## Asymptotic evaluation of physical optics for the analysis of on-board antennas

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A technique for the study of on-board antennas mounted over arbitrary structures is presented. The arbitrary shape of the bodies is achieved by employing parametric surfaces called nonuniform rational B-splines. The method solves the physical optics integral following an asymptotic technique known as the 'stationary phase method'. Both the comptutational efficiency and accuracy of this procedure increases as the electric size of the structure increases.

Introduction: The analysis of on-board antennas has been widely studied over recent decades. Low frequency techniques, such as the method of moments (MM), the finite element (FE) method, and the conjugate gradient method/fast Fourier transform (CGM/ FFT) are accurate and rigorous but when the structure begins to increase in size, the application of these techniques becomes unaffordable. Among the high frequency techniques, physical optics (PO) [1] has revealed itself as a good approach for evaluating the interactions between the antenna and the structure on which it is mounted. It is more accurate when the electrical size of the structure is larger and the body has a low number of discontinuities (wedges, edges, vertices, etc.). When the PO approach is considered, the current induced on the surface of the structure is due only to the impressed fields that come from the antenna. The radiated field produced by this current is obtained after its integration over the illuminated region of the body.

The integration can be performed rigorously: the current is sampled over the structure with a criterion of 10 samples per wavelength. If this operation line is followed, when the size of the structure reaches hundreds or thousands of  $\lambda^2$ , the processing is totally inefficient from a computational viewpoint. This Letter proposes the employment of an asymptotic technique to evaluate that integral. As an asymptotic procedure, it is more accurate and more efficient as the electric size of the body increases. The asymptotic procedure considered is the *stationary phase method* [2 – 4]. Following this line of operation, solution of the radiated field integral is restricted to the contribution of some specific points over the surface.

The environment or structure is modelled with a set of parametric surfaces called NURBS (nonuniform rational B-spline) [5]. The main advantage of this approach is its accurate representation of the real structure demanding low storage of data. These surfaces depend on polynomial functions called B-splines. As these functions are computationally unstable, the NURBS format is translated into the Bézier format through the Cox-de-Boor algorithm [5]. Bézier patches are also parametric surfaces, however, they are now represented in terms of a Berstein basis.

Stationary phase method: The stationary phase method [2-4] is an asymptotic technique that solves integrals of the shape

$$I = \int_{\mathcal{H}} g(\vec{x}) e^{jkf(\vec{x})} d\vec{x} \tag{1}$$

where  $g(\vec{x})$  is an analytic function with smooth variation within the integration domain H and  $f(\vec{x})$  has rapid variation over the same domain;  $\vec{x}$  represents the vector of independent variables, in our case, the parametric variables over the surface u and v.

The stationary phase method is applied to evaluate the radiated field due to the current induced on the surface of the structure. Conversely, the physical optics approach is adopted to account for the induced current. When this approximation is performed, only the current on the illuminated region of the structure is considered. The expression for the radiated field under far field conditions when the antenna is near the surface is

$$\vec{E}^{scatt} = -\frac{j}{\lambda} \frac{e^{-jkr}}{r} I$$

$$I = \int_{S^{illum}} \frac{\{ [\hat{k}_s \times [(\vec{E}^{dip} \times \hat{k}_i) \times \hat{n}]] \times \hat{k}_s \}}{h(\vec{r}')} e^{jk[\hat{k}_s \cdot \vec{r}' - h(\vec{r}')]} ds'$$
(2)

where  $\hat{k}_s$  is the scattering unitary direction,  $\hat{k}_i$  is the incident unitary direction,  $\vec{E}^{dip}$  is the impressed electric field, referred to the antenna phase centre,  $h(\vec{r}')$  is the distance from the antenna to

the current application point,  $\hat{n}$  is the vector normal to the surface at the current application point, and  $\vec{r}' = \vec{r}'(u,v)$  is the current application point, depending on the parametric co-ordinates.

When eqns. 1 and 2 are compared, the values for  $f(\vec{x})$  and  $g(\vec{x})$  can be deduced. The integration domain H is now the domain of definition of each Bézier surface.

The structures under consideration tend to be electrically large, with their shape being smooth in terms of wavelengths. Under these conditions, function g(u, v) varies slowly within the integration domain. The function f(u, v) represents the exponent of the phase term. As the frequency of analysis increases, the contributing terms of the exponential tend to cancel each other out, with the exception of an area denoted as the stationary phase point which coincides with the specular reflection point. That is, the contributions to the radiated field mainly come from specific points located over the surface. This effect complies with the local behaviour of the electromagnetic mechanisms when high frequency techniques are applied.

The stationary phase method gives a solution to the integral *I* by means of the contribution of several points on the surface that comply with some specified conditions. These points are classified into three types:

- (i) stationary phase points  $I_s(u_s, v_s)$ : these are within the domain of each Bézier surface
- (ii) boundary points  $I_b(u_c, v_c)$ : these are located over the boundaries of each parametric surface
- (iii) vertex points  $I_{\nu}(u_{\nu}, v_{\nu})$ : the four vertices of the parametric patch.

