STRUCTURAL RISK MINIMIZATION FOR ROBUST BLIND IDENTIFICATION OF SPARSE SIMO CHANNELS

Dongho Han, Jose C. Principe, Liuqing Yang

University of Florida
Dept. of Electrical & Computer Engineering
Gainesville, FL 32611-6130, USA

Ignacio Santamaría, Javier Vía

University of Cantabria
Dept. of Communications Engineering
39005 Santander, Spain

ABSTRACT

In this paper the structural risk minimization (SRM) principle is applied to derive an iterative algorithm for blind identification of sparse single-input multiple-output (SIMO) channels. The key idea consists of reformulating this problem as a support vector regression (SVR) problem in which the channel coefficients are the Lagrange multipliers of the dual problem. By employing the Vapnik's ϵ-insensitivity loss function, the solution can be expanded in terms of a reduced number of Lagrange multipliers (i.e., the nonzero filter coefficients) and then a sparse solution is found. This method can be also used for non-sparse channels when the channel order has been highly overestimated. In this situation, the SRM principle pushes zero to the small leading and trailing terms of the impulse response. Some simulation results are provided to demonstrate the performance of the method.

1. INTRODUCTION

Blind channel identification, which consists of identifying the channel from its output without using a training sequence, is a widely studied problem with many signal processing applications: channel equalization, sonar, seismic deconvolution, etc. Single-input multiple-output (SIMO) channels appear either when the signal is oversampled at the receiver or from the use of an array of antennas. The transmission channels in some applications such as high-definition television (HDTV) [1], hilly terrain delay profile or underwater acoustic channels [2], are sparse: that is, only a small number of their coefficients are non-zero.

Since the work by Tong, Xu and Kailath [3], it is well known that SOS are sufficient for blind identification when the input signal is informative enough and the channels do not share any common roots. Widely used SOS-based methods are the subspace approach (SS), the least squares (LS) technique and the linear prediction (LP) methods. However, a common drawback of SS and LS techniques is their poor performance when the channel order is overestimated. Some robust techniques to alleviate this problem have been recently proposed [4, 5]. Although these methods offer increased robustness against channel order overestimation, they still fail to identify sparse channels.

The structural risk minimization (SRM) principle is a criterion that establishes a trade-off between the complexity of the solution and the closeness to the data. In particular, the support vector machine (SVM) technique, which can be derived from the SRM principle, typically provides sparse solutions [6]. Specifically, the SVM solution can be expanded in terms of a reduced set of relevant input data samples (the so-called support vectors) [7]. Recently, the SVM approach was applied to blind identification of non-sparse SIMO channels [8]. This method can be viewed as a regularized version of the LS technique proposed in [9], but it does not exploit explicitly the sparsity provided by the SVM solution.

In this paper we present a new SVM-based blind identification algorithm for sparse SIMO channels. In the proposed formulation, the channel coefficients play the role of the Lagrange multipliers. By using the Vapnik's ϵ-insensitive loss function only those Lagrange multipliers corresponding to support vectors are nonzero and therefore a sparse solution is obtained. The ϵ parameter allows us to control the sparseness of the final solution. Finally, the proposed algorithm can be also used as a robust method to combat the channel overmodeling problem, even if the channel is not sparse.

2. BLIND SPARSE SIMO CHANNEL IDENTIFICATION

Without loss of generality, in this work we focus on the one-input, two-output SIMO system shown in Fig.1, where the FIR channels $(\mathbf{h}_1, \mathbf{h}_2)$ are known to be sparse in a pri-

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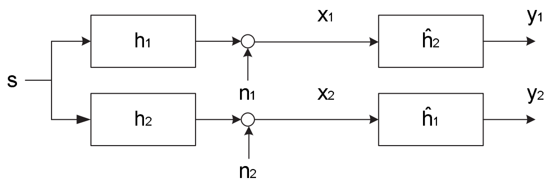

Fig. 1. Single input/two output channels

ori. In blind channel identification, we need to identify the unknown channel responses, $\mathbf{h}_1, \mathbf{h}_2$, from the observed receive signals only. If the order of the channels is M, then the received signal $\mathbf{x}_i$ from the *i*th channel is

$$x_i(n) = \sum_{k=0}^{M} h_i(k)s(n-k) + n_i(n), \quad i = 1, 2 \tag{1}$$

When we cast $x_i(n), h_i(k), s(n), n_i(n)$ into vectors $\mathbf{x}_i$, $\mathbf{h}_i$, $\mathbf{s}$ and $\mathbf{n}_i$, (1) becomes

$$\mathbf{x}_i = \mathbf{h}_i * \mathbf{s} + \mathbf{n}_i, \quad i = 1, 2 \tag{2}$$

where $*$ denotes convolution. As shown in Fig.1, using the channel outputs $(\mathbf{x}_1, \mathbf{x}_2)$ and the channel estimates $(\hat{\mathbf{h}}_1, \hat{\mathbf{h}}_2)$, one can form the following vectors.

