A SUFFICIENT CONDITION FOR BLIND IDENTIFIABILITY OF MIMO-OSTBC CHANNELS BASED ON SECOND ORDER STATISTICS

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ABSTRACT

In this paper the conditions for blind identifiability of multiple-input multiple-output (MIMO) channels under orthogonal space-time block coded (OSTBC) transmissions are studied. Specifically we prove that, regardless of the number of receive antennas, any real or complex OSTBC transmitting an odd number of real symbols permits the blind identification of the MIMO channel by only exploiting the second order statistics (SOS) of the received signal. This result extends to complex OSTBCs and provides an alternative proof of an identifiability theorem previously proved only for real OSTBCs in [1]. Furthermore, this sufficient condition suggests that any non-identifiable OSTBC can be made identifiable simply by not transmitting one real symbol of each block (either the real or imaginary part of a symbol in the case of complex OSTBCs). In order to minimize the reduction in the transmission rate, several OSTBC blocks are grouped before erasing one real symbol. It is shown in the paper that the proposed rate-reduction technique outperforms the differential OSTBC scheme in terms of capacity for a wide range of SNRs.

1. INTRODUCTION

In recent years, orthogonal space-time block coding (OSTBC) [2, 3] has emerged as one of the most promising techniques to exploit spatial diversity and to combat fading in multiple-input multiple-output (MIMO) systems. The special structure of OSTBCs implies that, assuming that the MIMO channel is known at the receiver, the optimal maximum likelihood (ML) decoder is a simple linear receiver, which can be seen as a matched filter followed by a symbol-by-symbol detector.

Training approaches are typically used to obtain an estimate of the channel at the receiver. However, the use of a training sequence implies a reduction on the bandwidth efficiency, which is avoided by other approaches like differential space-time codes [4] or blind channel estimation techniques [5, 6, 7]. Specifically, the blind channel estimation method proposed in [7] is based only on second order statistics (SOS) and it has a reduced computational complexity. However, there exist some OSTBCs (including the popular Alamouti code) that cannot be unambiguously identified using the SOS-based method proposed in [7]. The sufficient and necessary conditions (related to the underlying structure of the ST code) for the blind identifiability of the MIMO channel remain unclear. This problem has recently been studied in [1, 8], where some sufficient conditions ensuring the blind identifiability of OSTBCs have been found. Specifically, in [1] it is proved that, for real OSTBCs transmitting an odd number of real symbols, the channel can be unambiguously identified up to a scalar.

In this work we consider the blind identifiability of OSTBCs and provide several new results. First, we prove that if the channel is identifiable using only the SOS of the received signal, then the channel can be unambiguously estimated by the method proposed in [7]. This result shows that those OSTBCs for which the multiplicity order of the principal eigenvalue of the correlation matrix is larger than one are non-identifiable due to the underlying structure of the code and not to the applied estimation method. Second, we generalize the result in [1] to complex OSTBCs and provide an alternative proof. Finally, the new sufficient condition for blind identifiability suggests how a non-identifiable OSTBC can be transformed into an identifiable one. The proposed technique does not transmit one real symbol out of every B OSTBC blocks. Since the number of blocks B can be made arbitrarily large, this symbol erasure procedure only provokes a slight reduction in the transmission rate. The proposed technique is analyzed in terms of capacity, proving that, if the received signal to noise ratio (SNR) is under a specified threshold, the rate-reduction technique outperforms the differential scheme.

2. SOME BACKGROUND ON OSTBCS

Throughout this paper we will use bold-faced upper case letters to denote matrices, e.g., $\mathbf{X}$, with elements x_{ij}; bold-faced lower case letters for column vector, e.g., $\mathbf{x}$, and light-face lower case letters for scalar quantities. The superscripts $(\cdot)^T$ and $(\cdot)^H$ denote transpose and Hermitian, respectively. The real and imaginary parts will be denoted as $\Re(\cdot)$ and $\Im(\cdot)$, and superscript $\hat{(\cdot)}$ will denote estimated matrices, vectors or scalars. The trace, range (or column space) and Frobenius norm of matrix $\mathbf{A}$ will be denoted as $\mathrm{Tr}(\mathbf{A})$, $\mathrm{range}(\mathbf{A})$ and $\|\mathbf{A}\|$ respectively. Finally, the identity matrix of the required dimensions will be denoted as $\mathbf{I}$, and $E[\cdot]$ will denote the expectation operator.

