

Equal-Phase Beamforming Architecture for RF-MIMO Antenna Systems

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Abstract. This paper considers a novel multiple-input multiple-output (MIMO) architecture, which combines the signals in the radio-frequency (RF) domain. Unlike previous approaches, the proposed architecture is exclusively based on the application of different gain factors to the transmitted/received signals, and therefore it avoids the need of including a controllable phase-shifter (or sign switch) for each transmit/receive antenna. From a baseband point of view, the transceiver design consists in obtaining the optimal equal phase transmit (EPT) and equal-phase combining (EPC) beamformers. Interestingly, this problem can be exactly solved in the case of rank-one channels, which can be exploited to construct an iterative algorithm for the general MIMO case. The proposed architecture is evaluated by means of Monte Carlo simulations, which show that the slight performance degradation with respect to previous approaches is justified by the significant reduction in the hardware cost and power consumption.

Key words: RF-MIMO beamformer, equal-phase MIMO beamforming, semidefinite relaxation (SDR).

1 Introduction

One of the most important problems for the commercial deployment of new generation multiple-input multiple-output (MIMO) systems consists in the high hardware complexity and power consumption associated to conventional MIMO transceivers. This high costs are due to the need of replicating the up/down conversion RF chains for each transmit and receive antenna. In order to alleviate this drawback, several alternative architectures have been proposed in the last years. These solutions range from the idea of pre-FFT combining systems [4–6], which reduces the number of FFT blocks in multicarrier systems (but still requires the replication of the RF chains), to truly analog antenna combining systems [1, 7–10], which combine the signals in the RF domain and only require one RF chain.

In this paper we consider a new simplification of RF-MIMO transceivers. In particular, we show that the hardware complexity can be significantly reduced by removing the controllable phase shifters (or sign switch modules) required in previously proposed architectures. Obviously, the simplification of the RF hardware comes at a cost of a slightly reduced performance. However, we will show that, by properly selecting the amplification factors in each RF branch, the performance of this alternative architecture is close to that of previous approaches.

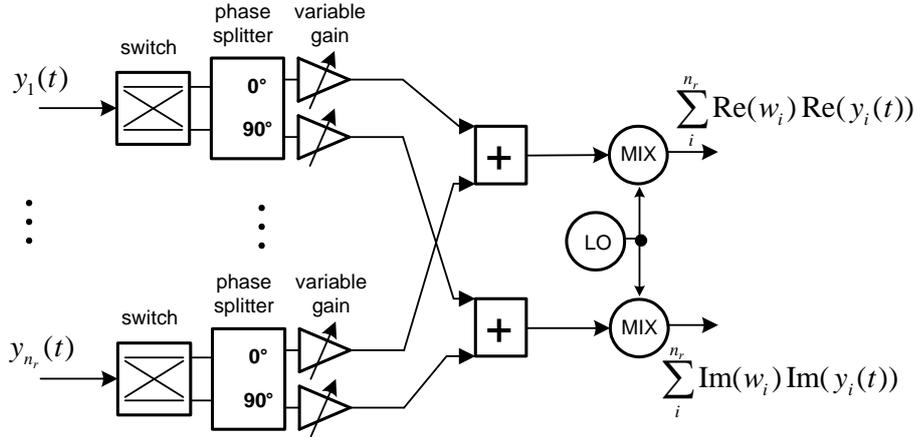


Fig. 1. RF-MIMO Architecture. Original design including variable gain amplifiers and phase shifters.

From a baseband point of view, the transceiver design consists in selecting the optimal amplification factors for each antenna. In other words, the proposed architecture forces us to work with equal phase transmit (EPT) and equal phase combining (EPC) beamformers, which are defined by the amplification factors. In the case of rank-one MIMO channels, which include the cases of single-input multiple-output (SIMO) and multiple-input single-output (MISO) systems, the optimal beamformer can be obtained by means of a semidefinite relaxation (SDR) approach [2]. Thus, in order to solve the general MIMO EPT/EPC beamforming problem, we propose an iterative algorithm consisting in alternating the optimization of the transmit and receive beamformers. Finally, the performance of the proposed architecture and algorithm is illustrated by means of some Monte Carlo simulations, which show that the slight performance degradation is justified by the significant decrease in the hardware complexity and power consumption.

