# Beamforming-based emulation of spatial and temporal correlated MISO channels

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***Abstract*—Temporal and spatial correlation are inherent mobile wireless channel characteristics that determine multiple-input multiple-output (MIMO) systems performance. It is widely known that in indoor laboratories with fixed conditions, wireless channels are intrinsically quasi-static. In this paper we propose a methodology for emulating multiple-input single-output (MISO) channels with arbitrary distributions as well as spatial and temporal correlation characteristics. This methodology, based on time-varying beamforming, allows us to evaluate MISO techniques in controlled channel conditions. To prove its feasibility, it has been implemented and tested over a commercial MIMO testbed.**

## I. Introduction

Spatial and temporal fading correlation are crucial issues for practical multiple-input multiple-output (MIMO) wireless communication systems. The performance of MIMO techniques is greatly affected by both correlation characteristics [1].

MIMO test beds [2], [3], allow system tests and evaluations which are less expensive than field trials. However, realistic channel conditions are difficult to reproduce in low-mobility indoor scenarios where the test beds are usually placed. Measurements corroborate that, in indoor sites with the absence of motion within the environment, the channel response is time-invariant [4].

In this paper we propose a method, based on time-varying beamforming, to emulate narrowband multiple-input single-output (MISO) channels with any fading distribution as well as any spatial and temporal correlation characteristics. It allows us working under controlled and repeatable conditions that would not normally be possible in actual field testings. The methodology works on block fading basis where beamformers change from block to block remaining fixed within a certain block. The method has been assessed for different channel distributions and spatial correlation matrices as well as time variation models [5], [6].

The remainder of the paper is organized as follows: Section II presents the proposed method for MISO channel emulation. The following steps towards its implementation on a certain equipment is summarized in Section III. Section IV presents the validation of the method by simulation results and by measurements making use of a commercial wireless testbed. Finally, Section V is devoted to the concluding remarks.

## II. MISO emulation

A multiple-input single-output (MISO) system with $n_T$ transmit antennas is considered. Let $\mathbf{h} = \left[h_1\, h_2 \ldots h_{nT}\right]^T$ represents the channel response, where $h_i$ is the complex channel gain from the $i$th transmit antenna to the receive antenna, assumed known and time invariant. A spatial-domain MISO beamforming scheme is depicted in Fig. 1. The transmit signals, $\mathbf{s} = [s_1\ s_2\ \ldots\ s_{n_T}]^T$, where $(\cdot)^T$ denotes transpose, are multiplied by a transmit weight vector (i.e. beamformer) $\mathbf{w} = [w_1\ w_2\ \ldots\ w_{n_T}]^T$ ($\mathbf{w} \in \mathbb{C}^{n_T}$). In absence of noise, the output signal at the receiver, $y$, can be expressed as

$$y = \mathbf{s}^T diag\left(\mathbf{w}\right)\mathbf{h} = \mathbf{s}^T \mathbf{h}_e, \qquad (1)$$

where $diag(\mathbf{w})$ is the diagonal matrix with the diagonal elements of vector $\mathbf{w}$. The MISO equivalent channel is

$$\mathbf{h}_e = diag(\mathbf{w})\mathbf{h}. \qquad (2)$$

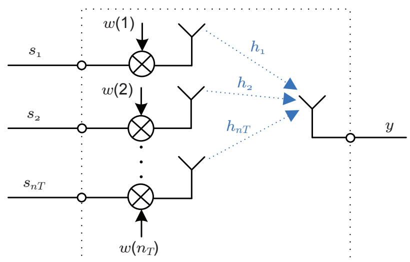


Fig. 1. MISO channel with complex weights at the transmitter.

In the following subsections we derive weights vector distribution to emulate a Rician fading channel with desired spatial and temporal correlations.