A flat fading MIMO system with n_T transmit and n_R receive antennas is assumed. The $n_T \times n_R$ complex channel matrix is

$$\mathbf{H} = [\mathbf{h}_1 \cdots \mathbf{h}_{n_R}],$$

where $\mathbf{h}_j = [h_{1j}, \ldots, h_{n_T j}]^T$ contains the channel responses associated with the j-th receive antenna. The complex noise at the receive antennas is considered both spatially and temporally white with variance σ^2.

2.1. Data Model for OSTBCs

Let us consider a space-time block code (STBC) transmitting M symbols during L slots and using n_T antennas at the transmitter side.

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The transmission rate is defined as $R = M/L$, and the number of real symbols transmitted in each block is

$$M' = \begin{cases} M & \text{for real constellations,} \\ 2M & \text{for complex constellations.} \end{cases}$$

For a STBC, the n-th block of data can be expressed as

$$\mathbf{S}[n] = \sum_{k=1}^{M'} \mathbf{C}_k s_k[n],$$

where $\mathbf{C}_k$ are the STBC code matrices,

$$s_k[n] = \begin{cases} \Re(r_k[n]), & k \leq M, \\ \Im(r_{k-M}[n]), & k > M, \end{cases}$$

and $r_k[n]$ denotes the k-th complex symbol of the n-th STBC block.

The combined effect of the STBC code and the j-th channel can be represented by means of the vectors

$$\mathbf{w}_k(\mathbf{h}_j) = \mathbf{C}_k \mathbf{h}_j, \qquad k = 1, \ldots, M',$$

and taking into account the isomorphism between complex vectors $\mathbf{w}_k(\mathbf{h}_j)$ and real vectors $\tilde{\mathbf{w}}_k(\mathbf{h}_j) = \left[\Re(\mathbf{w}_k(\mathbf{h}_j))^T, \Im(\mathbf{w}_k(\mathbf{h}_j))^T\right]^T$ we can define the extended code matrices

$$\tilde{\mathbf{D}}_k = \begin{bmatrix} \Re(\mathbf{C}_k) & -\Im(\mathbf{C}_k) \\ \Im(\mathbf{C}_k) & \Re(\mathbf{C}_k) \end{bmatrix},$$

which satisfy $\tilde{\mathbf{w}}_k(\mathbf{h}_j) = \tilde{\mathbf{D}}_k \tilde{\mathbf{h}}_j$, with $\tilde{\mathbf{h}}_j = \left[\Re(\mathbf{h}_j)^T, \Im(\mathbf{h}_j)^T\right]^T$. The signal at the j-th receive antenna is

$$\mathbf{y}_j[n] = \sum_{k=1}^{M'} \mathbf{w}_k(\mathbf{h}_j) s_k[n] + \mathbf{n}_j[n],$$

where $\mathbf{n}_j[n]$ is the white complex noise with variance σ^2.

Defining now the real vectors $\tilde{\mathbf{y}}_j[n] = [\Re(\mathbf{y}_j[n])^T, \Im(\mathbf{y}_j[n])^T]^T$ and $\tilde{\mathbf{n}}_j[n] = [\Re(\mathbf{n}_j[n])^T, \Im(\mathbf{n}_j[n])^T]^T$, the above equation can be rewritten as

$$\tilde{\mathbf{y}}_j[n] = \sum_{k=1}^{M'} \tilde{\mathbf{w}}_k(\mathbf{h}_j) s_k[n] + \tilde{\mathbf{n}}_j[n] = \tilde{\mathbf{W}}(\mathbf{h}_j)\mathbf{s}[n] + \tilde{\mathbf{n}}_j[n],$$

where $\mathbf{s}[n] = [s_1[n], \ldots, s_{M'}[n]]^T$ contains the M' transmitted real symbols and $\tilde{\mathbf{W}}(\mathbf{h}_j) = [\tilde{\mathbf{w}}_1(\mathbf{h}_j) \cdots \tilde{\mathbf{w}}_{M'}(\mathbf{h}_j)]$. Finally, stacking all the received signals into $\tilde{\mathbf{y}}[n] = \left[\tilde{\mathbf{y}}_1^T[n], \ldots, \tilde{\mathbf{y}}_{n_R}^T[n]\right]^T$, we can write

