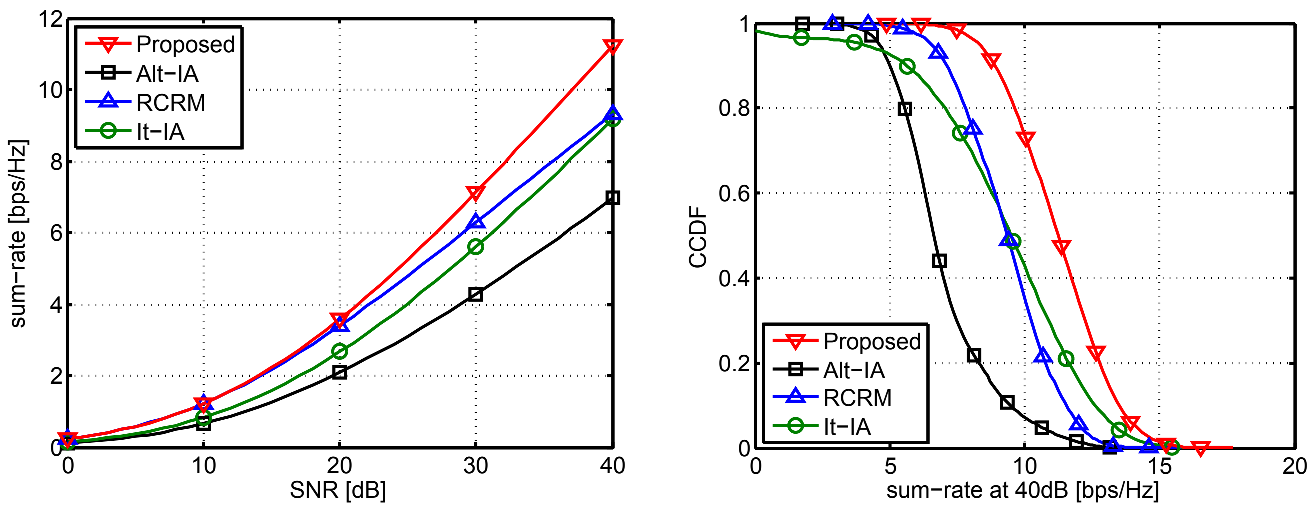
The average sum-rate of the proposed method is clearly higher than that obtained with the other algorithms. Alternatively, we show in Fig. 1(b) the complementary cumulative distribution function (CCDF) of the sum-rate when the SNR is 40dB. It can be observed that the proposed method is more likely to achieve higher sum-rate than the other algorithms. For instance, the proposed algorithm achieves a sum-rate of at least 13.7 bps/Hz in 81% of the channel realizations, whereas the percentages for the Alt-IA, RCRM and It-IA are 5%, 0% and 19%, respectively.



(a) Sum-rate performance

(b) CCDF of the sum-rate at 40dB

Fig. 1. Performance of the different algorithms in the $[(1 \times 1, 3)^4, 8]$ scenario.



