Frequency Sampling Design of Prototype Filters for Nearly Perfect Reconstruction Cosine-Modulated Filter Banks

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***Abstract*—A new approach to the design of prototype filters for conventional nearly perfect reconstruction (N-PR) cosine-modulated filter banks is presented. The new method is based on the frequency sampling approach for the design of finite-impulse response filters. In the proposed approach, the magnitude response values of samples in the transition band of the prototype filter are the only parameters to be optimized. The analytical and simulation results show that despite there being no direct control over the stopband attenuation of the prototype filter, the performance of the filter bank is extremely good, and in several cases, the whole system closely satisfies the PR property.**

***Index Terms*—Channel bank filters, cosine-modulated filter banks, filter bank design, filtering theory, frequency sampling technique.**

I. Introduction

Cosine-modulated finite-impulse response (FIR) filter banks are used extensively in applications such as speech and image coding, biomedical signal processing and transmultiplexer design. They offer two main advantages: 1) under certain conditions the analysis and synthesis stages can be implemented using fast algorithms; and 2) the design of the filter bank is very straightforward, as it is only necessary to design one or two prototype filters. In this letter, we focus on this second point.

Several efficient methods have been put forward that facilitate the design of prototype filters for conventional nearly perfect reconstruction (N-PR) cosine-modulated filter banks [1]. The technique proposed in [2] establishes a relationship between the $2M$th band filter and a set of m quadratic constraints in the impulse response coefficients of the prototype filter. These constraints and the stopband attenuation of the prototype filter are minimized. This technique makes the filter banks extremely close to the PR property. The programming of the algorithm however, is not simple. Furthermore, the design of the prototype filter requires a great deal of computational effort. As a result of continued efforts in this field, several techniques that have simpler minimizing cost functions have been developed, however they are not as close to the PR as the filter bank proposed in [2]. For its efficiency and simplicity, we mention, the method proposed by Creusere and Mitra (CAM) [3], the Kaiser window approach (KWA) [4], the design processes in [5], and the recent technique proposed in [6].

In this letter, we propose a new prototype filter design technique for conventional cosine-modulated filter banks. This method is based on the frequency sampling approach for designing FIR filters [7], [8], and only the magnitude response values of the samples in the transition band are optimized. As a result, the analysis and synthesis filters show good magnitude responses, and the overall system is very close to the PR property. The experimental results demonstrate the effectiveness of this new technique.

II. Frequency Sampling Approach

The frequency sampling approach is usually used to design recursive FIR filters, whose structure can be made computationally efficient. Let $p[n]$ be the impulse response coefficients of the N-length prototype filter and $P(e^{j\omega})$ be its frequency response. Let $P[k] \equiv P(e^{j\omega_k})$, where $\omega_k = (k+\alpha)\cdot 2\pi/N$, $0 \le k \le (N-1)$, and $\alpha = 0$ or $\alpha = 1/2$. The relation between the impulse response coefficients and the frequency response samples is given by

$$p[n] = \frac{1}{N}\sum_{k=0}^{N-1} P(e^{j\omega_k})\cdot e^{j2\pi/N(k+\alpha)n}, \qquad n = 0, 1\ldots, N-1. \tag{1}$$

For simplicity, we consider that N is an even number, $\alpha = 0$, and that $p[n]$ is real and symmetric, i.e., $p[n] = p[N-1-n]$. In this particular case, the filter coefficients can be obtained using the following equation [8]:

$$p[n] = \frac{1}{N}\left\{H[0] + 2\cdot\sum_{k=1}^{N/2-1} H[k]\cdot\cos\left(\left(n+\frac{1}{2}\right)\cdot\frac{2\pi k}{N}\right)\right\} \qquad n = 0, 1\ldots, N-1 \tag{2}$$

where $H[k] = P[k]\cdot e^{-jk\pi/N}$ and $0 \le k \le N-1$.