$$\tilde{\mathbf{y}}[n] = \tilde{\mathbf{W}}(\mathbf{H})\mathbf{s}[n] + \tilde{\mathbf{n}}[n],$$

where $\tilde{\mathbf{W}}(\mathbf{H}) = \left[\tilde{\mathbf{W}}^T(\mathbf{h}_1) \cdots \tilde{\mathbf{W}}^T(\mathbf{h}_{n_R})\right]^T$, and $\tilde{\mathbf{n}}[n]$ is defined analogously to $\tilde{\mathbf{y}}[n]$.

In the case of orthogonal STBCs (OSTBCs), the matrix $\tilde{\mathbf{W}}(\mathbf{H})$ satisfies

$$\tilde{\mathbf{W}}^T(\mathbf{H})\tilde{\mathbf{W}}(\mathbf{H}) = \|\mathbf{H}\|^2 \mathbf{I}, \tag{1}$$

which, considering $\mathbf{H}$ known and a Gaussian distribution for the noise, reduces the complexity of the ML receiver to find the closest symbols to the estimated signal [4]

$$\hat{\mathbf{s}}[n] = \frac{\tilde{\mathbf{W}}^T(\mathbf{H})\tilde{\mathbf{y}}[n]}{\|\mathbf{H}\|^2}.$$

The necessary and sufficient conditions on the code matrices $\mathbf{C}_k \in \mathbb{C}^{L \times n_T}$, $(k, l = 1, \ldots, M')$, to satisfy (1) are [4]

$$\mathbf{C}_k^H \mathbf{C}_l = \begin{cases} \mathbf{I} & k = l, \\ -\mathbf{C}_l^H \mathbf{C}_k & k \neq l, \end{cases}$$

which also imply $\mathbf{S}^H[n]\mathbf{S}[n] = \|\mathbf{s}[n]\|^2 \mathbf{I}$ and

$$\tilde{\mathbf{D}}_k^T \tilde{\mathbf{D}}_l = \begin{cases} \mathbf{I} & k = l, \\ -\tilde{\mathbf{D}}_l^T \tilde{\mathbf{D}}_k & k \neq l. \end{cases}$$

3. PREVIOUS WORK ON BLIND IDENTIFIABILITY OF OSTBCS

Recently, a new method for blind channel estimation of OSTBC MIMO channels, which is only based on SOS, has been proposed [7]. It has been shown that this method is able to blindly identify the channel up to a real scalar ambiguity in most of the analyzed OSTBCs when the number of receive antennas is $n_R > 1$. However, some OSTBCs (including the Alamouti code [2]) can not be identified by this method without assuming a certain structure on the correlation matrix of the information symbols $\mathbf{R}_s = E[\mathbf{s}[n]\mathbf{s}^T[n]]$ [7, 9].

In this section, the method proposed in [7] is summarized, and the conditions that result in non-identifiable channels are pointed out. Additionally, we prove that these ambiguities are due to the OSTBC and channel $\mathbf{H}$, and not to the particular blind identification method proposed in [7].

3.1. General formulation

Let us start by writing the correlation matrix of $\tilde{\mathbf{y}}[n]$ as

$$\mathbf{R}_{\tilde{\mathbf{y}}} = E[\tilde{\mathbf{y}}[n]\tilde{\mathbf{y}}^T[n]] = \tilde{\mathbf{W}}(\mathbf{H})\mathbf{R}_s\tilde{\mathbf{W}}^T(\mathbf{H}) + \frac{\sigma^2}{2}\mathbf{I}.$$

The method proposed in [7] is based on the following optimization problem

$$\underset{\hat{\mathbf{H}}}{\text{argmax}}\ \text{Tr}\left(\tilde{\mathbf{W}}^T(\hat{\mathbf{H}})\mathbf{R}_{\tilde{\mathbf{y}}}\tilde{\mathbf{W}}(\hat{\mathbf{H}})\right), \quad \text{s. t.} \quad \tilde{\mathbf{W}}^T(\hat{\mathbf{H}})\tilde{\mathbf{W}}(\hat{\mathbf{H}}) = \mathbf{I}, \tag{2}$$

whose solution is given by any estimated channel matrix $\hat{\mathbf{H}}$ with $\|\hat{\mathbf{H}}\| = 1$ satisfying

$$\text{range}(\tilde{\mathbf{W}}(\hat{\mathbf{H}})) = \text{range}(\tilde{\mathbf{W}}(\mathbf{H})). \tag{3}$$

The solution of (2) can be obtained by means of an eigenvalue (EV) problem [7]. Alternatively, we have shown in [9] that (2) can be reformulated as a principal component analysis (PCA) problem, which permits a straightforward derivation of adaptive algorithms.

