ANALOG ANTENNA COMBINING IN MULTIUSER OFDM SYSTEMS: BEAMFORMING DESIGN AND POWER ALLOCATION

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ABSTRACT

The goal of this paper consists in characterizing the capacity region of a broadcast system based on analog antenna combining and orthogonal frequency division multiplexing (OFDM). In particular, we consider the maximization of a weighted sum of the rates, which results in a highly non-convex optimization problem. The proposed iterative approach is based on the alternative optimization of the beamformers, and the power and subcarrier allocation. Finally, our experiments show that in most practical situations, the complexity of the optimal approaches (subcarrier sharing and power allocation) is not justified by the marginal gains in the capacity region.

Index Terms— Analog beamforming, power allocation, OFDM, multiuser, broadcast channel.

1. INTRODUCTION

Multiple-input multiple-output (MIMO) wireless radios that apply analog beamforming techniques already in the radio frequency domain have recently received renewed interest due to their reduced system size, cost and power consumption in comparison to conventional MIMO systems [1]. For point-to-point links, the design of the optimal Tx-Rx analog beamformers for multicarrier transmissions has been throughly considered in [2].

The goal of this paper consist in evaluating these kind of systems in a multiuser scenario. In particular, we consider a broadcast channel based on orthogonal frequency division multiplexing (OFDM), and try to numerically characterize the capacity region, which is accomplished by maximizing a weighted sum of the user's rates. Unfortunately, this optimization problem is highly non-convex, and we have to resort to an iterative suboptimal technique, which is based on the alternative solution of two optimization problems. On the one hand, the analog beamformers (RF weights) are optimized considering a virtual point-to-point channel [2], which is obtained from a fixed power allocation among users and subcarriers. On the other hand, the power allocation problem is solved (under different criteria) considering fixed beamformers, which allows us to resort to well-known results in the case of single-antenna OFDM-based broadcast channels. Specifically, in [3] the authors solve the optimal power and subcarrier allocation assuming successive decoding in the subcarriers. In [4], the authors propose a numerical method for characterizing the rate region achievable with frequency-division multiple access (FDMA) for a Gaussian multiple-access channel with intersymbol interference. In [5], the authors focus on an orthogonal frequency division multiple access (OFDMA) system (subcarrier sharing is not allowed), and the subcarrier and power allocation problem is designed for maximizing the sum capacity.

In this paper we also consider the more complicated problem of maximizing weighted-sum capacities in OFDMA systems with power allocation. In particular, an approximate solution to this general non-convex problem is obtained by means of an iterative approach. Finally, the main conclusion of this work, which is supported by simulation examples, is that for moderate signal to noise ratios, the complexity of subcarrier-sharing and power allocation approaches is not justified by the marginal gains observed in the capacity regions.

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2. PROBLEM FORMULATION

Let us consider a downlink channel with one base station (BS) and K users as the one in Fig.1. We assume that the system is based on analog antenna combining and OFDM with L subcarriers. The main idea of analog antenna combining consists in reducing the number of down/up conversion RF chains by applying analog weights in the RF domain [6]. Thus, from a baseband point of view, the main difference with a conventional MIMO system consists in the fact that the same beamformer (RF weights) is applied to all the subcarriers.

At the transmitter side the BS is equipped with n_T antennas and performs analog combining with the beamformer $\mathbf{u} \in \mathbb{C}^{n_T \times 1}$ to transmit a single OFDM data stream. At the receiver side, the users have n_R antennas and perform analog combining with beamformers $\mathbf{v} \in \mathbb{C}^{n_R \times 1}$.Therefore, after removing the cyclic prefix (which is assumed longer than the channel impulse response) and performing FFT, the signal viewed by the k-th user in the l-th subcarrier is given by

$$y_{k,l} = \mathbf{v}_k^H \mathbf{H}_{k,l} \mathbf{u} s_l + \mathbf{v}_k^H \mathbf{r}_{k,l},$$

where $\mathbf{H}_{k,l}$ represents the response of the MIMO channel for the k-th user and l-th subcarrier, s_l is the signal transmitted in the l-th subcarrier, and $\mathbf{r}_{k,l}$ is the additive noise vector whose components are i.i.d. zero-mean circular complex Gaussian random variables with variance σ^2.

