# Optimal rate and delay performance in non-cooperative opportunistic spectrum access

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Abstract—We study transmission rate control and performance delay in cognitive radio (CR) links from a cross-layer perspective. We assume a hierarchical CR network where the secondary users (SU) access the spectrum band in an opportunistic and noncooperative way. The SU goal is to transmit a fixed-size file (fixed amount of data packets) during the sojourn time of the primary users (PU's) idle state. We assume that the SU's support frames retransmission through an automatic repeat request (ARQ) mechanism. By formulating the problem as a Markov decision process, we demonstrate that there is always an optimal stationary rate adaptation policy, and we propose a simple algorithm to obtain it. We derive an exact closedform expression for the probability of successful transmission as a function of the PU's access probability and the signal-tonoise ratio at the link receiver. We also study the performance delay, understood as the time required to transmit the entire data file, taking into account frames retransmission. To do that, we analyze the Markov process associated with the optimal rate policy in the transform domain. Then, using probabilistic flowgraph techniques, we derive exact closed-form expressions for the statistical distribution of transmission delay.

#### I. INTRODUCTION

Recently, IEEE 802.22 working group has released the first cognitive radio standard for wireless regional area networks [1]. This standard supports rate adaptation using adaptive modulation and coding. It also allows SU's to support frames retransmission through an ARQ mechanism so that SU's can setup ARQ enabled connections.

This work focuses on opportunistic spectrum access (OSA) in hierarchical CR networks where the SU's only use the licensed spectrum when primary users PU's are not transmitting. We consider noncooperative spectrum sharing where each SU makes his/her own decision on the spectrum access strategy, based on local observation of the spectrum dynamics. We assume that the SU's can adapt the transmission rate according to the channel fading conditions and the PU's channel access statistics. We also assume that the SU's support ARQ protocol, so when a frame is decoded with error, its data is retransmitted in a further frame.

By formulating the rate adaptation problem as a Markov decision process (MDP), we demonstrate that there is always

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a stationary optimal policy that maximizes the probability of successful transmission, and we derive a simple algorithm to obtain such policy. Then, we derive a closed-form expression for the probability of successful transmission under the optimal policy. Also, we analyze the performance delay of the SU link for the optimal rate adaptation policy. By using probabilistic flow-graph techniques, we derive simple closed-from expressions for the distribution and moments of the total transmission delay for both successful and failed transmissions.

To the best of our knowledge, optimal rate adaptation while considering retransmissions of failed frames has not been addressed so far in the context of OSA. Rate adaptation in CR has been addressed in the technical literature, [2], [3], [4]. However, none of the above works consider frames retransmission. There are also a number of works that have studied the delay performance of SU links in the context of OSA [5], [6], [7], [8], but they do not consider frames retransmission either. Note that when ARQ is used the delay due to frames retransmission can be an important fraction of the transmission delay.

## II. SYSTEM MODEL

Let us consider a SU that periodically sense the spectrum band. Once it detects an idle channel, it starts the transmission with the goal to transmit a fixed-size file, comprising N packets. These packets have been previously arrived from the higher layer application and was placed in the buffer of the SU transmitter. During the transmission, the SU adapts the transmission rate with the goal to maximize the probability of transferring the entire file before a PU reclaims the channel.

All packets have the same number of information bits and the data of each packet is encoded in a single frame for transmission. We consider K available transmit rates. Therefore, the frames duration t(k) depends on the selected rate where,  $k=1,\ldots,K$ . We assume that the channel remains constant during the sojourn time of the PU's idle state. We also assume constant transmit power, so the SNR at the CR link receiver does not change during the transmission. Let  $p(k), k=1,\ldots,K$  denote the frame error rates (FER) for each rate according to the SNR.

We consider a conventional ARQ mechanism to overcome transmission errors. Once the receiver receives a frame, it

sends an ACK (acknowledgement) packet back to the transmitter through an instantaneous error-free feedback channel to inform whether the frame has been correctly decoded or not. If a frame is decoded with error, the corresponding packet must be retransmitted under a further frame. Therefore, a data packet remains in the transmit buffer until the receiver informs the transmitter of the corresponding frame has been successfully decoded.

We assume that an SU does not distinguish different PU's and treat the collective of all PU's as one "aggregated" PU. Let  $\beta(k)$  denotes the probability of the PU's do not access the spectrum band during the transmission of a frame of length t(k). We assume that these probabilities do not change during the file transmission.