3.2. Indeterminacy problems

The constraint $\tilde{\mathbf{W}}^T(\hat{\mathbf{H}})\tilde{\mathbf{W}}(\hat{\mathbf{H}}) = \mathbf{I}$ in (2), which implies $\|\hat{\mathbf{H}}\| = 1$, introduces a real scalar ambiguity in the estimation process. This is a common indeterminacy for all the blind estimation techniques, then in the sequel we will assume $\|\hat{\mathbf{H}}\| = \|\mathbf{H}\| = 1$. A more important indeterminacy results from (3). Here, we prove that this ambiguity is not due to the particular optimization criterion given by (2), but to the code and channel properties. Let us start by rewriting (3) as $\tilde{\mathbf{W}}(\hat{\mathbf{H}}) = \tilde{\mathbf{W}}(\mathbf{H})\mathbf{Q}$, where $\mathbf{Q}$ is an orthogonal matrix (i.e., real and unitary) of dimensions $M' \times M'$, and introducing the following Lemma:

Lemma 1 *In OSTBC systems, the MIMO channel* $\mathbf{H}$ *can be identified up to a real scalar based only on second order statistics iff the equality*

$$\tilde{\mathbf{W}}(\hat{\mathbf{H}}) = \tilde{\mathbf{W}}(\mathbf{H})\mathbf{Q}, \tag{4}$$

where $\mathbf{Q}$ *is an orthogonal matrix, holds only for* $\hat{\mathbf{H}} = \pm\mathbf{H}$ *and* $\mathbf{Q} = \pm\mathbf{I}$.

Proof: From (2) and (3) it is clear that, if the equality (4) is only satisfied by $\hat{\mathbf{H}} = \pm\mathbf{H}$ and $\mathbf{Q} = \pm\mathbf{I}$, the channel can be estimated up to a scalar ambiguity by means of the criterion (2). To prove the converse, we proceed by contradiction: let us assume that there exists an estimate $\hat{\mathbf{H}} \neq \pm\mathbf{H}$ and an orthogonal matrix $\mathbf{Q} \neq \pm\mathbf{I}$ such that (4) holds. Then, we can define $\hat{\mathbf{s}}[n] = \mathbf{Q}^T\mathbf{s}[n]$ such that

$$\begin{aligned}\tilde{\mathbf{y}}[n] &= \tilde{\mathbf{W}}(\mathbf{H})\mathbf{s}[n] + \tilde{\mathbf{n}}[n] = \tilde{\mathbf{W}}(\mathbf{H})\mathbf{Q}\mathbf{Q}^T\mathbf{s}[n] + \tilde{\mathbf{n}}[n] = \\ &= \tilde{\mathbf{W}}(\hat{\mathbf{H}})\hat{\mathbf{s}}[n] + \tilde{\mathbf{n}}[n],\end{aligned}$$

which implies that the observation vector $\tilde{\mathbf{y}}[n]$ could be the result of a channel $\hat{\mathbf{H}}$ and a signal $\hat{\mathbf{s}}[n]$ instead of the true channel and signal. ■

The above Lemma implies that the ambiguities appearing in the method proposed in [7] are due to properties of the code and the channel and not to the specific criterion (2). According to the work by Shahbazpanahi et. al. [7] it is known that the indeterminacy in (3) is traduced in the fact that the largest eigenvalue of the associated EV problem has multiplicity larger than one. This identifiability problem has also been pointed out in [6], where the authors propose a method similar to the relaxed blind ML estimator.