### A. Emulation

Assuming a fixed and known physical channel response, $\mathbf{h}$, according to (2), the weights distribution determines the distribution of the equivalent channel. To emulate Rician fading distributions, these weights can be designed as a sum of deterministic part, $\mathbf{w}_d$, and a zero-mean complex Gaussian term, $\mathbf{w}_s$, as

$$\mathbf{w} = \mathbf{w}_d + \mathbf{w}_s \qquad (3)$$

This leads to an equivalent channel, $\mathbf{h}_e$, consisting of a deterministic, also denoted as line of sight (LOS) component, and a zero mean diffuse part, or non-line of sight (NLOS),

$$\mathbf{h}_e = \mathbf{h}_{ed} + \mathbf{h}_{es}. \tag{4}$$

Let's suppose that we desire to emulate a MISO channel with the following spatial covariance matrix

$$\mathbf{R}_h^o = \begin{bmatrix} r_{11}^o & r_{12}^o & \dots & r_{1n_T}^o \\ r_{21}^o & r_{22}^o & \dots & r_{2n_T}^o \\ \vdots & \vdots & \ddots & \vdots \\ r_{n_T 1}^o & r_{n_T 2}^o & \dots & r_{n_T n_T}^o \end{bmatrix}, \tag{5}$$

and a certain deterministic component,

$$\mathbf{h}_{ed}^o = \left[ h_{ed}^o(1) \ \dots \ h_{ed}^o(n_T) \right]. \tag{6}$$

The Rician factor for the channel of antenna $i$th will be $k(i)^o = \frac{|h_{ed}^0(i)|^2}{r_{ii}^o}$, and the deterministic part of the weights should be

$$w_d(i) = E\left[w(i)\right] = \frac{h_{ed}^o(i)}{h_i}, \quad i = 1, \dots, n_T. \tag{7}$$

The spatial covariance matrix of the equivalent channel is

$$\begin{aligned} \mathbf{R}_s &= E\left[\mathbf{h}_{es}\mathbf{h}_{es}^H\right] \\ &= E\begin{bmatrix} |h_1|^2|w_s(1)|^2 & \dots & h_1 h_{n_T}^* w_s(1) w_s(n_T)^* \\ h_2 h_1^* w_s(2) w_s(1)^* & \dots & h_2 h_{n_T}^* w_s(2) w_s(n_T)^* \\ \vdots & \ddots & \vdots \\ h_{n_T} h_1^* w_s(n_T) w_s(1)^* & \dots & |h_{n_T}|^2 |w_s(n_T)|^2 \end{bmatrix}. \end{aligned} \tag{8}$$

where $E\left[\cdot\right]$ is the expectation operator. The equivalent channel will be distributed according to $\mathbf{h}_e \sim CN(\mathbf{h}_{ed}, \mathbf{R}_h^o)$. Then, from (8) and (5), the weights moments should be

$$E\left[w_s(i) w_s(j)^*\right] = \frac{r_{ij}^o}{h_i h_j^*}, \quad i, j = 1, \dots, n_T, \tag{9}$$

and the weights covariance matrix should be

$$\mathbf{R}_{w_s} = E\left[\mathbf{w}_s \mathbf{w}_s^H\right] = \begin{bmatrix} \frac{r_{11}^o}{|h_1|^2} & \frac{r_{12}^o}{h_1 h_2^*} & \dots & \frac{r_{1n_T}^o}{h_1 h_{n_T}^*} \\ \frac{(r_{12}^o)^*}{h_2 h_1^*} & \frac{r_{22}^o}{|h_2|^2} & \dots & \frac{r_{2n_T}^o}{h_2 h_{n_T}^*} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\left(r_{1n_T}^o\right)^*}{h_{n_T} h_1^*} & \frac{\left(r_{2n_T}^o\right)^*}{h_{n_T} h_2^*} & \dots & \frac{r_{n_T n_T}^o}{|h_{n_T}|^2} \end{bmatrix}. \tag{10}$$

Then, the weights will be obtained from

$$\mathbf{w} = \mathbf{w}_d + \mathbf{w}_s = \mathbf{w}_d + \mathbf{R}_{w_s}^{1/2} \mathbf{w}_w \tag{11}$$

where $w_d$ is given by (7)$\mathbf{w}_w \sim CN(0, \mathbf{I}_{n_T})$, being $\mathbf{I}_n$ the identity matrix of dimension $n$, $CN$ refers to complex normal distribution and we read the symbol $\sim$ as "is distributed as".

Note that the Rayleigh channel emulation is a particular case where $\mathbf{h}_{ed} = \mathbf{0} \Longrightarrow \mathbf{w}_d = \mathbf{0}$.

### B. Temporal correlation

To achieve temporal correlation in the random part of the equivalent channel, we must generate the realizations of $\mathbf{w}_w$ accordingly. For a given channel distribution, time correlation is characterized by the maximum Doppler frequency, $f_D$. We use the Lee model [5] to generate complex Gaussian random realizations of the elements of $\mathbf{w}_w$ according to a given $f_D$.