2 RF-MIMO Architectures

In order to reduce the hardware cost and power consumption associated to conventional MIMO transceivers, some recent works have considered the possibility of moving part of the signal processing from the baseband to the RF domain [1, 7–10]. Specifically, Fig. 1 shows an analog antenna combining system, which avoids the replication of the up (or down) conversion chains for all the transmit/receive antennas. Thus, the transmitted/received signals can be weighted (complex weights) and combined in the RF domain, which results in a simplified architecture with a performance close to that of conventional MIMO systems [7, 10].

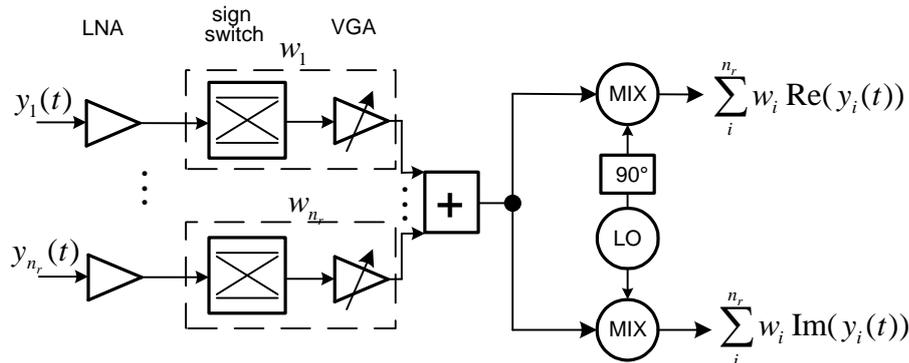


Fig. 2. Simplified RF-MIMO Architecture. The phase shifters are replaced by sign-switch modules, which translates into beamformers with real coefficients.

In order to further simplify the original RF-MIMO architecture, in [3] we have proposed a system based on the application of a real gain factor to each signal (see Fig. 2), which introduces the constraint of applying beamformers with real coefficients (RF weights). In this work we propose an additional simplification, which allows us to remove the sign-switch blocks in Fig. 2, and introduces the additional constraint of having equal-phase transmit (EPT) and equal-phase combining (EPC) beamformers. The proposed RF-MIMO architecture is illustrated in Fig. 3, and in this work we consider the problem of designing the transmit and receive beamformers under the assumption of perfectly known flat-fading and static¹ MIMO channels.

3 Design of the Beamformers

Throughout this paper we will use bold-faced upper case letters to denote matrices, bold-faced lower case letters for column vector, and light-faced lower case letters for scalar quantities. Superscripts $(\cdot)^T$ and $(\cdot)^H$ denote transpose and Hermitian respectively. $\|\mathbf{A}\|$, $\text{Tr}(\mathbf{A})$, $\text{rank}(\mathbf{A})$ and $\text{vec}(\mathbf{A})$ will denote, respectively, the Frobenius norm, trace, rank, and column-wise vectorized version of matrix \mathbf{A} . $\text{unvec}(\mathbf{a})$ is the inverse of the $\text{vec}(\mathbf{A})$ operation, i.e., $\text{unvec}(\text{vec}(\mathbf{A})) = \mathbf{A}$. $\mathbf{A} \succeq \mathbf{0}$ means that \mathbf{A} is symmetric and positive semidefinite, whereas $\mathbf{A} \geq \mathbf{0}$ means that the elements of \mathbf{A} are non-negative. $\Re(\mathbf{A})$ denotes the real part of the complex matrix \mathbf{A} , and $\mathbf{u}_{\max}(\mathbf{A})$ is the principal eigenvector of the positive semidefinite matrix \mathbf{A} . Finally, \mathbf{I} and $\mathbf{0}$ are the identity and zero matrices of the required dimensions.

Assuming n_T transmit and n_R receive antennas, the data model after Tx-Rx beamforming can be written as

$$y = hs + n,$$

¹ In the case of time-varying channels, the design of the beamformers would follow the same lines. However, the channel estimation process for the simplified architectures requires more training time than in the conventional MIMO case.