Recently, some works have studied the identifiability conditions of OSTBCs. In [5] the authors have pointed out that it is impossible to achieve blind equalization for the Alamouti code [2] without using some precoding or assuming a correlation matrix $\mathbf{R}_s$ with non-equal eigenvalues. In [8] the authors study the identifiability conditions under the assumptions of real OSTBCs and BPSK signals, introducing the definition of non-rotatable and strictly non-rotatable codes. Real OSTBCs have also been studied in [1], where it has been proved that, if the symbol dimension M is odd, or if it is even and the channel matrix $\mathbf{H}$ is full row rank (which implies $n_R \geq n_T$), then the channel is identifiable up to a scalar ambiguity based solely on SOS. In the following sections we extend the first result in [1] to complex OSTBCs and present a straightforward technique for designing identifiable complex OSTBCs.

4. SUFFICIENT IDENTIFIABILITY CONDITION

In this section we show that for any OSTBC (real or complex) transmitting an odd number of real symbols, the MIMO channel can be identified up to a real scalar. Let us start by introducing some properties of skew-symmetric matrices

4.1. Properties of Skew-Symmetric Matrices

A skew-symmetric matrix $\mathbf{A}$ is defined as a square matrix with real entries satisfying $\mathbf{A}^T = -\mathbf{A}$. Some well-known properties of skew-symmetric matrices are the following:

Property 1 *All eigenvalues of skew-symmetric matrices are purely imaginary or zero.*

Property 2 *If* $\mathbf{A}$ *is skew-symmetric the elements along its main diagonal are zero:* $a_{ii} = 0, \forall i$. *Consequently,* $Tr(\mathbf{A}) = 0$.

A proof of Properties 1 and 2 can be found in [10]. For orthogonal matrices is easy to prove the following properties

Property 3 *An orthogonal skew-symmetric matrix* $\mathbf{A}$ *has the same number of* $+j$ *and* $-j$ *eigenvalues. Therefore, there do not exist orthogonal and skew-symmetric matrices of odd order.*

Proof: Since $\mathbf{A}$ is orthogonal, the absolute value of all its eigenvalues is 1. Combining this fact with properties 1 and 2, and taking into account that the trace of a matrix is equal to the sum of its eigenvalues, it is clear than an orthogonal skew-symmetric matrix must have the same number of $+j$ and $-j$ eigenvalues, and hence its order must be even. ■

4.2. Sufficient OSTBC Channel Identifiability Condition

We first extend Lemma 1 by showing that the orthogonal matrix $\mathbf{Q}$ in (4) must also be skew-symmetric, i.e., $\mathbf{Q}^T = -\mathbf{Q}$.

Lemma 2 *In OSTBC systems, the MIMO channel* $\mathbf{H}$ *cannot be identified up to a real scalar based only on second order statistics iff there exists an orthogonal skew-symmetric matrix* $\mathbf{Q}$ *of dimensions* $M' \times M'$ *such that*

$$\tilde{\mathbf{W}}(\hat{\mathbf{H}}) = \tilde{\mathbf{W}}(\mathbf{H})\mathbf{Q}. \tag{5}$$

Conversely, if such a matrix does not exist, the MIMO channel is identifiable.

Proof: Rewriting (4) as $\tilde{\mathbf{W}}(\hat{\mathbf{h}}_j) = \tilde{\mathbf{W}}(\mathbf{h}_j)\mathbf{Q}$, for $j = 1, \ldots, n_R$, and multiplying from the right by $\tilde{\mathbf{W}}^T(\mathbf{h}_j)$ we obtain $\|\mathbf{h}_j\|^2\mathbf{Q} = \tilde{\mathbf{W}}^T(\mathbf{h}_j)\tilde{\mathbf{W}}(\hat{\mathbf{h}}_j)$. Taking into account that $\tilde{\mathbf{w}}_k(\mathbf{h}_j) = \tilde{\mathbf{D}}_k\tilde{\mathbf{h}}_j$ and $\tilde{\mathbf{W}}(\mathbf{h}_j) = [\tilde{\mathbf{w}}_1(\mathbf{h}_j) \cdots \tilde{\mathbf{w}}_{M'}(\mathbf{h}_j)]$, we can write the element $q_{k,l}$ in the k-th row and l-th column of $\mathbf{Q}$ as