In this work we try to characterize the capacity region, and follow the classical approach based on the maximization of the weighted-sum capacity. Thus, defining the user weights (or priori-

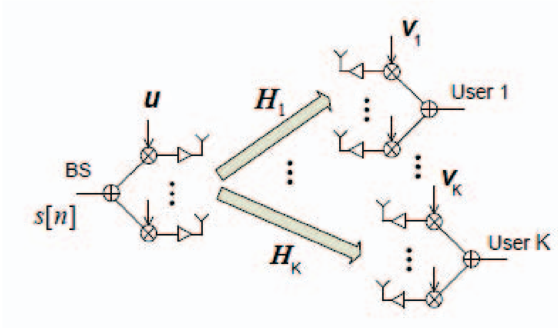

Fig. 1. Downlink OFDM multiuser system with analog beamforming

ties) as $\lambda_k \geq 0$, our optimization problem is

$$\underset{\mathbf{u},\mathbf{v}_k,P_{k,l}}{\text{maximize}} \sum_{k=1}^{K}\sum_{l=1}^{L}\lambda_k R_{k,l}, \tag{1}$$

$$\begin{aligned}\text{subject to } & \|\mathbf{u}\| \leq 1,\\ & \|\mathbf{v}_k\| \leq 1, \qquad \forall k,\\ & P_{k,l} \geq 0, \qquad \forall k,l,\\ & \sum_{k=1}^{K}\sum_{l=1}^{L} P_{k,l} = P,\end{aligned}$$

where $P_{k,l}$ is the fraction of the total available power P assigned to the l-th subcarrier and user k, and $R_{k,l}$ is the associated rate, which obviously depends on the particular transmission and decoding scheme.

A brief comment is in order here. We must note that the solutions of (1) will provide a set of Pareto-optimal points [7] of the multiobjective problem consisting in maximizing the users' rates. This Pareto optimal points could not completely characterize the capacity region, which is not necesarily convex. However, the Pareto optimal solutions are sufficient for obtaining the capacity region by means of time-sharing [8]

3. DESIGN OF THE BEAMFORMERS

Unfortunately, the optimization problem in (1) is highly non-convex in general, and we have to resort to a suboptimal technique. In this section we design the beamformers assuming a fixed power allocation scheme. In particular, we define the virtual channels $\tilde{\mathbf{H}}_{k,l}$ as

$$\tilde{\mathbf{H}}_{k,l} = \mathbf{H}_{k,l}\sqrt{P_{k,l}},$$

With this definition, the problem of designing the transmit and receive beamformers can be written as

$$\underset{\mathbf{u},\mathbf{v}_k}{\text{maximize}} \quad \sum_{k=1}^{K}\sum_{l=1}^{L}\lambda_{k,l}\log\left(1+\frac{\left|\mathbf{v}_k^H\tilde{\mathbf{H}}_{k,l}\mathbf{u}\right|^2}{\sigma_{k,l}^2}\right) \tag{2}$$

$$\begin{aligned}\text{subject to} \quad & \|\mathbf{u}\| \leq 1,\\ & \|\mathbf{v}_k\| \leq 1 \qquad k = 1,\ldots,K,\end{aligned}$$

where $\sigma_{k,l}^2$ is the noise plus interference seen by the k-th user in the l-th subcarrier. In general, there are three different strategies to treat the interference in OFDM scenarios. The values of the interference depend on the strategy adopted:

1. FDMA: In this case, each subcarrier is assigned to one user, and therefore, there is no interference, $\sigma_{k,l}^2 = \sigma^2$.
2. Subcarrier sharing: In this case, each user treats the interference received for all the other users as noise, that is

$$\sigma_{k,l}^2 = \sigma^2 + \left(\sum_{j\neq k, j=1}^{K} P_{j,l}\right)\left|\mathbf{v}_k^H\mathbf{H}_{k,l}\mathbf{u}\right|^2.$$