#### III. OPTIMAL RATE

In this section we formulate the rate adaptation problem as a MDP as follows,

- Stages: Each stage corresponds to a frame transmission.
   The process can finish in two different ways: 1) The SU has successfully transmitted the N packets, and 2) A PU has reclaimed the frequency band so the transmission has been interrupted.
- Controls: The controls at the stages are the available rates:  $k \in \{1, 2, ..., K\}$ .
- States: There are  $N_S=N+2$  possible states, that are indexed and classified as follows
  - Transient states:  $1 \le i \le N$ .
  - Success state (S): i = N + 1
  - Fail state (F): i = N + 2

Each transient state is defined by the number of packets successfully transmitted during the process, so the system is in state i when i-1 packets have been already transmitted. The success state (S) corresponds to the situation where all packets have been transmitted, whereas the system falls in the fail state (F) when a PU has reclaimed the frequency band before all packets have been transmitted. Both, S and F are absorbing states in the sense that once the system has fallen in one of these states, it remains in it indefinitely. To illustrates it, figure 1 shows the transition graph for K=2 available rates. The states are represented by labeled boxes and the arrows represent the possible transitions between states.

Transition probabilities: There are three types of transitions: 1) transitions from a transient state to itself when the transmitted frame has been decoded with error,
2) transition from a transient state to another transient state or to the success state when the frame has been successfully transmitted,
3) transitions from a transient state to the fail state when a PU reclaims the channel. Therefore, the transient probability from state i to j when control k is applied is

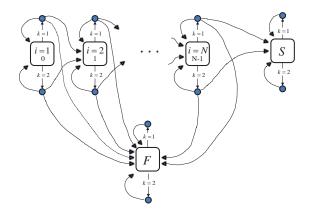


Fig. 1. Transition graph in the case of K=2 available rates

$$p_{i,j}^k = \begin{cases} 1, & i = j > N \\ \beta(k)p(k), & i = j \leq N \\ 1 - \beta(k), & i \leq N \ \land \ j = N+2 \end{cases} \quad \text{(1)} \\ \beta(k)(1 - p(k)), & i \leq N \ \land \ j = i+1 \\ 0, & \text{otherwise.} \end{cases}$$

 Rewards: We define the transition reward from state i to j when control k is applied as follows

$$r_{i,j}^k = \begin{cases} 1, & i = N \land j = N+1\\ 0, & \text{otherwise.} \end{cases}$$
 (2)

In words, there is not reward until all blocks have been successfully decoded. Then, the system remains indefinitely in the success state with zero reward. The expected immediate reward when the system is in state i and control k is applied will be

$$q_{i}^{k} = \sum_{j=1}^{N} p_{i,j}^{k} r_{i,j}^{k} = p_{i,N+1}^{k}$$

$$= \begin{cases} \beta(k)(1 - p(k)), & i = N \\ 0, & \text{otherwise.} \end{cases}$$
(3)

• Policies: A policy is defined by vector  $\mathbf{d} = \{d_1, d_2, ..., d_N\}$ , where  $d_i$  denotes the control to use when the system is in state i. Each policy determines a Markov chain with rewards, where the transition probabilities are  $p_{i,j}^{d_i}$  and the immediate rewards are  $q_i^{d_i}$ .

# A. Probability of successful transmission

Let us consider a given policy  $\mathbf{d}$ . Let us consider its value vector  $\mathbf{v^d} = \{v_1^\mathbf{d}, v_2^\mathbf{d}, ..., v_N^\mathbf{d}\}$ , where  $v_i^\mathbf{d}$  is the expected total reward when the system is in state i. In other words,  $v_i^\mathbf{d}$  is the probability of the system to reach the success at the end of the process, starting from state i. According to the Bellman equation [9], [10], [11]

$$v_i^{\mathbf{d}} = q_i^{d_i} + \sum_{j=1}^{N} p_{i,j}^{d_i} v_j^{\mathbf{d}}, \quad i = 1, 2, \dots, N,$$
 (4)