$$q_{k,l} = \frac{\tilde{\mathbf{w}}_k^T(\mathbf{h}_j)\tilde{\mathbf{w}}_l(\hat{\mathbf{h}}_j)}{\|\mathbf{h}_j\|^2} = \frac{\tilde{\mathbf{h}}_j^T\tilde{\mathbf{D}}_k^T\tilde{\mathbf{D}}_l\hat{\tilde{\mathbf{h}}}_j}{\|\mathbf{h}_j\|^2}, \qquad j = 1, \ldots, n_R,$$

and considering that, for $k \neq l$, $\tilde{\mathbf{D}}_k^T\tilde{\mathbf{D}}_l = -\tilde{\mathbf{D}}_l^T\tilde{\mathbf{D}}_k$, the above equation implies

$$q_{k,l} = \begin{cases} \frac{\tilde{\mathbf{h}}_j^T\hat{\tilde{\mathbf{h}}}_j}{\|\mathbf{h}_j\|^2} & k = l, \\ -q_{l,k} & k \neq l, \end{cases} \qquad j = 1, \ldots, n_R.$$

Hence, matrix $\mathbf{Q}$ can be written as $\mathbf{Q} = \alpha\mathbf{I} + \sqrt{1-\alpha^2}\mathbf{Q}_\perp$, where $\alpha = \tilde{\mathbf{h}}_j^T\hat{\tilde{\mathbf{h}}}_j/\|\mathbf{h}_j\|^2$ and $\mathbf{Q}_\perp$ is an orthogonal skew-symmetric matrix. Using this decomposition, (5) becomes

$$\tilde{\mathbf{W}}(\hat{\mathbf{H}}) = \alpha\tilde{\mathbf{W}}(\mathbf{H}) + \sqrt{1-\alpha^2}\tilde{\mathbf{W}}(\mathbf{H})\mathbf{Q}_\perp,$$

which implies that, if the channel cannot be identified (and, therefore, according to Lemma 1 $\hat{\mathbf{H}} \neq \pm\mathbf{H}$ and $\alpha \neq \pm 1$), we can find a channel $\hat{\mathbf{H}}_\perp = \frac{\hat{\mathbf{H}} - \alpha\mathbf{H}}{\sqrt{1-\alpha^2}}$ satisfying

$$\tilde{\mathbf{W}}(\hat{\mathbf{H}}_\perp) = \tilde{\mathbf{W}}(\mathbf{H})\mathbf{Q}_\perp.$$

To summarize, if the channel is non-identifiable then there exists an orthogonal and skew-symmetric matrix $\mathbf{Q}$ such that (5) holds. ■

The combination of Lemma 2 and Property 3 yields the following Theorem, which generalizes the first theorem in [1]:

Theorem 1 *If an OSTBC code transmits an odd number of real symbols (*M' *odd), then the channel can be identified regardless of the number of receiving antennas.*

Proof: The proof proceeds by contradiction. Let us assume that an OSTBC transmitting an odd number of real symbols (M' odd) is not identifiable, then from Lemma 2 there must exist an orthogonal and skew-symmetric matrix $\mathbf{Q}$ of dimensions $M' \times M'$ relating $\tilde{\mathbf{W}}(\hat{\mathbf{H}})$ and $\tilde{\mathbf{W}}(\mathbf{H})$. From Property 3 it is clear that an orthogonal skew-symmetric matrix of odd order can not exist and therefore the MIMO channel can be identified up to a real scalar. ■

5. DESIGN OF IDENTIFIABLE OSTBCS

Theorem 1 establishes that any OSTBC transmitting an odd number of real symbols permits the blind identification of the MIMO channel. For real OSTBCs, this result is only of limited value from a practical standpoint since most of the useful codes transmit an even number of real symbols (see [4, 7]). On the other hand, for complex OSTBCs we obviously have $M' = 2M$, where M is the number of complex symbols. Therefore M' is always even and the theorem does not apply. However, an interesting idea derived from this theorem is that a non-identifiable complex OSTBC can be made identifiable simply by not transmitting one real symbol (either the real or imaginary part of a symbol in the case of complex OSTBCs). Obviously, the price we pay is a reduction in the code rate: for a complex OSTBC transmitting $M = 4$ symbols the original code rate would be reduced by a factor $\beta = 7/8$. However, by grouping B consecutive OSTBC blocks the resulting matrix can be viewed as a new OSTBC with n_T antennas transmitting BM symbols in BL time slots. Applying the proposed technique to this new OSTBC, the rate-reduction factor is

$$\beta = \frac{BM' - 1}{BM'},$$

which increases with B, and tends to one for $BM' \gg 1$.

