

# AN IRWLS PROCEDURE FOR ROBUST BEAMFORMING WITH SIDELOBE CONTROL

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## ABSTRACT

Recently, the application of support vector machines (SVM) has been proposed for increasing the robustness of array beamforming against signal mismatch situations and providing additional sidelobe control. In this approach, the conventional Capon cost function is modified by including a regularization term that penalizes differences between the actual and the ideal array responses. The resulting cost function is convex with a unique global minimum that can be found using quadratic programming (QP) techniques. This paper expands this approach in order to reduce the computational cost of the SVM solution by means of an iterative re-weighted least squares (IRWLS) procedure. The robustness of the proposed IRWLS-based beamformer is examined in several mismatch scenarios, showing a good performance even when the signal DOA mismatch error is underestimated: a situation where other robust approaches typically fail.

## 1. INTRODUCTION

Increasing the robustness of adaptive beamforming techniques has focused considerable efforts during the recent years. Several works have shown that the traditional minimum variance distortionless response (MVDR) or Capon beamformer degrades considerably its performance in practical scenarios with limited number of available snapshots, mismatches between the assumed direction of arrival and the actual one, or calibration errors.

Several approaches have been proposed to improve the robustness of the MVDR beamformer, for instance, diagonal loading methods [1][2], and eigenspace-based beamformers [3]. In addition, when the number of snapshots used for covariance matrix estimation is insufficient or the scenario is favourable in terms of SNR, the MVDR beamformer can present unacceptably high sidelobes, which reduces its performance in the presence of unexpected interferences. To overcome this drawback, additional sidelobe

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control has been proposed in [4] and [5]. These approaches modify the MVDR beamforming problem to incorporate additional constraints outside the mainlobe beampattern. In [4], the corresponding optimization problem is solved by second-order cone (SOC) programming techniques. If a feasible solution does not exist, the sidelobe levels defining the constraints must be relaxed and a new optimization problem must be solved. In addition, this technique does not consider the signal mismatch problem.

In [5], the design of a robust adaptive beamformer is approached from the support vector machine (SVM) framework [6]. The MVDR beamforming problem is reformulated by incorporating an additional regularization term that penalizes sidelobe levels while, at the same time, allows a certain error in the desired signal direction. The resulting cost function can be interpreted as a support vector machine for regression (SVR), whose unique solution has traditionally been found using quadratic programming techniques. In comparison to [4], the robust SVM-based beamformer always provides a solution even if the problem is not feasible.

In this paper the SVM beamformer is obtained by means of an iterative re-weighted least square (IRWLS) algorithm, which considerably reduces the complexity of the conventional quadratic programming techniques. This procedure has been successfully applied to solve SVM's [7] and it has recently proven to converge to the SVM solution [8]. We present simulation examples where the performance of the proposed SVM beamformer solved via IRWLS is compared with the Capon beamformer and with other robust beamforming techniques. The IRWLS beamformer exhibits an increased robustness against the signal mismatch problem, even when the DOA estimation error is larger than expected.

## 2. BACKGROUND

Consider a narrowband beamformer with  $M$  sensors, its output is given by

$$y(k) = \mathbf{w}^H \mathbf{x}(k),$$

where  $k$  is the time index,  $\mathbf{x}(k) = [x_1(k), \dots, x_M(k)]^T \in C^{M \times 1}$  is the complex vector of array observations,  $\mathbf{w} =$

$[w_1, \dots, w_M]^T \in C^{M \times 1}$  is the complex vector of beamformer weights, and  $(\cdot)^T$  and  $(\cdot)^H$  denote the transpose and Hermitian transpose, respectively. The observation (snapshot) vector is given by

$$\mathbf{x}(k) = s(k)\mathbf{a}(\theta_s) + \sum_{j=1}^{N_i} i_j(k)\mathbf{a}(\theta_j) + \mathbf{n}(k),$$

where  $s(k)$  and  $i_j(k)$  are the signal of interest (SOI) and interference waveforms;  $\mathbf{n}(k)$  represents the noise signal. The signal and interference directions of arrival (DOA) are  $\theta_s$  and  $\theta_j$ ,  $j = 1, \dots, N_i$ , respectively, with corresponding steering vectors  $\mathbf{a}(\theta_s)$  and  $\mathbf{a}(\theta_j)$ .