The computational problem which must be solved consists of finding all of these points and, afterwards, employing closed form expressions in order to determine the contribution of each point [2-4]. For example, a stationary first-order phase point is that where  $f_u(u_s, v_s) = 0.0$  and  $f_v(u_s, v_s) = 0.0$ ; its contribution is

$$I_{c}(u_{s}, v_{s}) = \frac{2\pi}{k} g e^{jkf} \frac{|\vec{r}_{u} \times \vec{r}_{v}|}{|f_{uu}f_{vv} - f_{uv}^{2}|^{\frac{1}{2}}} e^{j\frac{\pi}{4}[\sigma(\delta+1)]} \Big|_{\substack{u=u_{s} \\ v=v_{s} \\ v=v_{s}}}$$

$$\sigma = \text{sign}(f_{vv}) \qquad \delta = \text{sign}(f_{vv}f_{vv} - f_{vv}^{2}) \qquad (3)$$

where  $f_u$ ,  $f_v$ , and  $f_{uu}$ ,  $f_{vv}$ ,  $f_{uv}$  are, respectively, the first-order and second-order partial derivatives of f(u,v) with respect to the parametric coordinates u and v,  $\overrightarrow{r}_{uv}$ ,  $\overrightarrow{r}_{vv}$  are the parametric derivatives of the position vector of a point on a Bézier surface.

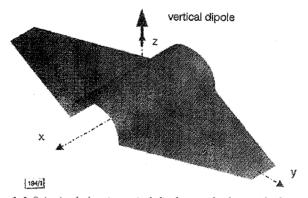
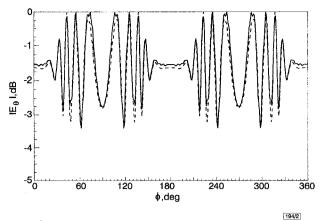


Fig. 1 Infinitesimal electric vertical dipole over fuselage with electrical dimensions  $120\lambda\times40\lambda\times10\lambda$ 

Results: Considerable concordance of results has been obtained, with important improvements in the time efficiency. Fig. 1 shows a structure composed of curved and flat surfaces. The antenna is an infinitesimal electric dipole located 1.0m above the structure. The dipolar moment for the dipole is  $I_0 \cdot l = 1.0, 0^{\circ}$ . The operation frequency is 3.0GHz. The radiation pattern for a cut at  $\theta = 90^{\circ}$  is illustrated in Fig. 2. The Figure shows a comparison between the rigorous evaluation (continuous line) of the PO integral and its evaluation using the SPM (stationary phase method) (dashed line). The maximum dimensions of the structure are  $120\lambda \times 40\lambda \times 10\lambda$ .

The execution time for the rigorous evaluation of the PO integral in this case was 1h, 4min and 40s; the stationary phase method took only 1m and 55s. Both executions were carried out

on a PC Pentium 200MHz with 128Mbytes of RAM. This reduction in computation time has been observed in all the structures analysed.



**Fig. 2**  $|\vec{E}_{\theta}|$  field for radiation pattern at  $\theta = 90^{\circ}$ 

----- PO

Conclusions: Reliable results have been obtained when the asymptotic integration (stationary phase method) of the PO current is combined with geometric modelling in terms of NURBS surfaces. The new technique is much more efficient in terms of computation time. Both accuracy of results and efficiency of computation time increase as the electrical size of the structure increases.

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## Proximity-coupled stacked circular-disc microstrip antenna with slots

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The design and measured results of a proximity-fed stacked circular-disc antenna are described. The antenna has an impedance bandwidth of 26%, centred around 4.2GHz, a gain of 8dBi, and -30dB cross-polarisation. An essential feature of the design is the presence of four linear slots in the bottom patch of the stacked arrangement.

Introduction: Bandwidth broadening of microstrip antennas using stacked patches has been a topic of considerable interest in recent years. The feeding methods that have been extensively studied in connection with stacked patches are coaxial probe [1, 2], and stripline through an aperture [3, 4]. Typical bandwidths obtained using coaxial feeds are 10–20%, while those obtained using aperture coupling are in the range 20–40%. The disadvantage of a coaxial feed is that the feed has to be physically soldered to the patch. The

disadvantage of aperture coupling is the relatively high backlobe radiation. A feeding method which does not have the above short-comings is proximity coupling. This method was introduced by Pozar and Kaufman [5] for the single layer rectangular patch. Using a tuning stub, a 13% impedance bandwidth was obtained, with a centred frequency of ~3.4GHz. The cross-polarisation level was ~20–25dB below that of the co-polarisation radiation. To the authors' knowledge, there has been relatively little attention paid to proximity-fed stacked patches in the literature.

This Letter presents the design and measured results of a proximity-coupled stacked circular-disc microstrip antenna with 26% impedance bandwidth, centred around 4.2 GHz. At the centre of the band, the co-polarisation gain in the broadside direction is ~8 dBi and ~30 dB higher than the cross-polarisation gain. An essential feature in the design is that there are four linear slots in the bottom patch of the stacked arrangement.

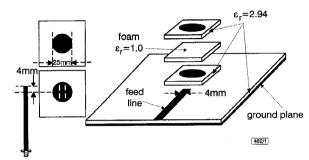


Fig. 1 Geometry of proximity coupled stacked circular-disc microstrip

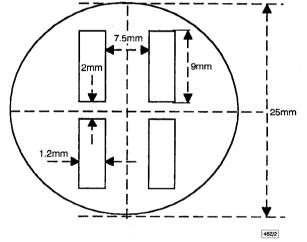
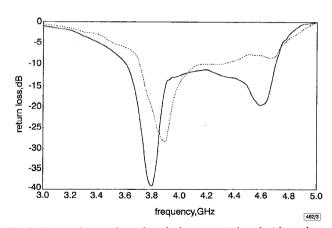


Fig. 2 Orientation and dimensions of the four slots



**Fig. 3** Measured return loss of stacked antenna with and without slots — — — without 4 slots

Description of antenna: The geometry of the antenna is shown in Fig. 1. The three substrates have the same relative permittivity of 2.94. The diameters of the upper and lower circular patches are 25mm. The thickness of the upper substrate is 1.57mm and that of

with 4 slots