5.1. A comparison in terms of capacity

Here we compare the proposed rate-reduction technique with the well known differential OSTBC technique [4] in terms of capacity. Considering i.i.d Gaussian noise with variance σ^2, and assuming without loss of generality that the average transmitted energy per antenna and time interval is $1/n_T$, the capacity of the OSTBC-MIMO channel $\mathbf{H}$ for unity bandwidth is [4]

$$C_{\text{OSTBC}}(R, \text{SNR}) = R \log_2 \left(1 + \frac{\text{SNR}}{R}\right),$$

where $\text{SNR} = \frac{\|\mathbf{H}\|^2}{n_T \sigma^2}$ is the received signal to noise ratio. In the case of the proposed technique, the capacity reduces to

$$C_{\text{Red}} = C_{\text{OSTBC}}(\beta R, \text{SNR}),$$

and considering the 3-dB penalty incurred by differential schemes, the capacity of a differential OSTBC is given by [4]

$$C_{\text{Diff}} = C_{\text{OSTBC}}(R, \text{SNR}/2).$$

Thus, considering $\text{SNR} \gg 1$, it can be readily proved that

$$\begin{cases} C_{\text{Red}} > C_{\text{Diff}} & \text{if} \quad \text{SNR} < \text{SNR}_{\text{th}}, \\ C_{\text{Red}} < C_{\text{Diff}} & \text{if} \quad \text{SNR} > \text{SNR}_{\text{th}}, \end{cases}$$

where SNR_{th} is a threshold given by

$$10 \log_{10}(\text{SNR}_{\text{th}}) = 10 \log_{10}(R) + \frac{3 + \beta 10 \log_{10}(\beta)}{1 - \beta}. \quad (6)$$

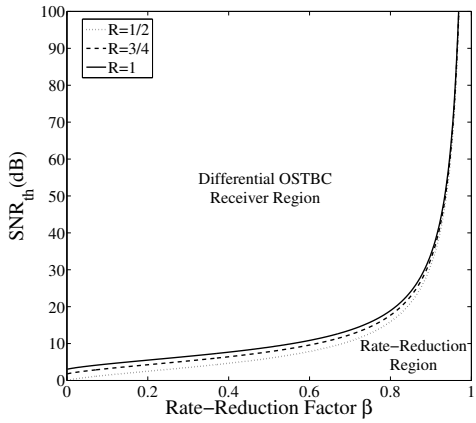

Fig. 1. Theoretical SNR threshold and capacity versus transmission rate R and rate-reduction parameter β.

The above threshold increases with the rate-reduction factor β and, correspondingly, with the number of OSTBC blocks B. Therefore, by increasing B, the proposed method outperforms the differential receiver. However, we must point out that this capacity analysis has been carried out assuming perfect channel estimation (even for the blind method), which is only true for noise free scenarios or when an infinite number of blocks is available at the receiver. Furthermore, increasing β (and B) implies a reduction on the number of available composed blocks at the receiver (for a fixed number of time slots). Therefore there exists a trade-off regarding the selection of B: a large B is required to increase the capacity, but a small B provides a better channel estimate. This idea will be illustrated by computer simulations in the next Section.

6. SIMULATION RESULTS

In this section the performance of the proposed method is evaluated through some simulation examples. In all the simulations, the results of 1000 independent realizations are averaged. The elements of the flat fading MIMO channels are zero-mean, circular, complex Gaussian random variables with variance $\sigma^2_{\mathbf{H}}$, the averaged transmitted energy per antenna and time interval is $1/n_T$, and the SNR at the transmitter side is defined as $10 \log_{10}(\sigma^2_{\mathbf{H}}/\sigma^2)$.

The i.i.d source signal belongs to a 16-QAM constellation. We have tested the 3/4 OSTBC code for $M = 3$ complex symbols, $L = 4$ time slots and $n_T = 4$ transmit antennas, which is presented in Eq. (7.4.10) of [4]. The number of receive antennas is $n_R = 1$, which provokes an ambiguity problem in the channel estimation [7].