The classical formulation for the MVDR beamformer is

$$\min_{\mathbf{w}} E[|y(k)|^2] \quad \text{subject to} \quad \mathbf{w}^H \mathbf{a}(\theta_s) = g \quad (1)$$

whose solution is given by

$$\mathbf{w}_0 = \frac{g^* \mathbf{R}_x^{-1} \mathbf{a}(\theta_s)}{\mathbf{a}(\theta_s)^H \mathbf{R}_x^{-1} \mathbf{a}(\theta_s)}, \quad (2)$$

where  $*$  denotes conjugate. In practice, the exact covariance  $M \times M$  matrix  $\mathbf{R}_x$  is not available and is replaced by the sampled covariance matrix  $\hat{\mathbf{R}}_x$  which is given by

$$\hat{\mathbf{R}}_x = \frac{1}{N} \sum_{k=1}^N \mathbf{x}(k)\mathbf{x}(k)^H, \quad (3)$$

where  $N$  is the number of observed snapshots.

### 3. ROBUST BEAMFORMING WITH SIDELOBE CONTROL

In this section, the conventional MVDR beamforming problem is modified by incorporating additional constraints in order to increase the robustness against mismatches in the SOI steering vector as well as to control the sidelobe level. Likewise [4], the additional constraints are included as a regularization term of the array output power [5].

Let us consider a grid of directions of arrival  $\theta_i$ ,  $i = 1, \dots, P$ ; which sample the beampattern in  $[-90^\circ, 90^\circ]$ . We define an angular mainlobe beamwidth  $\Delta$  centered at the assumed SOI direction of arrival  $\theta_s$ .  $P_1$  from the total set of angles sample the mainlobe beamwidth, including  $\theta_s$ : in this way, we increase the mainlobe beampattern to account for a possible signal mismatch error. The remainder  $P_2 = P - P_1$  angles sample the beampattern outside the mainlobe. Based on the formulation of SVMs [6], we consider the following regularized MVDR problem

$$\min_{\mathbf{w}} \frac{1}{2} \mathbf{w}^H \mathbf{R}_x \mathbf{w} + C \sum_{i=1}^P |d_i - \mathbf{w}^H \mathbf{a}(\theta_i)|_\epsilon, \quad (4)$$

where

$$|d_i - \mathbf{w}^H \mathbf{a}(\theta_i)|_\epsilon = \max\{0, |d_i - \mathbf{w}^H \mathbf{a}(\theta_i)| - \epsilon\}, \quad (5)$$

is the so-called Vapnik's  $\epsilon$ -insensitive loss function [6] and  $d_i$  is the desired beamformer output

$$d_i = \begin{cases} 0 & \text{if } |\theta_i - \theta_s| > \Delta, \\ g_R + jg_I & \text{if } |\theta_i - \theta_s| \leq \Delta, \end{cases} \quad (6)$$

Note that the regularized cost function (4) establishes a trade-off between the array output power and a term that penalizes mismatches larger than  $\epsilon$  between the actual and desired array responses for the given angle grid. Therefore, the procedure can be interpreted as a regression problem where the parameter  $\epsilon$  acts as a sidelobe control parameter while, at the same time, allows some tolerance in the array response for the assumed signal arrival angle. The optimal values of  $C$  and  $\epsilon$  must be obtained for each scenario, depending on the number of sensors, the noise level, the required sidelobe level and the DOA estimation error.