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III. Prototype Filter Design

Based on the technique mentioned in the previous section, we can design a lowpass linear-phase FIR prototype filter. Moreover, when the cutoff frequency of the stopband attenuation ω_s is located at approximately π/M, i.e.,

$$\left|P(e^{j\omega})\right| \approx 0, \qquad |\omega| > \frac{\pi}{M} \tag{3}$$

the distortion transfer function $T(e^{j\omega})$ is [2]

$$\begin{aligned} T(e^{j\omega}) &= \sum_{k=0}^{M-1} F_k(e^{j\omega}) \cdot H_k(e^{j\omega}) \\ &= e^{-j\omega(N-1)} \sum_{k=0}^{2M-1} \left|P(e^{j(\omega-k\cdot\pi/M)})\right|^2 \end{aligned} \tag{4}$$

where $H_k(e^{j\omega})$ and $F_k(e^{j\omega})$ are, respectively, the analysis and synthesis filters, which are obtained by applying the conventional cosine modulation to the prototype filter $p[n]$ [1]. This modulation provides alias cancellation between adjacent channels, and if the prototype filter has linear phase, the distortion transfer function $T(e^{j\omega})$ will also have linear phase. Therefore, the reconstructed signal does not suffer from phase distortion and the aliasing error is reduced.

The prototype filter is designed so that the magnitude response $\left|T(e^{j\omega})\right|$ is approximately flat for all values of ω. To this end, several methods have been proposed that are based on different objective functions. For example, the objective function proposed in [3] is defined as follows. Find the prototype filter coefficients that minimize

$$\phi = \max_{\omega} \left\{ \left|P(e^{j\omega})\right|^2 + \left|P(e^{j(\omega-\pi/M)})\right|^2 - 1 \right\} \tag{5}$$

where $\omega \in (0, \pi/M)$. This function guarantees that $\left|T(e^{j\omega})\right| \approx 1$.

On the other hand, the objective function ϕ proposed in [4] is defined as follows. Find the prototype filter coefficients that minimize

$$\phi = \max_{n, n\neq 0} |g[2Mn]| \tag{6}$$

where $g[n]$ is the inverse Fourier transform of $G(e^{j\omega}) = \left|P(e^{j\omega})\right|^2$. This technique provides prototype filters that are spectral factors of approximately $2M$th band filters, and therefore $\left|T(e^{j\omega})\right| \approx 1$.

If we design the prototype filters using the objective functions proposed in (5) or (6), the conditions for approximate reconstruction in the M-channel cosine-modulated filter bank are fulfilled.

A. Initial Specifications

In this subsection we indicate the initial values that $P[k]$ must have, i.e., the samples of the frequency response of the prototype filter before the optimizing process starts. Before indicating these values it is necessary to make certain observations. First, the number $L \geq 1$ of samples in the transition band—its width being approximately $(L+1)\cdot 2\cdot\pi/N$—must be selected. Next, the center of the transition band must be approximately situated at the frequency $\pi/(2M)$. Therefore, $\omega = r\cdot 2\pi/N$ is placed at the value closest to $\omega_c = \pi/(2M)$, where N is the length of the prototype filter, and M the number of channels and $r \in Z^+$. We proceed in this way because the value of the magnitude response at $\omega_c = \pi/(2M)$ plays an important role in obtaining an approximately PR filter bank [6].

Given L, r, and N, the value of the magnitude response for each sample is initially defined as[1]

$$|P[k]| = \begin{cases} 1, & 0 \leq k \leq r - \left\lceil \frac{L}{2} \right\rceil \text{ (passband)} \\ \left(\frac{\omega_s - k\cdot 2\cdot\frac{\pi}{N}}{\omega_s - \omega_p}\right), & r - \left\lceil \frac{L}{2} \right\rceil + 1 \leq k \leq r + \left\lfloor \frac{L}{2} \right\rfloor \text{ (transition band)} \\ 0, & r + \left\lfloor \frac{L}{2} \right\rfloor + 1 \leq k \leq \frac{N}{2} - 1 \text{ (stopband)} \end{cases} \tag{7a}$$

$$|P[N-1-k]| = |P[k]| \quad \frac{N}{2} \leq k \leq (N-1) \tag{7b}$$

where ω_p is the frequency corresponding to the last sample of the passband, and ω_s is the frequency corresponding to the first sample of the stopband. The value of the phase response for each sample is defined in the following way:

$$\arg\{P[k]\} = \begin{cases} -\frac{(N-1)}{2} \cdot \frac{2\pi}{N} \cdot k & k = 0, \ldots, \frac{N}{2} - 1 \\ \frac{(N-1)}{2} \cdot \frac{2\pi}{N} \cdot (N-k) & k = \frac{N}{2}, \ldots, N-1 \end{cases} \tag{8}$$

B. Optimization Procedure

The optimization procedure consists of the following steps:

Step 1). Select the filter length N.