In words, the expected total reward from state i equals the immediate expected reward plus the expected total reward from the subsequent state out state i. As it was mentioned, when the system reach an absorbing state it remains in it indefinitely with no additional reward so  $v_{N+1}^{\mathbf{d}} = v_{N+2}^{\mathbf{d}} = 0$ . From (4), considering (1) and (3)

$$v_i^{\mathbf{d}} = \begin{cases} \beta(d_i) p(d_i) \ v_i^{\mathbf{d}} + \beta(d_i) (1 - p(d_i)) \ v_{i+1}^{\mathbf{d}}, & i < N \\ \beta(d_i) p(d_i) \ v_i^{\mathbf{d}} + \beta(d_i) (1 - p(d_i)), & i = N. \end{cases},$$

Then,

$$v_i^{\mathbf{d}} = a(d_i) \begin{cases} 1, & i = N \\ v_{i+1}^{\mathbf{d}}, & i < N. \end{cases}$$
 (5)

where

$$a(k) = \frac{\beta(k)(1 - p(k))}{1 - \beta(k)p(k)}, \quad k = 1, 2, \dots, K.$$
 (6)

Let us consider now the transition probability from a transient state i to the subsequent state i+1 (distinct to F), considering frames retransmission, when frames of type k are used. It will be

$$\beta(k)(1-p(k))\sum_{n=0}^{\infty}(\beta(k)p(k))^n,$$
 (7)

where the n-th term in (7) is the transition probability from i to i+1 when the frame has been retransmitted n times. Note that (7) equals (6), so a(k) is merely the packet transmission probability (considering frames retransmission) when frames of type k are used.

## B. Optimal policy

We are interested in policies that maximize the probability of successful transmission  $v_1^{\mathbf{d}}$ . Equation (5) shows that the values  $v_i^{\mathbf{d}}$  are simply the product of the factors  $a(d_i)$ , which do not depend on the state. Therefore, the optimal control will be identical for all states (stationary), regardless the number of packets N to transmit. Among the K stationary policies, the optimal one and the corresponding probability of successful transmission will be

$$d^* = \arg\max_{L} a(k), \ P\{S\} = v_1^* = a(d^*)^N.$$
 (8)

#### IV. ANALYSIS OF TRANSMISSION DELAY

The transition diagram of the Markov process for the optimal stationary policy  $(d^*)$  is depicted in figure 2, where  $\beta=\beta(d^*)$  and  $p=p(d^*)$ . Under the optimal stationary policy the total transmission delay equals the number of transitions before entering a trapping state multiplied by the frames duration  $t(d^*)$ . Therefore, to analyze the transmission delay we will focus exclusively on the number of transitions to reach the absorbing states. Since there are two absorbing states, we distinguish between successful transmissions and failed transmissions. Let n/S denote the number of transitions required to enter state S in successful transmissions, and let

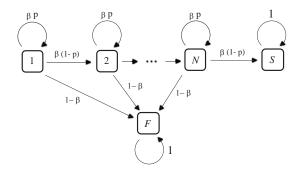


Fig. 2. Transition diagram of the Markov chain under optimal policy.

n/F denote the number of transitions to reach state F in failed transmissions. In the following sections we will derive closed-form expressions for the probability distributions and moments of these random variables.

#### A. Delay of successful transmissions

Let  $p_S(n)$  the probability of enter state S in the n-th transition. In the appendix we derive an expression for the geometric transform of  $p_S(n)$  by analyzing the Markov process in the geometric transform domain,

$$p_S(z) = \frac{\beta^N (1 - p)^N z^N}{(1 - \beta p z)^N}$$
 (9)

Then,  $p_S(n)$  can be obtained by inverting the geometric transform of (9)

$$p_S(n) = \beta^N (1 - p)^N (\beta p)^{n - N} \binom{n - 1}{n - N}, \qquad (10)$$

where we assume that the binomial coefficient equals zero when the second entry (n-N) is negative.  $p_S(n)$  is, in fact, the joint probability of reaching state S at transition n. Then, the conditional probability distribution  $p_S(n/S)$  that the process will reach state S on his n-th transition given that the transmission will finish successfully is

$$p_S(n/S) = \frac{p_S(n)}{P\{S\}} = (1 - \beta p)^N (\beta p)^{n-N} \binom{n-1}{n-N}$$
 (11)