In terms of real variables we can write

$$\mathbf{w}^H \mathbf{R}_x \mathbf{w} = \tilde{\mathbf{w}}^T \tilde{\mathbf{R}}_x \tilde{\mathbf{w}}$$

where

$$\tilde{\mathbf{w}}^T = [\mathbf{w}_R^T \quad \mathbf{w}_I^T]. \quad (7)$$

and

$$\tilde{\mathbf{R}}_x = \begin{bmatrix} \text{Re}(\mathbf{R}_x) & -\text{Im}(\mathbf{R}_x) \\ \text{Im}(\mathbf{R}_x) & \text{Re}(\mathbf{R}_x) \end{bmatrix}. \quad (8)$$

Likewise, the beamformer output for each DOA can be written in terms of real variables as

$$\mathbf{w}^H \mathbf{a}(\theta_i) = \tilde{\mathbf{w}}^T \tilde{\mathbf{a}}(\theta_i) + j \tilde{\mathbf{w}}^T \tilde{\mathbf{a}}'(\theta_i) \quad (9)$$

where  $\tilde{\mathbf{a}}(\theta_i)$  and  $\tilde{\mathbf{a}}'(\theta_i) \in R^{2M \times 1}$  are given by

$$\begin{aligned} \tilde{\mathbf{a}}(\theta_i)^T &= [\mathbf{a}_R^T(\theta_i) \quad \mathbf{a}_I^T(\theta_i)] \\ \tilde{\mathbf{a}}'(\theta_i)^T &= [\mathbf{a}_I^T(\theta_i) \quad -\mathbf{a}_R^T(\theta_i)]. \end{aligned} \quad (10)$$

For notational simplicity, we define the following compact variable  $\bar{\mathbf{a}} \in R^{2M \times 2P}$ ,

$$\bar{\mathbf{a}}(i) = \begin{cases} \tilde{\mathbf{a}}(\theta_i), & i = 1, \dots, P, \\ \tilde{\mathbf{a}}'(\theta_{i-P}), & i = P+1, \dots, 2P. \end{cases}$$

Hence, the initial complex formulation (4) can be written in terms of real variables as

$$J(\tilde{\mathbf{w}}) = \frac{1}{2} \tilde{\mathbf{w}}^T \tilde{\mathbf{R}}_x \tilde{\mathbf{w}} + C \sum_{i=1}^{2P} |y_i - \tilde{\mathbf{w}}^T \bar{\mathbf{a}}(i)|_\epsilon \quad (11)$$

where the real variable  $y_i = \text{Re}(d_i)$  for  $i = 1, \dots, P$ , and  $y_i = \text{Im}(d_i)$  for  $i = P+1, \dots, 2P$ , represents the desired output for each product  $\tilde{\mathbf{w}}^T \bar{\mathbf{a}}(i)$ .

Introducing a set of slack positive variables  $\xi$  and  $\tilde{\xi}$ , the cost function (11) can be written as the following optimization problem with constraints [6]: minimize

$$L(\tilde{\mathbf{w}}, \xi, \tilde{\xi}) = \frac{1}{2} \tilde{\mathbf{w}}^T \tilde{\mathbf{R}}_x \tilde{\mathbf{w}} + C \sum_{i=1}^{2P} (\xi_i + \tilde{\xi}_i) \quad (12)$$

subject to

$$\tilde{\mathbf{w}}^T \tilde{\mathbf{a}}(i) - y_i \leq \epsilon + \xi_i, \quad (13)$$

$$y_i - \tilde{\mathbf{w}}^T \tilde{\mathbf{a}}(i) \leq \epsilon + \tilde{\xi}_i, \quad (14)$$

$$\xi_i, \tilde{\xi}_i \geq 0 \quad (15)$$

for  $i = 1, \dots, 2P$ .

#### 4. IRWLS-PROCEDURE

Introducing the constraints into (12), the solution of the optimization problem is a saddle point of the following Lagrange function [6]:

$$\begin{aligned} L(\tilde{\mathbf{w}}, \xi, \tilde{\xi}, \alpha, \tilde{\alpha}, \gamma, \tilde{\gamma}) &= \frac{1}{2} \tilde{\mathbf{w}}^T \tilde{\mathbf{R}}_x \tilde{\mathbf{w}} - \sum_{i=1}^{2P} (\tilde{\gamma}_i \tilde{\xi}_i + \gamma_i \xi_i) \\ &+ C \sum_{i=1}^{2P} (\xi_i + \tilde{\xi}_i) - \sum_{i=1}^{2P} \alpha_i (y_i - \tilde{\mathbf{w}}^T \tilde{\mathbf{a}}(i) + \epsilon + \xi_i) \\ &- \sum_{i=1}^{2P} \tilde{\alpha}_i (\tilde{\mathbf{w}}^T \tilde{\mathbf{a}}(i) - y_i + \epsilon + \tilde{\xi}_i), \quad (16) \end{aligned}$$