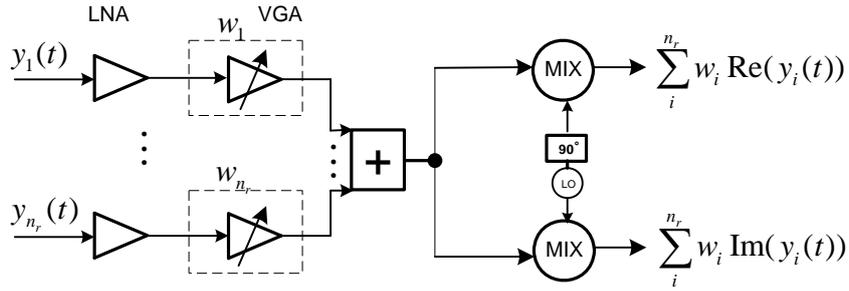


Fig. 3. Proposed simplification of the RF-MIMO Architecture. The phase shifters and sign-switch modules are avoided, which comes at the price of working with equal phase transmit (EPT) and equal phase combining (EPC) beamformers.

where $s \in \mathbb{C}$ represents the transmitted signal (information symbols), $y \in \mathbb{C}$ is the observation, $n \in \mathbb{C}$ represents the noise,

$$h = \mathbf{w}_R^T \mathbf{H} \mathbf{w}_T,$$

is the equivalent channel after Tx-Rx beamforming, $\mathbf{H} \in \mathbb{C}^{n_R \times n_T}$ is the MIMO channel, and $\mathbf{w}_T \in \mathbb{R}^{n_T \times 1}$, $\mathbf{w}_R \in \mathbb{R}^{n_R \times 1}$ represent the transmit and receive beamformers, which are defined by the gain factors applied to each antenna. Thus, the optimization problem associated to the design of the beamformers can be written as

$$\begin{aligned} & \underset{\mathbf{w}_T, \mathbf{w}_R}{\text{maximize}} && |\mathbf{w}_R^T \mathbf{H} \mathbf{w}_T| && (1) \\ & \text{subject to} && \|\mathbf{w}_T\| \leq 1, \\ & && \|\mathbf{w}_R\| \leq 1, \\ & && \mathbf{w}_T \geq \mathbf{0}, \\ & && \mathbf{w}_R \geq \mathbf{0}, \end{aligned}$$

where we can readily identify the conventional energy constraints, as well as the additional positiveness constraints due to the removal of the sign-switch blocks.

3.1 Beamformer Design in the SIMO and MISO cases

Unfortunately, the optimization problem in (1) is not convex, which precludes its solution by means of standard convex optimization tools [2]. However, in the case of rank-one channels (which includes the SIMO and MISO cases), the optimal solution of (1) can be obtained by means of a semidefinite relaxation (SDR) approach [2].

Let us consider the SIMO case² and write the optimization problem in (1) as

² The analysis for the MISO case (or the more general case of rank-one MIMO channels) can be made in a similar manner.

$$\begin{aligned} & \underset{\mathbf{w}_R}{\text{maximize}} && \mathbf{w}_R^T \mathbf{R} \mathbf{w}_R \\ & \text{subject to} && \|\mathbf{w}_R\| \leq 1, \\ & && \mathbf{w}_R \geq \mathbf{0}, \end{aligned}$$

where

$$\mathbf{R} = \Re(\mathbf{h}\mathbf{h}^H),$$

and $\mathbf{h} \in \mathbb{C}^{n_R \times 1}$ is the the SIMO channel.

Defining now the beamforming matrix $\mathbf{W}_R = \mathbf{w}_R \mathbf{w}_R^T$, it is easy to see that the above optimization problem can be rewritten as

$$\begin{aligned} & \underset{\mathbf{W}_R}{\text{maximize}} && \text{Tr}(\mathbf{R}\mathbf{W}_R) \\ & \text{subject to} && \text{Tr}(\mathbf{W}_R) \leq 1, \\ & && \mathbf{W}_R \geq \mathbf{0}, \\ & && \mathbf{W}_R \succeq \mathbf{0}, \\ & && \text{rank}(\mathbf{W}_R) = 1, \end{aligned} \tag{2}$$

where, excluding the rank-one constraint, we have three convex constraints and a concave objective function. Therefore, dropping the non-convex rank-one constraint, we obtain the following convex optimization problem