#### B. Delay of failed transmissions

Analogously, we can obtain the distribution and moments of n/F. First, let us consider the probability of enter state F in the n-th transition,  $p_F(n)$ . In the appendix we derive the following closed-form expression for its geometric transform

$$p_F(z) = (1 - \beta) \sum_{i=1}^{N} \frac{\beta^{i-1} (1 - p)^{i-1} z^i}{(1 - \beta p z)^i}$$
 (12)

Then, by transform inversion

$$p_F(n) = (1 - \beta) \sum_{i=1}^{N} \beta^{i-1} (1 - p)^{i-1} (\beta p)^{n-i} \binom{n-1}{n-i}$$
 (13)

TABLE I
RATE ADAPTATION PARAMETERS

| k | Modulation | Coding rate | Data rate (Mbps) | Frames duration $(\mu s)$ |
|---|------------|-------------|------------------|---------------------------|
| 1 | BPSK       | 1/2         | 6                | 2020                      |
| 2 | BPSK       | 3/4         | 9                | 1352                      |
| 3 | QPSK       | 1/2         | 12               | 1020                      |
| 4 | QPSK       | 3/4         | 18               | 688                       |
| 5 | 16-QAM     | 1/2         | 24               | 520                       |
| 6 | 16-QAM     | 3/4         | 36               | 352                       |
| 7 | 16-QAM     | 2/3         | 48               | 272                       |
| 8 | 16-QAM     | 3/4         | 54               | 244                       |

The probability of F being the final state is

$$P\{F\} = \sum_{n=1}^{\infty} p_F(n) = p_F(z=1)$$

$$= (1-\beta) \sum_{i=1}^{N} \beta^{i-1} (1-p)^{i-1} (1-\beta p)^{-i},$$
(14)

which also equals  $1 - P\{S\}$ . The quantity  $p_F(n)$  given by equation (13) is, in fact, the joint probability of reaching state F at the n-th transition and of failed transmission. Then, the conditional probability distribution  $p_F(n/F)$  that the process will reach state F on his n-th transition given that the transmission fails will be

$$p_F(n/F) = \frac{p_F(n)}{P\{F\}} = \frac{\sum_{i=1}^N \beta^{i-1} (1-p)^{i-1} (\beta p)^{n-i} \binom{n-1}{n-i}}{\sum_{i=1}^N \beta^{i-1} (1-p)^{i-1} (1-\beta p)^{-i}}$$
(15)

## V. NUMERICAL RESULTS

Let that the SU link uses the rate coding-modulation adaptation scheme emthe **IEEE** 802.11 wireless standards (http://standards.ieee.org/about/get/802/802.11.html). based on a set of K = 8 RCPC codes (rate-compatible punctured convolutional codes) whose characteristics are shown in table I. The data rate values assume 20 MHz channel bandwidth and the frames duration,  $t_C(k)$ , have been obtained assuming fixed packets length of 1500 bytes, which is a common value in many practical systems.

Figure 3 shows the FER's (p(k)) for the different rates as a function of the SNR (they have been obtained after intensive simulations).

In the following simulations we model the PU's access to the spectrum band as a Poisson process with access rate  $\lambda$ , so the probability of PU's do not accessing the band during the transmission of a frame of type k is  $\beta(k) = \exp^{-\lambda t(k)}$ .

Figure 4 shows the probability of successful transmission  $(P\{S\})$  as function of the number of packets for optimal rate adaptation, for different values of SNR, assuming that the number of packets is N=50.

Figure 5 shows the probability of successful transmission  $(P\{S\})$  as function of the PU's access parameter  $\lambda$ , for different values of SNR. The figure also shows the optimal policy in each case. The number of packets is N=50.

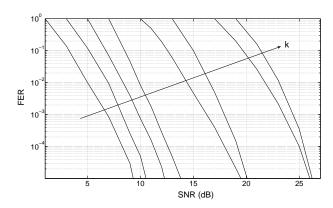


Fig. 3. Frame error probability vs. SNR for the eight available rates.

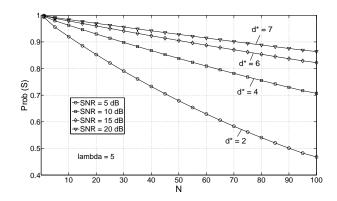


Fig. 4. Probability of successful transmission vs. number of packets (N). The PU's access rate is  $\lambda = 5$ . The optimal rate  $d^*$  is shown for each case.