minimum with respect to the primal variables  $\tilde{\mathbf{w}}$ ,  $\xi_i$  and  $\tilde{\xi}_i$ ; and maximum with respect to the Lagrange multipliers  $\alpha_i \geq 0$ ,  $\tilde{\alpha}_i \geq 0$ ,  $\gamma_i \geq 0$  and  $\tilde{\gamma}_i \geq 0$ , for  $i = 1, \dots, 2P$ .

Similarly to other SVM-based problems, here the optimal beamformer can be expanded in terms of a set of steering vectors, which are the support vectors for the problem. However, due to the minimum output power term in the cost function, now the support vectors in the expansion are transformed by the inverse of the autocorrelation matrix; in particular we have

$$\tilde{\mathbf{w}} = \sum_{i=1}^{2P} (\alpha_i - \tilde{\alpha}_i) \tilde{\mathbf{R}}_x^{-1} \tilde{\mathbf{a}}(i).$$

The support vectors and their corresponding Lagrange multipliers are typically found by means of quadratic programming (QP) techniques. Recently, it has been shown [7] that the high computational cost of the original procedure can be reduced by transforming the QP problem into an equivalent least squares problem [7]. By applying an Iterative Re-Weighted Least Square (IRWLS) algorithm the requirements both in time and memory are reduced without any loss of performance [8].

To obtain an IRWLS procedure, we need to apply Karush-Kuhn-Tucker (KKT) conditions, which impose that the terms depending on  $\xi_i$  and  $\tilde{\xi}_i$  must be removed at the solution [7]. Therefore, the cost function (16) can be written as

$$\begin{aligned} L(\tilde{\mathbf{w}}, \alpha, \tilde{\alpha}) &= \frac{1}{2} \tilde{\mathbf{w}}^T \tilde{\mathbf{R}}_x \tilde{\mathbf{w}} - \sum_{i=1}^{2P} \alpha_i (y_i - \tilde{\mathbf{w}}^T \tilde{\mathbf{a}}(i) + \epsilon) \\ &- \sum_{i=1}^{2P} \tilde{\alpha}_i (\tilde{\mathbf{w}}^T \tilde{\mathbf{a}}(i) - y_i + \epsilon) \\ &= \frac{1}{2} \tilde{\mathbf{w}}^T \tilde{\mathbf{R}}_x \tilde{\mathbf{w}} - \frac{1}{2} \sum_{i=1}^{2P} (f_i e_i^2 + \tilde{f}_i \tilde{e}_i^2), \quad (17) \end{aligned}$$

where

$$\begin{aligned} e_i &= \epsilon + y_i - \tilde{\mathbf{w}}^T \tilde{\mathbf{a}}(i), & f_i &= \frac{2\alpha_i}{\epsilon + y_i - \tilde{\mathbf{w}}^T \tilde{\mathbf{a}}(i)} \\ \tilde{e}_i &= \epsilon - y_i + \tilde{\mathbf{w}}^T \tilde{\mathbf{a}}(i), & \tilde{f}_i &= \frac{2\tilde{\alpha}_i}{\epsilon - y_i + \tilde{\mathbf{w}}^T \tilde{\mathbf{a}}(i)} \end{aligned}$$

Observe that (17) can be understood as a weighted least square equation with  $e_i$  and  $\tilde{e}_i$  as prediction errors and  $f_i$  and  $\tilde{f}_i$  are its corresponding weights. The minimization of (17) must be carried out iteratively [7].