$$\begin{aligned} & \underset{\mathbf{W}_R}{\text{maximize}} && \text{Tr}(\mathbf{R}\mathbf{W}_R) \\ & \text{subject to} && \text{Tr}(\mathbf{W}_R) \leq 1, \\ & && \mathbf{W}_R \geq \mathbf{0}, \\ & && \mathbf{W}_R \succeq \mathbf{0}, \end{aligned} \tag{3}$$

whose solution can be obtained by means of standard convex optimization tools. More importantly, it can be proved³ that there exists a rank-one solution for the relaxed problem in (3), or in other words, we can obtain the optimal solution of the non-convex optimization problem in (2) by solving the relaxed problem in (3).

3.2 Design of the MIMO Beamformers

In the general MIMO case, the problem of obtaining the optimal beamformers is much more complicated, and we have to resort to suboptimal approaches. Here, we propose an iterative algorithm, which is based on alternating minimizations on the transmit and receive beamformers. That is, once the transmit (respectively receive) beamformer has been fixed, the receive (resp. transmit) beamformer can be obtained as explained in the previous subsection. Thus, the overall technique is summarized in Algorithm 1, and as a convergence criterion we can use the Euclidean distance between the beamformers in two consecutive iterations. Finally, in order to guarantee the fast convergence of the algorithm, we propose an initialization approach based on a semidefinite relaxation technique.

³ Although we do not provide the details here, the proof is based on the fact that there exists a rank-one matrix \mathbf{W}_R satisfying the KKT conditions for the problem in (3).

Initialize the beamformers $\mathbf{w}_T, \mathbf{w}_R$.

repeat

Update of the receive beamformer

Obtain the equivalent SIMO channel $\mathbf{h}_{\text{SIMO}} = \mathbf{H}\mathbf{w}_T$.

Solve the optimization problem in (2) for the SIMO channel \mathbf{h}_{SIMO}

Use the obtained solution as the new receive beamformer \mathbf{w}_R .

Update of the transmit beamformer

Obtain the equivalent MISO channel $\mathbf{h}_{\text{MISO}} = \mathbf{H}^T \mathbf{w}_R$.

Solve the optimization problem in (2) for the MISO channel \mathbf{h}_{MISO}

Use the obtained solution as the new transmit beamformer \mathbf{w}_T .

until Convergence

Algorithm 1: Proposed iterative EPT-EPC beamforming algorithm.

Let us start by rewriting the optimization problem in (1) as

$$\begin{aligned}
 & \underset{\mathbf{w}_T, \mathbf{w}_R, \mathbf{w}}{\text{maximize}} && |\mathbf{h}^T \mathbf{w}| \\
 & \text{subject to} && \|\mathbf{w}_T\| \leq 1, \\
 & && \|\mathbf{w}_R\| \leq 1, \\
 & && \mathbf{w}_T \geq \mathbf{0}, \\
 & && \mathbf{w}_R \geq \mathbf{0}, \\
 & && \mathbf{w} = \mathbf{w}_T \otimes \mathbf{w}_R,
 \end{aligned}$$

where $\mathbf{h} = \text{vec}(\mathbf{H})$ is the vectorized version of the MIMO channel. Equivalently, the above problem can be rewritten as

$$\begin{aligned}
 & \underset{\mathbf{w}_T, \mathbf{w}_R, \mathbf{W}}{\text{maximize}} && \text{Tr}(\mathbf{R}\mathbf{W}) \\
 & \text{subject to} && \text{Tr}(\mathbf{W}) \leq 1, \\
 & && \mathbf{W} \geq \mathbf{0}, \\
 & && \mathbf{W} \succeq \mathbf{0}, \\
 & && \text{rank}(\mathbf{W}) = 1, \\
 & && \mathbf{u}_{\max}(\mathbf{W}) = \mathbf{w}_T \otimes \mathbf{w}_R,
 \end{aligned} \tag{4}$$

where $\mathbf{R} = \Re(\mathbf{h}\mathbf{h}^H)$, and $\mathbf{u}_{\max}(\mathbf{W})$ denotes the principal eigenvector of matrix \mathbf{W} . Obviously, the above problem is not convex due to the two last constraints. However, if we relax the last constraint, we obtain the non-convex optimization problem