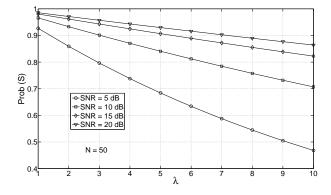


Fig. 5. Probability of successful transmission vs. PU's access parameter  $\lambda$ . The number of packets is N=50.

Figure 6 show  $p_S(n)$  and  $p_F(n)$  (the probabilities of reaching states S and F when the n-th frame has been transmitted) as function of n, for the optimal policy. In this case we consider SNR= 15dB,  $\lambda=5$  and N=20, so the optimal policy is  $d^*=6$  (see figure 4).

## VI. APPENDIX

In this appendix we derive the expressions (9) and (12) for  $p_S(z)$  and  $p_F(z)$ , respectively. In a Markov process, the

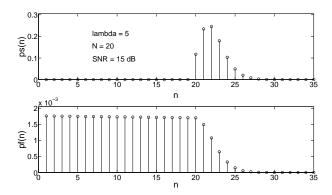


Fig. 6. Probability of reaching the absorbing states when the n-th frame has been transmitted.

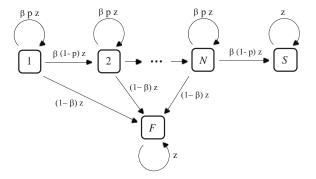


Fig. 7. Flow graph of the Markov chain for the optimal stationary policy.

geometric transform of the probability of reaching state j from state i in n transitions equals the transmission factor,  $t_{i,j}$ , from node i to node j in the probabilistic flow graph associated with the Markov process [12]. The probabilistic flow graph of the Markov process for the optimal stationary policy is depicted in Fig. 7, where  $\beta = \beta(d^*)$  and  $p = p(d^*)$ . Therefore,  $p_S(z)$  and  $p_F(z)$  will be the transmission factors  $t_{1,S}$  and  $t_{1,F}$  in Fig. 7, respectively.

The transmission factors can be calculated using the Mason's method [13]. According to this, the transmission factor between nodes i and j is given by

$$t_{i,j} = \frac{1}{\Delta} \sum_{k \in S_{i,j}} t_{i,j}^k \Delta_{i,j}^k,$$
 (16)

where  $S_{i,j}$  is the set of paths leading from node i to node j and  $t_{i,j}^k$  is the product of the branches gains along the k-th path. The quantities  $\Delta$  and  $\Delta_{i,j}^k$  are related to the loops in the graph. The loop product of a simple loop is defined as the product of all branches gains of the loop with minus algebraic sign. There are also loop products associated with all sets of simple loops that have not common nodes. The loop products have an algebraic sign which is plus (or minus) for an even (or odd) number of loops in the set. The quantity  $\Delta$  is one plus the sum of all loop products in the graph, whereas  $\Delta_{i,j}^k$  is equal to one plus the sum of the loop products of all

loops that share no node with the k-th path. The one-branch loops associated with absorbing nodes (with gain z) are not considered to compute  $t_{i,j}$ .

In our flow graph there are N simple loops, all of them with loop factor  $-\beta pz$ . Considering them and all possible set of loops

$$\Delta = (1 - \beta pz)^N. \tag{17}$$

Let us consider the transmission factor  $t_{1,S}$ . There is only one path connecting 1 and S with the following parameters

$$t_{1,S}^{1} = \beta^{N} (1-p)^{N} z^{N}, \qquad (18)$$
  
$$\Delta_{1,S}^{1} = 1.$$

Then, from (16), (17) and (18),

$$t_{1,S} = p_S(z) = \frac{\beta^N (1-p)^N z^N}{(1-\beta pz)^N}.$$

Let us consider now the transmission factor  $t_{1,F}$ . There are N different paths connecting nodes 1 and F with the following parameters

$$t_{1,F}^{k} = (1-\beta)\beta^{k-1}(1-p)^{k-1}z^{k},$$

$$\Delta_{1,F}^{k} = (1-\beta pz)^{N-k}, k = 1, 2, ..., N.$$
(19)

Then, from (19), (16) and (17),

$$t_{1,F} = p_F(z) = (1 - \beta) \sum_{k=1}^{N} \beta^{k-1} (1 - p)^{k-1} \frac{z^k}{(1 - \beta pz)^k}.$$

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