According to the Representer Theorem [6][7], it can be shown that the coefficients  $\tilde{\mathbf{w}}$ , which minimize (16), can be expressed as:

$$\tilde{\mathbf{w}} = \sum_{i=1}^{2P} (\tilde{\beta}_i - \beta_i) \tilde{\mathbf{R}}_x^{-1} \tilde{\mathbf{a}}(i) = \tilde{\mathbf{R}}_x^{-1} \Phi^T \gamma, \quad (18)$$

where  $\Phi$  and  $\gamma$  are given by

$$\Phi = [\tilde{\mathbf{a}}(1), \tilde{\mathbf{a}}(2), \dots, \tilde{\mathbf{a}}(2P)]^T \quad (19)$$

$$\gamma = [\beta_1 - \tilde{\beta}_1, \beta_2 - \tilde{\beta}_2, \dots, \beta_{2P} - \tilde{\beta}_{2P}]^T. \quad (20)$$

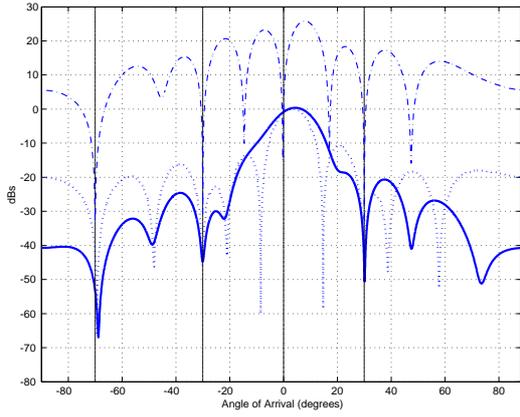
When the IRWLS procedure converges, the variables  $\tilde{\beta}_i$  and  $\beta_i$  achieve the same values as the Lagrange multipliers  $\tilde{\alpha}_i$  and  $\alpha_i$ .

Substituting (18) into (17) the minimum of the cost function with respect to  $\gamma$  for fixed  $f_i$  and  $\tilde{f}_i$  is found solving the linear equation system [7]

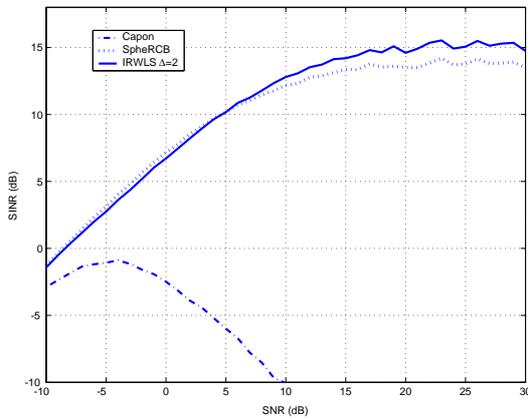
$$[\mathbf{H} - \mathbf{D}_{\tilde{\mathbf{f}}+\mathbf{f}}^{-1}] \gamma = [\mathbf{y} - \epsilon \mathbf{E}] \quad (21)$$

where

$$\begin{aligned} (\mathbf{H})_{i,j} &= \tilde{\mathbf{a}}(i)^T \tilde{\mathbf{R}}_x^{-1} \tilde{\mathbf{a}}(j), \quad i, j = 1, \dots, n \\ (\mathbf{D}_{\tilde{\mathbf{f}}+\mathbf{f}})_{i,j} &= \delta[i-j](\tilde{f}_i + f_i), \quad i, j = 1, \dots, n \\ \mathbf{y} &= [y_1, y_1, \dots, y_{2P}]^T, \\ \mathbf{E} &= \left[ \begin{array}{cccc} \tilde{f}_1 - f_1 & \tilde{f}_2 - f_2 & & \tilde{f}_{2P} - f_{2P} \\ \tilde{f}_1 + f_1 & \tilde{f}_2 + f_2 & \dots & \tilde{f}_{2P} + f_{2P} \end{array} \right]^T. \end{aligned}$$