3. Subcarrier sharing with successive interference cancellation: In this case, each user decodes and cancels the interference of the users with worse channels

$$\sigma_{k,l}^2 = \sigma^2 + \left(\sum_{P_{j,l}\leq P_{k,l}} P_{j,l}\right)\left|\mathbf{v}_k^H\mathbf{H}_{k,l}\mathbf{u}\right|^2.$$

The optimization problem in (2) is non-convex, but can be approximately solved following the lines in [2] for the point-to-point case. In particular, the transmit and receive vectors are updated by means of the following rules

$$\begin{aligned}\mathbf{u}(t+1) &= \mathbf{u}(t) + \mu\sum_{k=1}^{K}\sum_{l=1}^{L}\tilde{\mathbf{R}}_{\text{MISO}_{k,l}}(t)\mathbf{u}(t),\\ \mathbf{v}_k(t+1) &= \mathbf{v}_k(t) + \mu\sum_{l=1}^{L}\tilde{\mathbf{R}}_{\text{SIMO}_{k,l}}(t)\mathbf{v}_k(t),\end{aligned}$$

where μ is a step-size, t denotes the iteration index, the matrices

$$\begin{aligned}\tilde{\mathbf{R}}_{\text{MISO}_{k,l}} &= \lambda_k\tilde{\mathbf{h}}_{\text{MISO}_{k,l}}\tilde{\mathbf{h}}_{\text{MISO}_{k,l}}^H,\\ \tilde{\mathbf{R}}_{\text{SIMO}_{k,l}} &= \lambda_k\tilde{\mathbf{h}}_{\text{SIMO}_{k,l}}\tilde{\mathbf{h}}_{\text{SIMO}_{k,l}}^H,\end{aligned}$$

can be seen as weighted covariance matrices, and

$$\tilde{\mathbf{h}}_{\text{MISO}_{k,l}} = \tilde{\mathbf{H}}_{k,l}^H\mathbf{v}_k, \quad \tilde{\mathbf{h}}_{\text{SIMO}_{k,l}} = \tilde{\mathbf{H}}_{k,l}\mathbf{u},$$

are the virtual multiple-input single-output (MISO) and single-input multiple-output (SIMO) channels after fixing the receive and transmit beamformers.

Although we can not guarantee the convergence of this iterative technique to the global optimal beamformers, following the lines in [1] it can be proved that it converges to a solution of the KKT conditions, and in practice provide very satisfactory results. Furthermore, in our numerical examples, the convergence is achieved with only a few iterations.

4. POWER AND SUBCARRIER ALLOCATION

Once the transmit and receive beamformers have been fixed, the problem reduces to the MIMO-OFDM single-antenna case. In this section, the power allocation problem will be solved by three different methods: 1) Subcarrier sharing and water-filling power allocation (OFDM-SS), 2) OFDMA with water-filling power allocation (OFDMA-WF) and 3) OFDMA with uniform power allocation (OFDMA-UPA).

4.1. OFDM-SS

In this scenario, the receivers perform successive decoding and, after fixing the beamformers, the original optimization problem reduces to

$$\begin{aligned} \underset{P_{k,l},\sigma_{k,l}}{\text{maximize}} \quad & \sum_{k=1}^{K}\sum_{l=1}^{L}\lambda_k \log\left(1+\frac{P_{k,l}|h_{k,l}|^2}{\sigma_{k,l}^2}\right), \\ \text{subject to} \quad & P_{k,l}\geq 0, \forall k,l, \\ & \sum_{k=1}^{K}\sum_{l=1}^{L}P_{k,l}=P, \\ & \sigma_{k,l}^2=\sigma^2+\sum_{\alpha_{j,l}\leq\alpha_{k,l}}P_{j,l}|h_{k,l}|^2, \end{aligned}$$

where $h_{k,l}=\mathbf{v}_k^H\mathbf{H}_{k,l}\mathbf{u}$ is the equivalent SISO channel after fixing the beamformers. This problem has been addressed in [3], where the authors obtain the optimal power distribution based on a two-level water-filling approach. Therefore, the results in [3] can be directly applied in our case.