$$\mathbf{y}_1 = \mathbf{x}_1 * \hat{\mathbf{h}}_2,$$
$$\mathbf{y}_2 = \mathbf{x}_2 * \hat{\mathbf{h}}_1 \ .$$

In the absence of noise, and if the nontrivial channel estimates are exact, then we have $\mathbf{y}_1 = \mathbf{y}_2$. This is because

$$\begin{aligned} \mathbf{y}_1 &= \mathbf{h}_2 * \mathbf{x}_1 = \mathbf{h}_2 * (\mathbf{h}_1 * \mathbf{s}) \\ &= \mathbf{h}_1 * (\mathbf{h}_2 * \mathbf{s}) = \mathbf{h}_1 * \mathbf{x}_2 = \mathbf{y}_2 \ . \end{aligned}$$

This relationship can be re-expressed in a matrix-vector form as:

$$\mathbf{y}_1 = \mathbf{X}_1 \hat{\mathbf{h}}_2 = \mathbf{X}_2 \hat{\mathbf{h}}_1 = \mathbf{y}_2 \ , \tag{3}$$

where $\mathbf{X}_i$'s are Toeplitz matrix defined as

$$\mathbf{X}_i = \begin{pmatrix} x_i(M) & \cdots & x_i(0) \\ x_i(M+1) & \cdots & x_i(1) \\ \vdots & \ddots & \vdots \\ x_i(M+N-1) & \cdots & x_i(N-1) \end{pmatrix}, \tag{4}$$

or, equivalently,

$$\mathbf{X} \, \hat{\mathbf{h}} = \mathbf{0} \ , \tag{5}$$

where

$$\mathbf{X} = \left[\ \mathbf{X}_2 \quad -\mathbf{X}_1 \ \right], \quad \hat{\mathbf{h}} = \begin{bmatrix} \hat{\mathbf{h}}_1 \\ \hat{\mathbf{h}}_2 \end{bmatrix} .$$

If we solve (5) by minimizing $\hat{\mathbf{h}}^T \mathbf{X}^T \mathbf{X} \hat{\mathbf{h}}$ with the constraint $\parallel \hat{\mathbf{h}} \parallel = 1$, then $\hat{\mathbf{y}}$ is the LS solution which is the eigenvector corresponding to the minimum eigenvalue of $\mathbf{X}^T \mathbf{X}$. Based on (5), we will next develop a blind identification method for sparse SIMO channels using SVMs.

2.1. Support Vector Regression Approach

In [8] the authors propose a blind iterative procedure in which, at each iteration, a desired output is constructed as $\mathbf{y}_d = \frac{\mathbf{y}_1 + \mathbf{y}_2}{2}$, and then a pair of uncoupled SVM-based regression problems is solved. As it was shown in [8], this procedure offers an increased robustness in comparison to [9]; however, it does not take full advantage of the SVM framework to get a sparse solution.

To fully exploit the sparse approximation characteristics provided by SVMs, we first propose the following regression problems

$$\mathbf{X}_1 \hat{\mathbf{h}}_2 \simeq \mathbf{y}_d, \tag{6}$$
$$\mathbf{X}_2 \hat{\mathbf{h}}_1 \simeq \mathbf{y}_d, \tag{7}$$

and premultiplying Eqs. (6) and (7) by $\mathbf{X}_1^T$ and $\mathbf{X}_2^T$,

$$\mathbf{X}_1^T \underbrace{\mathbf{X}_1 \hat{\mathbf{h}}_2}_{\mathbf{w}_1} = \underbrace{\mathbf{X}_1^T \mathbf{y}_d}_{\tilde{\mathbf{y}}_1},$$
$$\mathbf{X}_2^T \underbrace{\mathbf{X}_2 \hat{\mathbf{h}}_1}_{\mathbf{w}_2} = \underbrace{\mathbf{X}_2^T \mathbf{y}_d}_{\tilde{\mathbf{y}}_2},$$

or, more compactly,

$$\mathbf{X}_1^T \mathbf{w}_1 = \tilde{\mathbf{y}}_1, \tag{8}$$
$$\mathbf{X}_2^T \mathbf{w}_2 = \tilde{\mathbf{y}}_2. \tag{9}$$

These are two new regression problems for which the input matrix is now the transposed input matrix $\mathbf{X}_i^T$, and the new output vector is $\mathbf{X}_i^T \mathbf{y}_d$. Moreover, the new regressor $\mathbf{w}_i$ admits an expansion in terms of the filter coefficients, which, in this way, become the Lagrange multipliers of the SVM formulation.