Step 2). Select the required number L of samples in the transition band.

Step 3). Let $\mathbf{t} = [t[1] \ \ t[2] \ \ \cdots \ \ t[L]]$ be a vector whose elements are the samples of the magnitude response in the transition band. Initialize the N samples of the frequency response as described in the preceding subsection [**(7)** and **(8)**]. The resulting vector $|\mathbf{P}|$, with corresponding magnitude response values $|P[k]|$, $0 \leq k \leq N/2 - 1$, is

$$|\mathbf{P}| = \Big[\underbrace{1 \quad 1 \quad \cdots \quad 1}_{\text{passband}} \quad \underbrace{t[1] \quad \cdots \quad t[L]}_{\text{transition band}} \quad \underbrace{0 \quad \cdots \quad 0}_{\text{stopband}}\Big]. \tag{9}$$

Step 4). Find $\mathbf{t}_{\text{opt}}$, i.e., the values of the components of $\mathbf{t}$ that minimize the objective function defined in **(5)** or **(6)**.

Step 5). Calculate the optimum values of the frequency response $P_{\text{opt}}[k]$. These values are obtained from **(7)** and **(8)**, by replacing the initial values of the magnitude response in the transition band in **(7a)** by the optimized values $\mathbf{t}_{\text{opt}}$ obtained in the Step 4).

Step 6). Obtain the prototype filter coefficients $p[n]$ through the Inverse Discrete Fourier Transform (IDFT) of $P_{\text{opt}}[k]$, that is, by using **(1)** and $\alpha = 0$.

The objective function ϕ to be minimized in **Step 4)** can be defined as in [3] [(5)], as in [4] [(6)], or by defining a different cost function that yields a prototype filter that is a spectral factor

[1] $\lfloor\cdot\rfloor$ and $\lceil\cdot\rceil$ mean rounding to the next larger/smaller integer, respectively.

TABLE I
EXAMPLE 2. RESULTS OBTAINED USING 64-CHANNEL CONVENTIONAL COSINE-MODULATED FILTER BANKS DESIGNED WITH DIFFERENT PROTOTYPE FILTERS

Prototype Filter Design Technique	Stopband Attenuation (dB)	*Rpp*	*Ea*	*PSNR*
$U\omega_{C3dB}$- Blackman window	75.3	$1.8444 \cdot 10^{-3}$	$4.0139 \cdot 10^{-7}$	80.0011
$U\omega_{C3dB}$- Kaiser window (β=4.3124)	48	0.0315	$1.1455 \cdot 10^{-5}$	54.9925
KWA (β=4.3124)	48	0.0309	$1.1332 \cdot 10^{-5}$	55.3517
CAM	85	0.0022	$9.6294 \cdot 10^{-6}$	75.8461
Proposed Technique	48	$1.6061 \cdot 10^{-3}$	$3.5665 \cdot 10^{-6}$	82.4336

of an approximately $2M$th band filter. The main difference between the method that we present here and the methods in [3] and [4] is the technique used in designing the prototype FIR filter. In this letter, we use the frequency sampling approach and optimize the magnitude response of the samples in the transition band. In [3], the FIR filter is designed using the Parks–McClellan's method and the stopband cutoff frequency is optimized. In [4], the adopted design technique is the Kaiser windowing method, where the cutoff frequency of the ideal filter that is being windowed is optimized.

We would like to emphasize some important aspects of the optimization mentioned earlier. First, since the number of samples L in the transition band is low and the optimization is only performed on these samples, the complexity is reduced. This number L is obtained in an empirical way, and is limited by the number $\lfloor N/2M \rfloor$ due to the fact that ω_s must be located approximately at π/M. Second, as shown in the following section, the filter bank is very close to the PR property. It should be stressed that with this technique there is no direct control over the stopband attenuation in the prototype filter, which means that the technique proposed here is not efficient when this characteristic is fixed in advance, as is the case in the examples collected in [4] and [6].