4.2. OFDMA-WF

In order to avoid the complexity of the successive decoding approach, we will consider a more practical scheme based on OFDMA. Obviously, this means that we will not allow subcarrier sharing among users, and therefore our optimization problem can be written as

$$\begin{aligned} \underset{P_{k,l}}{\text{maximize}} \quad & \sum_{k=1}^{K}\sum_{l=1}^{L}\lambda_k \log\left(1+\frac{P_{k,l}|h_{k,l}|^2}{\sigma^2}\right), \qquad (3) \\ \text{subject to} \quad & P_{k,l}\geq 0, \forall k,l, \\ & \sum_{k=1}^{K}\sum_{l=1}^{L}P_{k,l}=P, \\ & \sum_{l=1}^{L}\max_k\left(P_{k,l}\right)=P. \end{aligned}$$

Again, excluding the case of equal priorities λ_k [5], this optimization problem is very difficult to solve in general [4]. Here, in order to solve the general problem we propose an alternative method consisting in an iterative technique. It is based on the two following stages:

- **Fixed power distribution**: If we assume that the power in each subcarrier is fixed, the solution of the problem reduces to selecting (in each subcarrier) the user with the highest contribution to the weighted-sum capacity. That is, the l-th subcarrier will be assigned to the user k maximizing

$$\lambda_k \log\left(1+\frac{P_l|h_{k,l}|^2}{\sigma^2}\right),$$

  where P_l denoted the available power for the l-th subcarrier.

- **Fixed user allocation**: Assuming that the subcarriers have been previously assigned to the users, the problem reduces to maximizing the capacity of a virtual SISO channel. That is, if the channel response for the selected user in the l-th subcarrier is denoted as h_l, we have

$$\begin{aligned} \underset{P_l}{\text{maximize}} \quad & \sum_{l=1}^{L}\lambda_k \log\left(1+\frac{P_l|h_l|^2}{\sigma^2}\right), \\ \text{subject to} \quad & \sum_{l=1}^{L}P_l=P, \end{aligned}$$

  and the solution reduces to a *weighted* water-filling.

In our simulations, we have verified that this iterative approach converges very fast to a satisfactory (local) solution of the problem in (3).

4.3. OFDMA-UPA

In the single-user case, [9] gives solid arguments for using uniform power allocation (UPA) instead of water-filling. Following this idea, once the waterfilling algorithm solves the subcarrier and power allocation, the total power assigned to the subcarriers of each user is reallocated uniformly between the subcarriers maintaining the separation between users. This approach is interesting in high SNR scenarios.

5. SIMULATION

In our experiments we consider a two user system with four antennas in transmission and four antennas in reception, $L = 256$ subcarriers, and Rayleigh channels with exponential power delay profile of

$$\text{PDP}_n = E\left[|h_k[n]|^2\right] = (1-\rho)\,\rho^n$$

for $n = 1, \cdots, N$ the number of temporal lags, and ρ is the parameter that controls the power delay profile. We have considered two different scenarios, one with low frequency selectivity ($\rho = 0.4$) and the other one with high frequency selectivity ($\rho = 0.7$). In our experiments we fix the total power $P = 1$ and define the SNR as

$$\text{SNR} = \frac{\sum_{k=1}^{K}\sum_{l=1}^{L}|\mathbf{H_{k,l}}|^2}{KL\sigma^2}$$

Normalizing the channels we obtain different values of SNR.

Figures 2-5 show the obtained results in the different scenarios with two random channel realizations.