The proposed SVR method minimizes the following cost function

$$J(\mathbf{w}_i) = C \sum_{n=0}^{M} (\xi_n + \xi_n^*) + \frac{1}{2} \parallel \mathbf{w}_i \parallel^2 \tag{10}$$

subject to

$$\begin{aligned} \tilde{y}_i(n) - \mathbf{w}_i^T \mathbf{x}_i(n) &\leq \epsilon + \xi_n, & n &= 0, \ldots, M \\ \mathbf{w}_i^T \mathbf{x}_i(n) - \tilde{y}_i(n) &\leq \epsilon + \xi_n^*, & n &= 0, \ldots, M \\ \xi_n &\geq 0, & n &= 0, \ldots, M \\ \xi_n^* &\geq 0, & n &= 0, \ldots, M \end{aligned}$$

for $i = 1, 2$, and where $\mathbf{x}_i(n)$ denotes the n-th column of $\mathbf{X}_i$.

In (10) the regularization parameter C controls the trade-off between the training error and the complexity of the solution. On the other hand, ϵ is a parameter that determines the precision of the regression and therefore controls the sparseness of the final solution. Then, the solution is a linear combination of input data

$$\mathbf{w}_i = \sum_{n=0}^{M} (\alpha_i^*(n) - \alpha_i(n))\mathbf{x}_i(n) \qquad (11)$$

for $i = 1, 2$. In (11) only a small number of Lagrange multipliers $(\alpha_i^*(n) - \alpha_i(n))$, which corresponds to the channel coefficients $h_i(0), h_i(1), \ldots, h_i(M)$, will be nonzero. The dual problem (10) is a quadratic programming (QP) problem, which can be efficiently solved [10].

To summarize, the proposed SVM-based method to blindly identify a sparse SIMO channel can be described as follows:

Algorithm 1 Summary of the SVM based blind sparse channel identification

Choose initial value for C, ϵ and $\hat{h}_1, \hat{h}_2$

repeat

Calculate $\mathbf{y}_1 = \mathbf{X}_1\hat{\mathbf{h}}_2$ and $\mathbf{y}_2 = \mathbf{X}_2\hat{\mathbf{h}}_1$.

Calculate $\mathbf{y}_d = \frac{\mathbf{y}_1 + \mathbf{y}_2}{2}$

Obtain (8),(9) and solve (10) for $i = 1, 2$

Obtain sparse channel coefficients $\mathbf{h}_i$ for i=1,2 from (11)

until Convergence

3. SIMULATION RESULTS

Several simulations have been conducted to test the performance of the proposed algorithm by varying different observation parameters such as signal-to-noise ratio (SNR), sample size, and overestimated channel order. The algorithm performance is measured in terms of normalized mean squared error (NMSE) defined in [5]

$$NMSE = \frac{1}{\| \mathbf{h} \|^2} \min_{\alpha, k \geq 0} \left\| \alpha\hat{\mathbf{h}} - \begin{bmatrix} \mathbf{0}_{k,1} \\ \mathbf{h} \\ \mathbf{0}_{M'-M-k} \end{bmatrix} \right\|^2 \qquad (12)$$

where $M' \geq M$ is the estimated channel order.

In the first simulation we consider a sparse SIMO system which consists of a single transmit antenna and two receive antennas. The two sparse channels are respectively, $\mathbf{H}_1(z) = 1 - 0.62z^{-5} - 0.33z^{-14} + 0.08z^{-24}$, $\mathbf{H}_2(z) = 0.91 + 0.56z^{-11} - 0.28z^{-17}$. Input of this system is $N = 100$, i.i.d. BPSK signals. In Fig.2, we plot the zeros of $\mathbf{h}_1, \mathbf{h}_2$. Note that there are pairs of close zeros which impair subspace based method because of a badly conditioned input correlation matrix. Fig.3 is the result of blind sparse SIMO channel identification with varying SNR for exact channel order estimate and overestimated channel order. We can see that the proposed blind method is superior to other methods particulary when the channel is overestimated.