IV. DESIGN EXAMPLES

In this section, we show three examples designed with MATLAB. The minimization algorithm has been implemented using the function `fminsearch`, which finds the minimum of an unconstrained multivariable function.

A. Example 1

The optimization algorithm provides generally good prototype filters, and sometimes, the characteristic of the resulting filter banks are very close to the PR property. We include as the first example the results obtained when designing a *64*-length prototype filter for a 16-channel filter bank. The design parameters are: $N = 64$, $L = 2$, $r = 1$ and the cost function in (5). The resulting filter bank shows a peak amplitude distortion $R_{pp} = 2.2188 \cdot 10^{-9}$, a maximum aliasing error $E_a = 2.4208 \cdot 10^{-10}$ (defined as in [1]), and a peak signal-to-noise ratio PSNR $= 181.1631$ dB. With these results, the filter bank can be considered very near to the PR property. The optimized values of the magnitude response of the transition band samples are $|P[1]| = 0.707\,106\,782\,338\,73 \approx 1/\sqrt{2}$ and $|P[2]| = 0.000\,052\,333\,576\,72$.

B. Example 2

In this example, we have designed filter banks with *64* channels by applying the conventional cosine modulation to different 768-length prototype filters. The methods used for the design of each prototype filter are: the CAM [3], the KWA [4], the location of the ω_{c3} dB $(U\omega_{c3}$ dB$)$ [6] and the proposed (PT) techniques. In the last method, we have the following design parameters: $N = 768$, $L = 5$, $r = 3$, and the cost function of (5). The results relating to the minimum stopband attenuation of the prototype filter, the peak amplitude distortion R_{pp} and the maximum aliasing error E_a are shown in Table I. The peak signal-to-noise ratio (PSNR) is also included, obtained after applying an electrocardiogram signal as the input to the filter banks. The optimized values of the magnitude response of the transition band samples are $|P[1]| = 0.998\,666$, $|P[2]| = 0.960\,34$, $|P[3]| = 0.707\,12$, $|P[4]| = 0.278\,74$ and $|P[5]| = 0.052\,078$.

C. Example 3

As the third example we show the results obtained when designing a *3072*-length prototype filter for a *256*-channel filter bank. The design parameters are $N = 3072$, $L = 5$, $r = 3$ and the cost function in (5). The resulting filter bank shows a peak amplitude distortion $R_{pp} = 8.4144 \cdot 10^{-4}$, a maximum aliasing error $E_a = 4.764\,83 \cdot 10^{-7}$, and a peak signal-to-noise ratio PSNR $= 89.6366$ dB. The optimized values of the transition band samples are $|P[1]| = 0.996\,801\,3$, $|P[2]| = 0.950\,208$, $|P[3]| = 0.707\,143$, $|P[4]| = 0.311\,684\,5$ and $|P[5]| = 0.079\,422$. Fig. 1(a) shows the prototype filter magnitude response, and the resulting function $\left|T(e^{j\omega})\right|$ of the filter bank is plotted in Fig. 1(b). Finally, Fig. 1(c) shows the aliasing error function $T_{a\ell}(e^{j\omega})$, defined as in [1, eq. (8.2.10)].

From the simulations performed we observe that when the rate (prototype filter length)/(number of channels), i.e., N/M is low, the best results are obtained by using the cost function in (5). On the other hand, if the rate N/M increases, it is preferable to use the function of (6). Taking into consideration the above arguments, it can be stated that in general the convergence of the algorithm is ensured and the behavior of the resulting filter bank is very good.