As can be seen, when compared with a conventional SISO system, analog beamforming provides an important improvement in the capacity region. Furthermore, it is clear that the capacity region of the OFDM-SS and OFDMA-WF methods are quite similar, and the drastic complexity reduction makes the OFDMA-WF a preferable option. Moreover, as one could expect, the gap between OFDM-UPA and the alternative approaches decreases when the SNR increases.

6. CONCLUSIONS

In this paper we have evaluated the performance of analog-antenna combining systems in a multiuser scenario. In particular, considering a broadcast system based on OFDM, we have proposed iterative algorithms for the design of the beamformers and the power allocation. The obtained results highlight the benefits of analog antenna combining in multiuser systems, and suggest the use of the most simple technique, which is based on OFDMA with waterfilling or uniform power allocation.

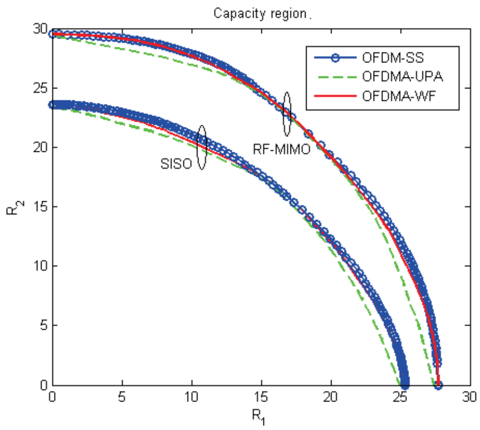

Fig. 2. Capacity region for a 2-user RF-MIMO system with SNR = 20dB and low frequency ($\rho = 0.4$)

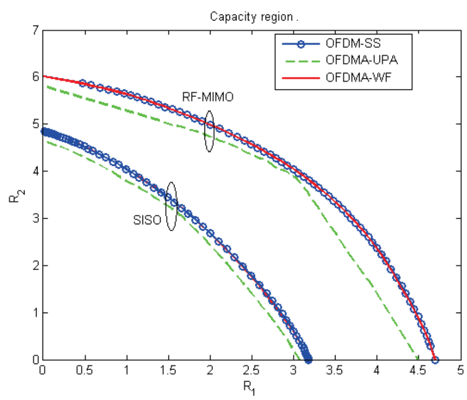

Fig. 3. Capacity region for a 2-user RF-MIMO system with SNR = 5dB and low frequency selectivity ($\rho = 0.4$)

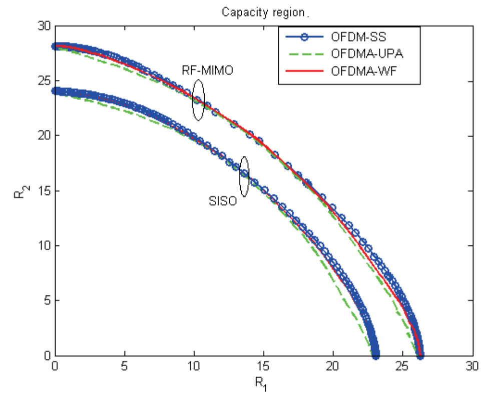

Fig. 4. Capacity region for a 2-user RF-MIMO system with SNR = 20dB and high frequency selectivity ($\rho = 0.7$)

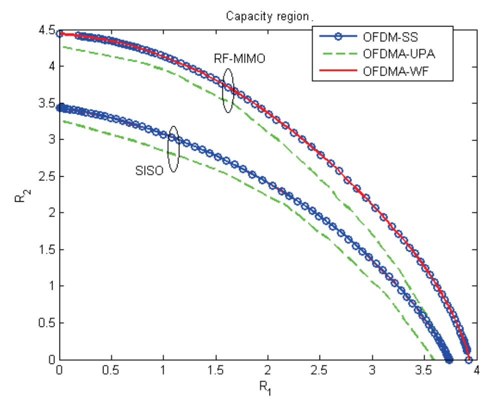

Fig. 5. Capacity region for a 2-user RF-MIMO system with SNR = 5dB and and high frequency selectivity ($\rho = 0.7$)

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