## III. Implementation of the MISO channel emulator

Starting on the basis that we have a transmitter and a receiver device under test (DUT) and our aim is to emulate the MISO channel among them, weights application should be performed at the transmit base band (BB) processor. Fig. 2 presents the general diagram of a typical implementation of our method.

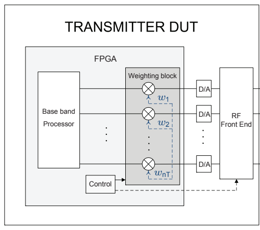


Fig. 2. MISO implementation scheme.

We add a new block called *Weighting Block* to the existing BB processor. It consists of $n_T$ complex multipliers, each consisting of 4 multipliers and 2 adders. The baseband in-phase and quadrature (IQ) signal samples are generated by the BB processor and the real-time weights are fed, along with the signal, to the complex multipliers. The outputs of the BB processor are converted to the analog domain by $n_T$ dual digital to analog converters (D/A) and then upconverted by the Analog Front End (AFE).

The initial estimation of the channel, typically performed by the BB processor, can be carried out as described in Section IV. The weights are defined within the range $[-1, 1]$ to avoid saturation. If a higher dynamic range is desired, there exist a control that allows varying the transmit power amplifier (PA) gain at the AFE. The implementation of the proposed method relies on the common block fading assumption. During each channel state (block) the signals are transmitted with fixed weights and the equivalent channel remains constant within a block. From one state to another, the weights change according to a certain spatial covariance following any distribution and temporal correlation among states.

## IV. Validation and test

In this section we first validate the proposed method by means of simulation results. Then, it has been also implemented in our MIMO testbed [7]. The *Weighting Block* is fitted into the transmit Field Programmable Gate Array (FPGA) (Xilinx Virtex II). The D/A has 14 bits of resolution so the output of the *Weighting block* must be truncated to 14 bits.

The transmit weight vector falls in the range $[-1, 1]$ with one bit for the sign and 15 bits for the decimal part.

The transmitter was placed 4 meters away from the receiver. For the measurements used herein, the testbed had $n_T = 4$ and $n_R = 1$ transmit and receive antennas respectively using a uniform linear antenna array at the transmitter. It was operated with a carrier frequency of 5.6 GHz. Making use of the channel estimation methodology presented in [4], we obtained the physical channel response, $\mathbf{h}$, fixing the weights value to one in order to bypass the *Weighting block*. Since there not exist any movement or people around the scenario, the channel can be considered time-invariant. We carried out this channel estimation 1000 times to prove the lack of variability (see Fig. 3). The last channel estimation was carried out 10 minutes later than the first one.

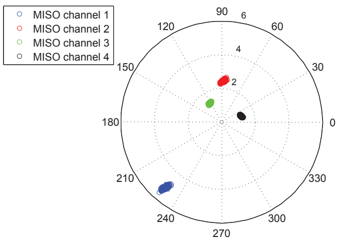


Fig. 3. Initial channel estimation with weights set to one, resulting time-invariant channels.

The measured magnitude of the MISO channel to be considered for both simulations and measurement was

$$|\mathbf{h}| = \begin{bmatrix} 5.0750 \\ 2.4195 \\ 1.2912 \\ 1.2102 \end{bmatrix} \qquad \arg(\mathbf{h}) = \begin{bmatrix} -128.6567 \\ 89.4564 \\ 123.9377 \\ 18.2808 \end{bmatrix}.$$

Let assume we desire to emulate a Rician fading channel with the following LOS part

$$|\mathbf{h}_{ed}^o| = \begin{bmatrix} 0.5500 \\ 0.8803 \\ 0.8944 \\ 0.6708 \end{bmatrix} \qquad \arg(\mathbf{h}_{ed}^o) = \begin{bmatrix} 14.8969 \\ 7.1620 \\ 11.4592 \\ 46.9825 \end{bmatrix}$$

and the following spatial covariance

$$|\mathbf{R}_h^o| = \begin{bmatrix} 0.0605 & 0 & 0.0583 & 0 \\ 0 & 0.1550 & 0.1360 & 0 \\ 0.0583 & 0.1360 & 0.1600 & 0 \\ 0 & 0 & 0 & 0.0900 \end{bmatrix} \tag{12}$$