$$\begin{aligned}
 & \underset{\mathbf{W}}{\text{maximize}} && \text{Tr}(\mathbf{R}\mathbf{W}) \\
 & \text{subject to} && \text{Tr}(\mathbf{W}) \leq 1, \\
 & && \mathbf{W} \geq \mathbf{0}, \\
 & && \mathbf{W} \succeq \mathbf{0}, \\
 & && \text{rank}(\mathbf{W}) = 1,
 \end{aligned}$$

which is identical to that in (2), and can be exactly solved by means of a SDR approach. Of course, the solution \mathbf{W} of the relaxed problem does not need to satisfy the constraint $\mathbf{u}_{\max}(\mathbf{W}) = \mathbf{w}_T \otimes \mathbf{w}_R$, and therefore it is not a solution of the original problem in (4). However, we can obtain an approximated solution by first obtaining $\mathbf{w} = \mathbf{u}_{\max}(\mathbf{W})$ and forming the $n_R \times n_T$ matrix $\tilde{\mathbf{W}} = \text{unvec}(\mathbf{w})$. Thus, a good approximation to the optimal EPT and EPC beamformers can be obtained from the best (in the Euclidean-norm sense) rank-one approximation of $\tilde{\mathbf{W}}$, which is given by $\mathbf{w}_R \mathbf{w}_T^T$, where \mathbf{w}_R and \mathbf{w}_T are the singular vectors associated to the largest singular value of $\tilde{\mathbf{W}}$. Finally, we must note that the solution \mathbf{W} satisfies the constraint $\mathbf{W} \geq \mathbf{0}$, which also implies $\mathbf{w} \geq \mathbf{0}$, $\tilde{\mathbf{W}} \geq \mathbf{0}$, and $\mathbf{w}_T \geq \mathbf{0}$, $\mathbf{w}_R \geq \mathbf{0}$, i.e., the obtained beamformers are feasible points of the original optimization problem in (1). Thus, with this initialization stage the proposed iterative EPT-EPC beamforming algorithm converges in a few iterations.

4 Simulation Results

We present below several numerical examples to demonstrate the performance of the proposed RF-MIMO architecture with EPT/EPC beamforming. In all the simulations, the transmitted signals belong to a QPSK constellation, and the flat-fading channel is generated according to an i.i.d. Rayleigh channel model. The performance is evaluated in terms of bit error rate (BER), and we have compared the following systems:

- Proposed scheme: Denoted as EPT/EPC and based on the simplified architecture shown in Fig. 3.
- Maximum ratio transmission and maximum ratio combining (denoted as MRT/MRC). This is the optimal beamforming strategy, which requires a conventional MIMO system or the RF-MIMO architecture shown in Fig. 1.
- Real beamforming (referred to as RB). This is the optimal beamforming for the simplified architecture in Fig. 2.
- Equal gain transmission and equal gain combining (EGT/EGC). This is an alternative simplification based on the use of phase shifters, and the elimination of the variable gain amplifiers.
- Selection diversity transmission and selection diversity combining (SDT/SDC). This technique consists in the selection of the best transmit/receive pair, i.e., the antennas associated to the coefficient with largest absolute value in the MIMO channel \mathbf{H} .

In the first simulation example we consider an scenario with only one transmit antenna. Therefore, we are obtaining the optimal receive EPC beamformer by solving the relaxed problem in (3). Fig. 4 shows the obtained results in the case of $n_R = 4$ and $n_R = 10$ receive antennas, where we can see that the proposed scheme is able to exploit all the spatial diversity in the system, and that the performance degradation with respect to more complicated RF-MIMO architectures is not greater than 3 dB.