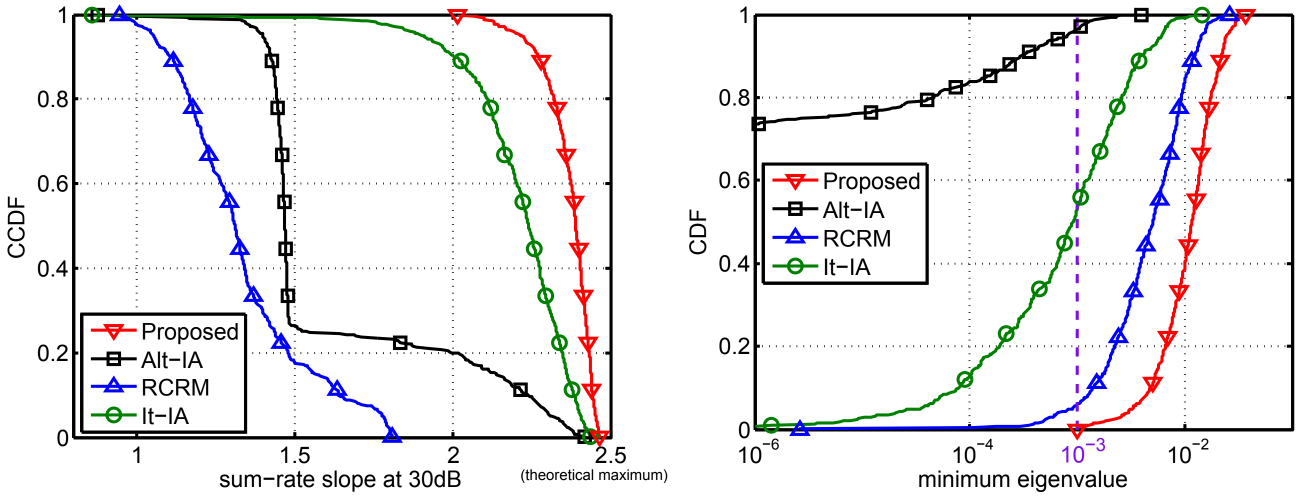
(a) Sum-rate performance

(b) CCDF of the sum-rate at 40dB

Fig. 2. Performance of the different algorithms in the $[(1 \times 1, 1)^4, 3]$ scenario with constant channels and improper signaling.

Then we consider the 4-user SISO interference channel with constant channel coefficients. According to [13], for this system it is feasible that each user transmits a complex data stream using $L = 3$ symbols extensions and improper signaling [14], thus achieving $4 \cdot 1/3 \simeq 1.33$ DoF per channel use. Fig. 2 shows the average sum-rate (a) and the CCDF of the sum-rate at 40dB (b). It can be observed that the proposed method provides a significant gain in terms of average sum-rate. Moreover, as shown in Fig. 2(b), the statistics of the sum-rate are more favorable than those obtained with the other algorithms. For instance, the proposed method is able to achieve at least 9.7 bps/Hz in 80% of the channel realizations, while Alt-IA, RCRM and It-IA in 8%, 43% and 46%, respectively.

Finally, we evaluate the algorithms for the $[(2 \times 1, 2)^3 (2 \times 1, 3)^3, 6]$ scenario with time-varying channels, which has $(3 \cdot 2 + 3 \cdot 3)/6 = 2.5$ DoF. In Fig. 3(a) we show the CCDF of the sum-rate slope at 30dB. This measure provides us with an idea of whether the algorithms converge to a perfect alignment solution that extracts all available DoF or not. Hence, if perfect alignment is achieved, i.e., the interference is perfectly suppressed and the direct channels are full-rank, the slope of the sum-rate at high SNR should be equal to 2.5. Notice that, in practice, since the interference cannot be completely nullified, an IA solution is expected to achieve a slightly less sum-rate slope, but close to the theoretical value of 2.5. It can be observed that the proposed method provides the highest slope among all considered algorithms. For instance, the proposed algorithm provides a slope higher than 2.25 (i.e., 90% of the theoretical value) in 93% of the channel realizations, whereas the percentage for the It-IA is 48%. Moreover, the minimum slope achieved by the proposed algorithm is equal to 2, which is more than twice the minimum value achieved by the other algorithms. These results indicate that the proposed method finds IA solutions in many more channel realizations than the rest algorithms. To illustrate this fact, we show in Fig. 3(b)



(a) CCDF of the sum-rate slope at 30dB

(b) CDF of the minimum eigenvalue of the direct channel among all users

Fig. 3. Performance of the different algorithms in the $[(2 \times 1, 2)^3(2 \times 1, 3)^3, 6]$ scenario.

the CDF of the minimum eigenvalue of the direct channels ($\mathbf{U}_i^H \mathbf{Q}_{D_i} \mathbf{U}_i$, for all i) among all users. As expected, the proposed method ensures the eigenvalues to be equal to or greater than 10^{-3}, thus successfully preserving the rank of the signal subspace. The RCRM and It-IA methods, however, do not guarantee the dimensionality of the signal subspace.

V. Conclusions

In this work we have proposed a new algorithm for computing IA solutions in structured channels. We have expressed the rank constraint as a minimum eigenvalue constraint, which has allowed us to minimize the total interference leakage while ensuring full-rank direct channels. A power constraint has been also incorporated into the optimization problem, avoiding the need of a normalization step that might reduce the numerical rank of the signal subspace. The proposed algorithm is based upon alternating optimization, solving a generalized eigenvalue problem at each step. We have shown through simulations that the proposed method provides a significant gain in the sum-rate performance in different structured scenarios. Moreover, simulations have also shown that the proposed method is more likely to find IA solutions than other existing algorithms when structured channels are considered.

Acknowledgments

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