$$\arg(\mathbf{R}_h^o) = \begin{bmatrix} 0 & 0 & -59.0362 & 0 \\ 0 & 0 & 36.0274 & 0 \\ 59.0362 & -36.0274 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Once we obtain the $N$ weights realizations, we get the equivalent channel according to (2). To estimate the LOS component and the covariance matrix we use

$$\hat{\mathbf{h}}_{ed} = \frac{1}{N}\sum_{1}^{N} \mathbf{h}_e \tag{13}$$

$$\hat{\mathbf{R}}_{he} = \frac{1}{N}\sum_{1}^{N} (\mathbf{h}_e - \mathbf{h}_{ed}) \left(\mathbf{h}_e^H - \mathbf{h}_{ed}\right) \tag{14}$$

### A. *Simulation results*

First, from (10) we computed the weights covariance matrix. Then, by using (11) we obtained $N = 3 \cdot 10^4$ weight vector realizations. From them we obtain the equivalent channels according to (2) and we estimate the covariance matrix making use of (14), resulting

$$\left|\hat{\mathbf{R}}_{he}\right| = \begin{bmatrix} 0.0706 & 0.0078 & 0.0558 & 0.0030 \\ 0.0078 & 0.1381 & 0.1309 & 0.0031 \\ 0.0558 & 0.1309 & 0.1563 & 0.0026 \\ 0.0030 & 0.0031 & 0.0026 & 0.0876 \end{bmatrix} \tag{15}$$

$$\arg\left(\hat{\mathbf{R}}_{he}\right) = \begin{bmatrix} 0 & -98.2188 & -59.5610 & -15.3398 \\ 98.2188 & 0 & 35.7360 & -24.9667 \\ 59.5610 & -35.7360 & 0 & -18.6121 \\ 15.3398 & 24.9667 & 18.6121 & 0 \end{bmatrix}.$$

By comparing (12) and (15) we can verify the proper behavior of our method since the results match up very well with the desired covariance matrix. Obviously, the comparison among phase results whose elements are zero in the magnitude matrix are not of interest. Fig. 4 shows the histograms for the magnitudes of the entries of the equivalent channel, $\mathbf{h}_e$.

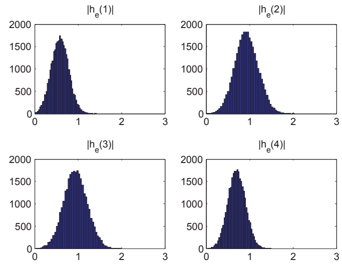


Fig. 4. Distributions of the $4 \times 1$ MISO channel (simulation).

The estimated absolute value and the phase of the LOS component are

$$\left|\hat{\mathbf{h}}_{ed}\right| = \begin{bmatrix} 0.5498 \\ 0.8874 \\ 0.8966 \\ 0.6719 \end{bmatrix} \qquad \arg\left(\hat{\mathbf{h}}_{ed}\right) = \begin{bmatrix} 14.8170 \\ 7.0065 \\ 11.1829 \\ 47.2795 \end{bmatrix} \tag{16}$$

which match up very well with (IV).

### B. *Measurements*

In this case, we just obtain $N = 1000$ temporally correlated weights realizations from (3), following a Rician distribution. Making use of the channel estimation methodology presented in [4], we estimate, for all the weights realizations, the equivalent channel. To estimate the empirical covariance matrix from those equivalent channels we use (14). The estimated covariance matrix is

$$\left|\hat{\mathbf{R}}_{he}\right| = \begin{bmatrix} 0.0829 & 0.0084 & 0.0530 & 0.0104 \\ 0.0084 & 0.1726 & 0.1399 & 0.0134 \\ 0.0530 & 0.1399 & 0.1600 & 0.0049 \\ 0.0104 & 0.0134 & 0.0049 & 0.0949 \end{bmatrix} \tag{17}$$

$$\arg\left(\hat{\mathbf{R}}_{he}\right) = \begin{bmatrix} 0 & 0.9116 & -47.6868 & -47.8626 \\ -0.9116 & 0 & 31.1895 & -76.9322 \\ 47.6868 & -31.1895 & 0 & -61.5086 \\ 47.8626 & 76.9322 & 61.5086 & 0 \end{bmatrix},$$

and the estimated deterministic component is

$$\left|\hat{\mathbf{h}}_{ed}\right| = \begin{bmatrix} 0.5580 \\ 0.8979 \\ 0.8647 \\ 0.6622 \end{bmatrix} \quad \arg\left(\hat{\mathbf{h}}_{ed}\right) = \begin{bmatrix} 16.2647 \\ 5.4304 \\ 9.1129 \\ 45.5734 \end{bmatrix}. \tag{18}$$