V. CONCLUSION

We have presented a new technique for obtaining prototype filters for nearly PR cosine-modulated filter banks based on the frequency sampling approach for the design of FIR filters. Filter banks designed by using the proposed algorithm have very

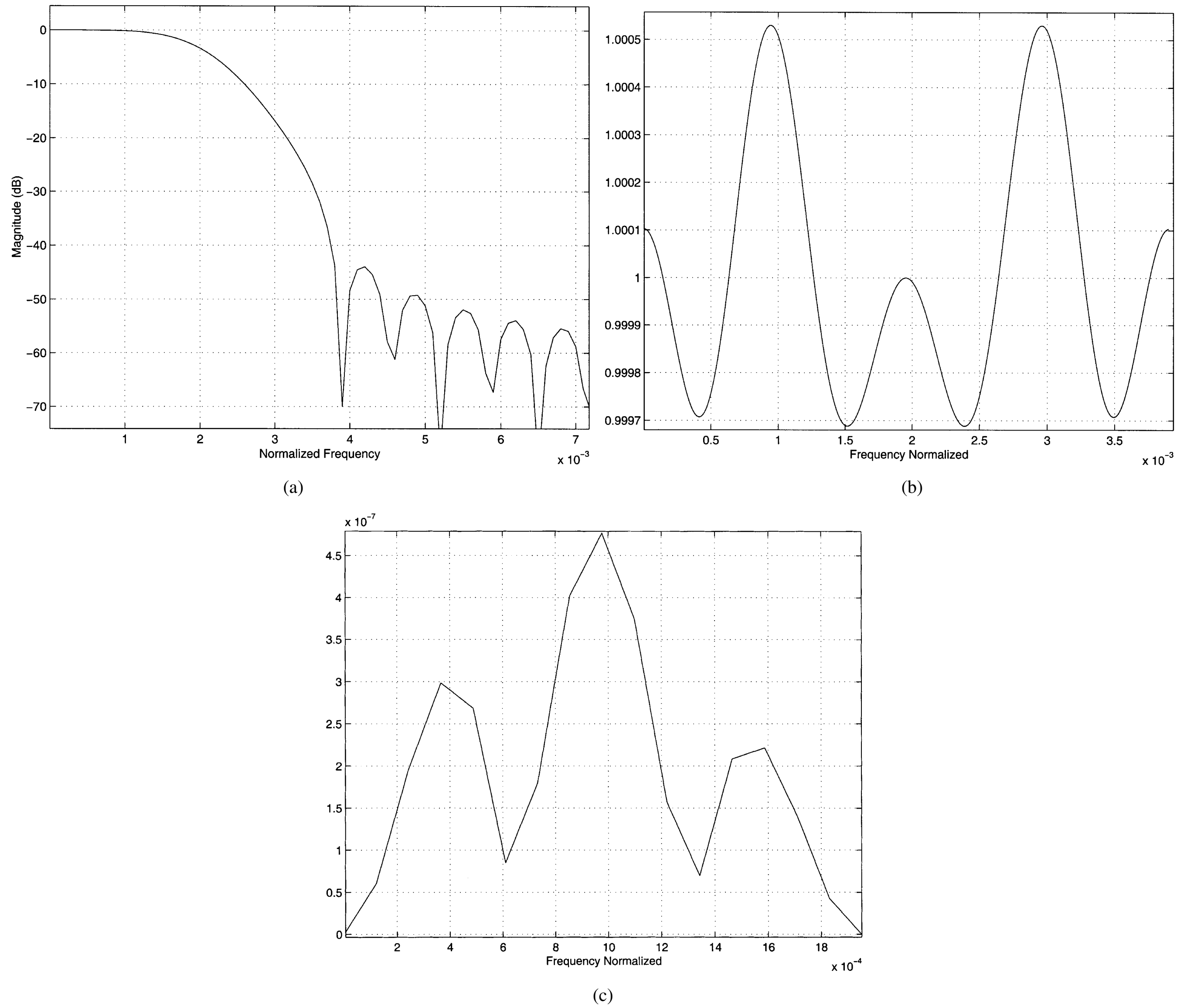

Fig. 1. Example 3. (a) Detail of the magnitude response of the prototype filter $P(z)$. (b) Magnitude response plot of $T(e^{j\omega})$ (period $\pi/256$) in the interval [0, $\pi/256$]. (c) Aliasing error function $T_{a\ell}(e^{j\omega})$ (period $\pi/512$) in the interval [0, $\pi/512$].

good characteristics that closely follow the PR property. This is achieved when the cost function to be optimized and the number of samples in the transition band are properly chosen in order for the algorithm to converge to optimum solutions. A drawback of this method is that there is no direct control over the stopband attenuation of the prototype filter and, in some applications this can be an essential requirement.

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