**Fig. 1.** Direction mismatch of  $2^\circ$ : SVM (solid line), SpheRCB (dotted line) and Capon (dash-dotted line).



**Fig. 2.** Direction mismatch of  $2^\circ$ : SINR versus SNR.

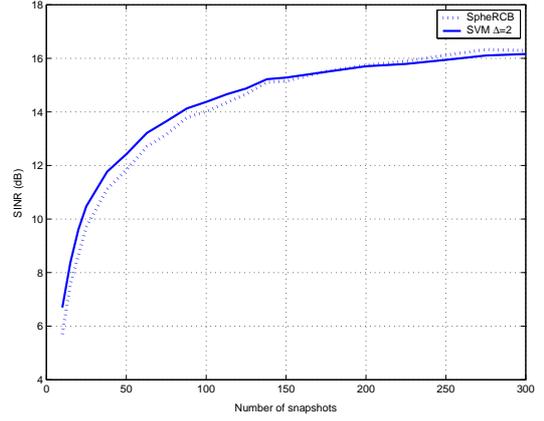
Finally, since  $f_i$  and  $\tilde{f}_i$  depend on the Lagrange multipliers  $\alpha_i$  and  $\tilde{\alpha}_i$ , we can substitute them using the KKT conditions

$$f_i = \begin{cases} 0 & \text{if } e_i < 0, \\ \frac{2C}{e_i} & \text{if } e_i \geq 0, \end{cases} \quad (22)$$

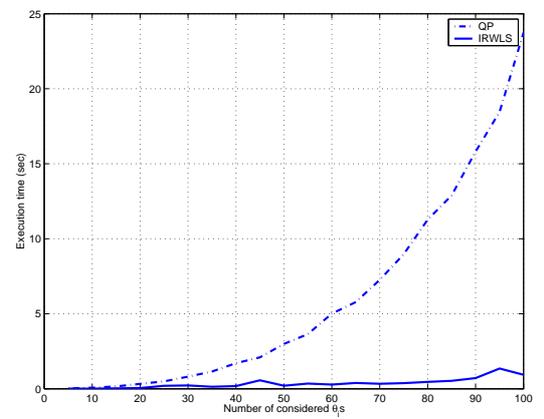
and equally for  $\tilde{f}_i$ .

## 5. SIMULATION RESULTS

To evaluate the performance of the proposed beamforming technique, some computer experiments are carried out in scenarios with source steering vector mismatches. We assume a uniform linear array with  $M = 10$  sensors and half-wavelength sensor spacing. All signal waveforms are i.i.d. QPSK. Spatially white Gaussian noise is assumed ( $\mathbf{Q} = \sigma_n^2 \mathbf{I}$ ). The power of the signal of interest (SOI) is  $\sigma_s^2 = 10$  dB, and the power of the interferences is  $\sigma_j^2 = 30$  dB,  $\forall j$ . The actual source DOA is  $\theta_s = 0^\circ$  and the DOAs of the interferences are  $\theta_1 = -30^\circ$ ,  $\theta_2 = 30^\circ$  and  $\theta_3 = 70^\circ$ . The signal-to-noise ratio SNR is 10 dB. In order to compute  $\hat{\mathbf{R}}_x$ ,  $N = 50$  snapshots are used. In all scenarios, each point is the average from 200 simulations.