By comparing (17) and (15), we realize that the measurement results do not match very well with the simulated ones. Noise and impairments affect the channel estimation, but the gap between them is due to the different number of realizations. To verify this fact, we took those 1000 weights realizations and we calculate, by simulation, the equivalent channel. Finally, we obtain the covariance matrix making use of (14)

$$\left|\hat{\mathbf{R}}_{he}\right| = \begin{bmatrix} 0.0805 & 0.0054 & 0.0536 & 0.0090 \\ 0.0054 & 0.1613 & 0.1369 & 0.0103 \\ 0.0536 & 0.1369 & 0.1600 & 0.0037 \\ 0.0090 & 0.0103 & 0.0037 & 0.0953 \end{bmatrix} \tag{19}$$

$$\arg\left(\hat{\mathbf{R}}_{he}\right) = \begin{bmatrix} 0 & -27.47 & -55.4035 & -48.6718 \\ 27.4713 & 0 & 36.0939 & -105.7887 \\ 55.4035 & -36.0939 & 0 & -80.2164 \\ 48.6718 & 105.7887 & 80.2164 & 0 \end{bmatrix}.$$

and the estimated deterministic component

$$\left|\hat{\mathbf{h}}_{ed}\right| = \begin{bmatrix} 0.5589 \\ 0.9105 \\ 0.8995 \\ 0.6738 \end{bmatrix} \quad \arg\left(\hat{\mathbf{h}}_{ed}\right) = \begin{bmatrix} 16.3674 \\ 4.8773 \\ 9.2676 \\ 45.5806 \end{bmatrix}. \tag{20}$$

As can be seen, regarding covariance matrix, (17) and (19) match very well; as well as the deterministic components, (18) and (20). If we compare these results with those obtained by simulation we appreciate differences mostly caused by the number of realizations. This means that asymptotically all the results would match up.

On the other hand, we need to verify that the distributions of the equivalent channels follow the Rician distribution. Fig. 5 shows the histograms for the magnitudes of the entries of the equivalent channel, $\mathbf{h}_e$.

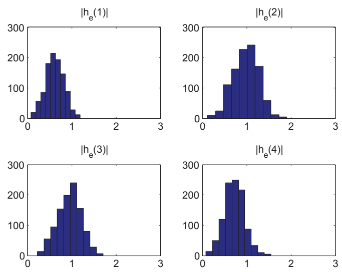


Fig. 5. Distributions of the $4 \times 1$ MISO channel (measurements).

In Fig. 6, the temporal evolution of one of the estimated channels, $\hat{\mathbf{h}}_{ed}(i)$, in order to verify that we get, both in simulation and empirically, similar results. Making use of the methodology presented in [4], the time among one channel estimation to the subsequent is approximately 1 sec. in mean. Therefore, the channel sampling frequency is 1 Hz and, assuming a normalized Doppler frequency of 0.1, we deal with a Doppler frequency $f_D = 0.1$ Hz.

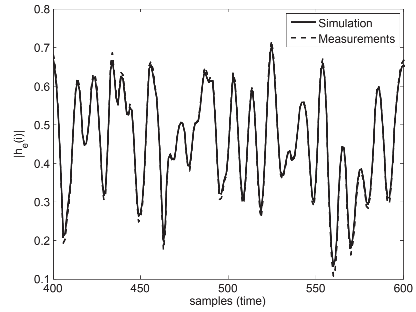


Fig. 6. Temporal evolution comparison.

## V. Conclusions

In this paper, a new framework for emulating spatially and temporal correlated MIMO Rician fading channels based on time-varying beamforming is proposed. It can be applied on arbitrary fading parameters and pre-specified spatio-temporal correlation, allowing the evaluation of MISO algorithms under controlled and repeatable arbitrary conditions. This is the first stage in order to tackle MIMO channel emulation controlling both spatial and temporal correlation, whose development is in progress, following the same principles as the ones presented in this paper.

## VI. Acknowledgments

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