Fig. 5 shows the results in a MIMO case with $n_T = n_R = 4$ antennas and ten iterations of the proposed iterative beamforming algorithm. Here we can see that the proposed method provides satisfactory results, and that the performance of the proposed architecture is better than that of an antenna selection method. Furthermore, we can

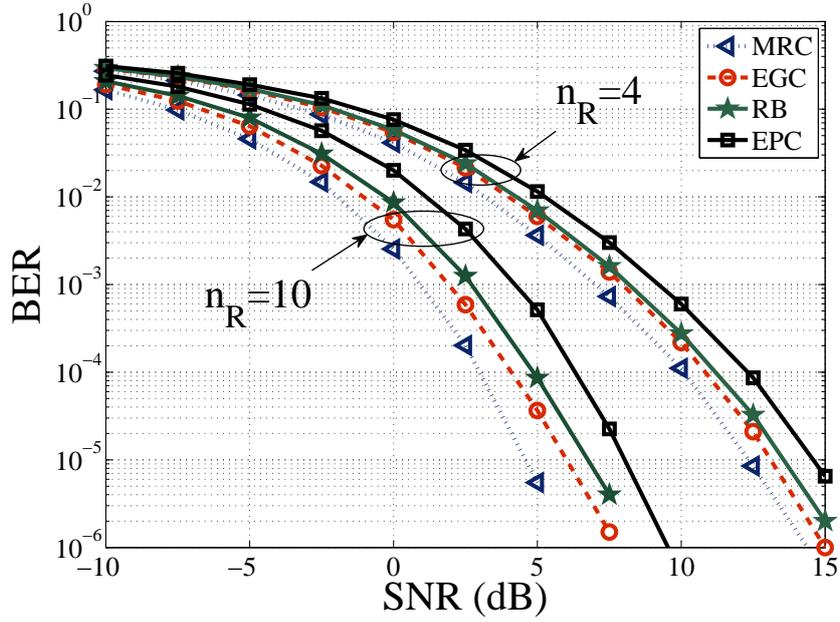


Fig. 4. Performance of the different architectures in a SIMO case.

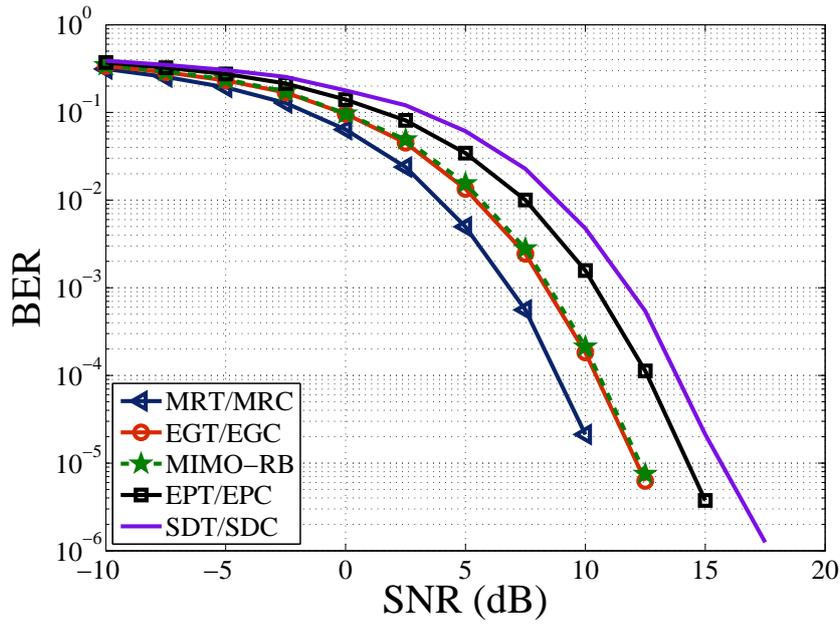


Fig. 5. Performance of the different architectures in a 4×4 MIMO case.

observe that the performance of the proposed system architecture is not very far from that of other RF-MIMO systems with a higher hardware cost and power consumption.

5 Conclusion

In this paper we have presented a simplified architecture for analog antenna combining, which is based on the application of different gain factors to each RF branch. From a baseband point of view, the proposed architecture imposes the use of equal phase transmit (EPT) and equal phase combining (EPC) beamformers. In general, the design of the optimal beamformers results in a non-convex optimization problem, which can not be exactly solved by means of standard optimization techniques. However, in the case of rank-one MIMO channels, which includes the SIMO and MISO cases, the optimal beamformers can be obtained by means of a semidefinite relaxation approach. This fact is exploited to propose an iterative algorithm based on alternating optimizations of the transmit and receive beamformers. Several simulation examples illustrate the good performance of the proposed algorithm, and they also show that the significant decrease in the system cost and power consumption justifies the slight performance degradation with respect to other alternative MIMO architectures.

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