**Fig. 3.** Direction mismatch of  $2^\circ$ : SINR versus number of snapshots.

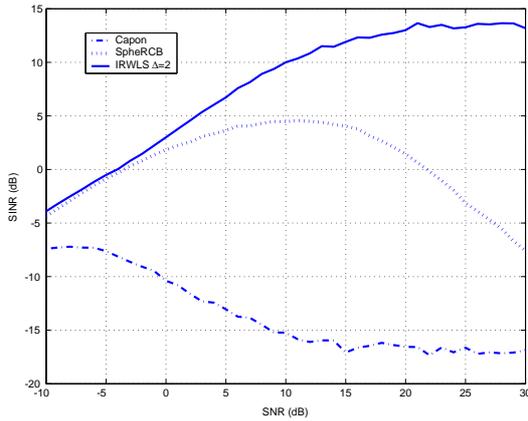


**Fig. 4.** Computational cost of QP and IRWLS.

### Example I

In the first scenario, the presumed source signal DOA is  $\theta_a = 2^\circ$ . Defining an angular beamwidth around  $\theta_a$  of  $\Delta = 2^\circ$ , our IRWLS approach employs a uniform grid with  $P_1 = 20$  DOAs inside the mainlobe and  $P_2 = 40$  outside the mainlobe. Fig.(1) shows a single realization of the beampatterns of the proposed IRWLS approach ( $C = 1$  and  $\epsilon = 0.001$ ) compared to the Capon beamformer and the robust array beamformer with spherical constraint proposed in [1] with control parameter  $\epsilon = 4.5$  (denoted as SpheRCB). It is demonstrated in [1] that the SpheRCB beamformer is equivalent to the SOC method proposed in [2]. Clearly, we observe that the SOI is considered to be an interference by the Capon beamformer. On the other hand the source signal is preserved by the IRWLS and the SpheRCB approaches, but the former has lower sidelobe level.

Fig.(2) depicts the output SINR versus the input SNR. As can be seen, the IRWLS method behaves similar to the SpheRCB beamformer in the low SNR region, but the IRWLS approach achieves better performance for high SNR. In Fig.(3), we illustrate SINR versus the number of snap-



**Fig. 5.** Direction mismatch of  $5^\circ$ : SINR versus SNR.

shots  $N$ , used to estimate  $\hat{\mathbf{R}}_x$ . It shows our IRWLS proposal operates slightly better than the SpheRCB beamformer when the number of snapshots  $N$  is less than 150. On the contrary, if  $N$  increases, the SpheRCB algorithm outperforms the IRWLS-based beamformer. Hence, we can conclude that the IRWLS beamformer is appropriate for scenarios when the number of available snapshots is scarce.

Compared to QP techniques [5], as shown in Fig.(4), an IRWLS procedure reduces considerably the computational cost of the SVM problem. This reduction is especially noticeable when the number of considered angles  $P$  increases. However, preliminary results have shown satisfactory performance if around 100 DOAs are considered to sample the angular grid.

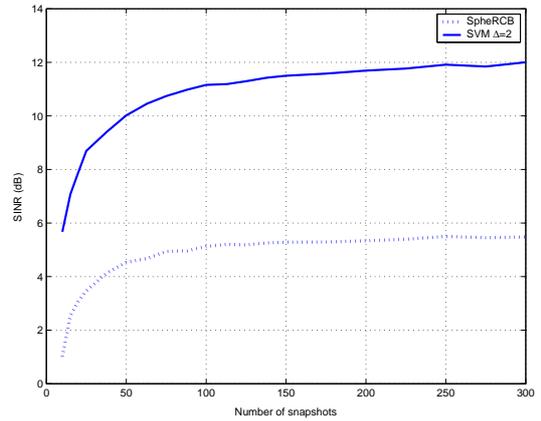
### Example II

In the second example, we simulate a scenario where error in the source DOA is underestimated for both the SVM and SpheRCB beamformer. We assume that the presumed source DOA is  $\theta_s = 5^\circ$  (the actual one is  $\theta_s = 0^\circ$ ). For the SpheRCB approach, we use  $\epsilon = 4.5$ , which is not large enough for an error of  $5^\circ$  in the source signal DOA. On the other hand, for the SVM beamformer the maximum error is controlled by the mainlobe beamwidth, which is  $\Delta = 2^\circ$ .

Figs.(5) and (6) are similar to Figs.(2) and (3). In both examples, the performance of the SpheRCB beamformer degrades considerably; however, the IRWLS beamformer still achieves good results even when the maximum mismatch error is underestimated. This increased robustness can be also explained by looking at the broad mainlobe provided by the SVM beamformer in Fig 1.

## 6. CONCLUSIONS

The SVM-beamformer has been recently proposed as a way to achieve increased robustness against the signal mismatch problem as well as sidelobe control. In this paper the high computational cost of the original SVM solution has been significantly reduced by transforming the original QP prob-



**Fig. 6.** Direction mismatch of  $5^\circ$ : SINR versus number of snapshots.

lem into an equivalent reweighted least squares problem. The robust performance of the proposed beamformer, even when the mismatch error is underestimated, has been demonstrated by computer simulations.

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