Figure 1 shows the theoretical SNR_{th} curves given by (6) for three different transmission rates ($R = 1$, $R = 3/4$ and $R = 1/2$). We can see that the curves divide the plane in two regions, in the upper region (labeled as Differential OSTBC Receiver Region) the differential scheme has more capacity than the proposed method (referred to as Rate-Reduction), whereas the converse is true in the lower region. It must be also noted that for rate-reduction factors $\beta > 0.9$, the threshold SNR_{th} is above 30dB, which implies that in practice the proposed technique is a better approach than the differential OSTBC scheme in terms of capacity.

In the second example we consider a more realistic scenario in which the channel estimate is obtained from $N = 40$ received

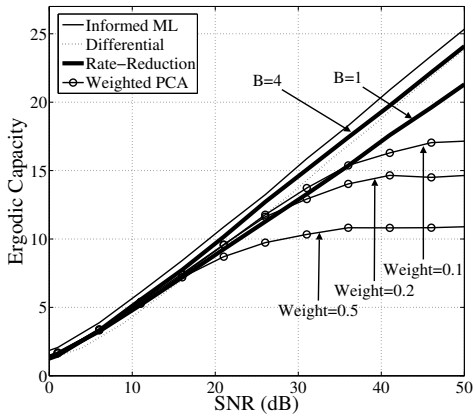

Fig. 2. Ergodic capacity including channel estimation.

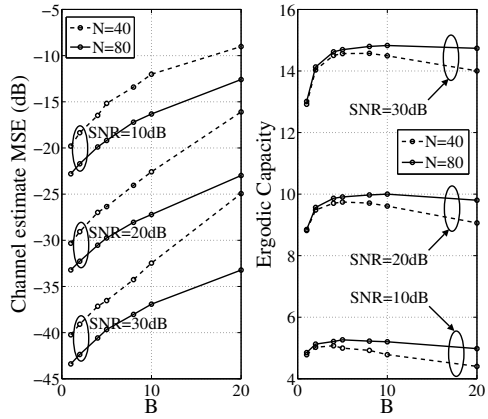

Fig. 3. Effect of the parameter B and number of available blocks N on channel estimation and ergodic capacity.

OSTBC blocks (160 time-slots). We compare the performance of the proposed rate-reduced method, the informed ML (perfect channel state information), the differential OSTBC receiver proposed in [4] and the linear precoding technique proposed in [7] (referred to as Weighted-PCA). For the Weighted-PCA, the first $M' - 1$ weights have been selected to be equal to 1, and the remaining one is selected as 0.1, 0.2 or 0.5 (always normalizing to transmit the same averaged energy per antenna and time interval). This means that one of the M' real source signals is transmitted with less energy than the rest. Note also that for $B = 1$ the proposed rate-reduction technique can be considered as a limiting case of the Weighted-PCA in which the weight assigned to one of the sources is zero (i.e., the symbol is not transmitted at all). For the proposed rate-reduction technique we consider $B = 1$ ($\beta = 5/6$) and $B = 4$ ($\beta = 23/24$). Fig. 2 shows the ergodic capacity obtained with the different methods, where we can see that the proposed technique with $B = 4$ outperforms the differential receiver for a large range of SNRs.

The final example illustrates the trade-off between the number of available OSTBC blocks (N), the rate-reduction parameter β (or B), the transmitted SNR, and the ergodic capacity. Figure 3 shows the MSE of the channel estimate (left) and the ergodic capacity (right) for different values of B, number of available blocks and SNRs. It can be noted that, for a given N, the MSE of the channel estimate increases with B, which is due to the reduction of the number of available composite blocks (N/B). On the other hand, the increase of B yields a higher transmission rate βR, and the combination of these effects implies the existence of an optimum parameter B, which maximizes the ergodic capacity, and depends on the SNR and the number of available OSTBC blocks N.

7. CONCLUSIONS

In this paper, the problem of blind identifiability of MIMO channels under OSTBC transmissions has been analyzed. We have derived a sufficient condition for blind identifiability based solely on second order statistics. Specifically, we have proved that any (real or complex) OSTBC transmitting an odd number of real symbols permits the blind identification of the MIMO channel up to a real scalar regardless of the number of receiving antennas. This condition is exploited to obtain identifiable OSTBCs from non-identifiable codes by means of a slight reduction of the transmission rate. The proposed method has been analyzed in terms of capacity, and we have shown by means of some simulation examples that it outperforms other previously proposed techniques.

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