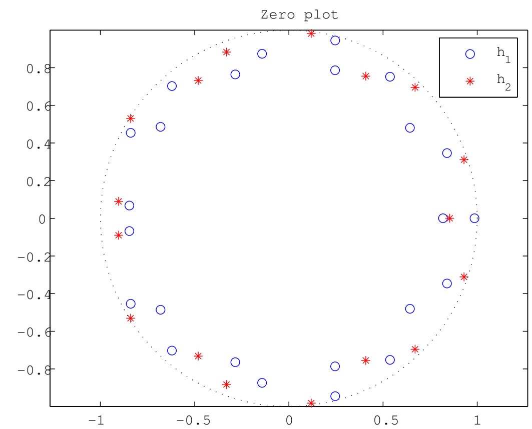

Fig. 2. Zeros of subchannel $\mathbf{h}_1, \mathbf{h}_2$

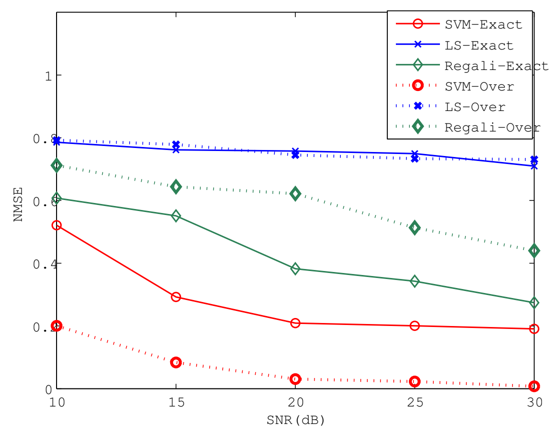

Fig. 3. Comparison with other methods when the channel order is exact (solid) or overestimated by 10 taps (dashed) in a sparse SIMO simulation

Fig.5 shows the robustness to order overestimation when SNR is 20dB. It is evident that the proposed method outperforms other methods in highly overestimated channel order estimate. 50 trials of the proposed algorithm and Regalia method is shown in Fig.6 when the order overestimated by 10 taps and SNR is 30dB. Proposed method performs much

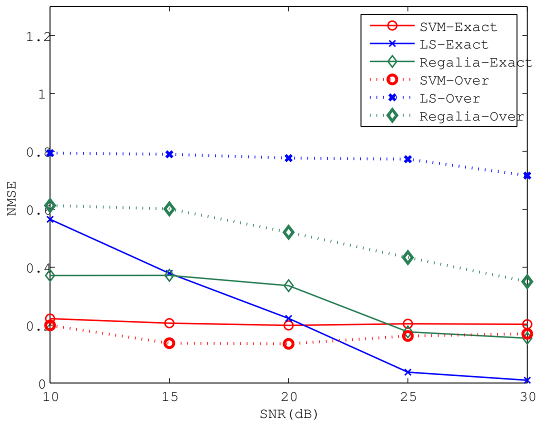

Fig. 4. Comparison with other methods when the channel order is exact (solid) or overestimated by 20 taps (dashed) in a raised-cosine pulse with multipath simulation

better than Regalia method in identifying the coefficients of zero taps or very small taps.

In the second example we consider a raised-cosine pulse limited in 4T (T is the symbol period) with roll-off factor 0.1 and the multipath channel $h(t) = \delta(t) - 0.7\delta(t - \frac{T}{4})$. The input signal is also i.i.d. BPSK signal and the received data were sampled at twice the symbol rate to obtain a SIMO system. Fig.4 shows the performance at different SNRs. Note that LS method and Regalia methods perform better than the proposed method at high SNR when the channel order is known a priori. However with overestimated channel order, proposed method outperforms other methods. 50 trials of the proposed algorithm and Regalia method is shown in Fig.7 when the order overestimated by 20 taps and SNR=20dB. The computational cost to be paid for this robustness to channel order overestimation is solving a QP problem of size N at each iteration which is much higher than LS based method and Regalia method. In its current implementation of the proposed method, its application is limited to the use of small dataset ($N \leq 100$ symbols).

4. CONCLUSIONS

In this paper we have developed a new Support Vector Regression based algorithm for blind identification of sparse SIMO channels. Sparsity property of SVM method was exploited to identify sparse channel coefficients and the good generalization performance of SVM leads a robust solution when the channel order is overestimated. Due to the high computational cost of the proposed iterative algorithm, the use of this SVM based algorithm is advisable in applications when the data samples are small ($N \leq 100$ symbols) and the way to reduce this computational cost should be investigated.

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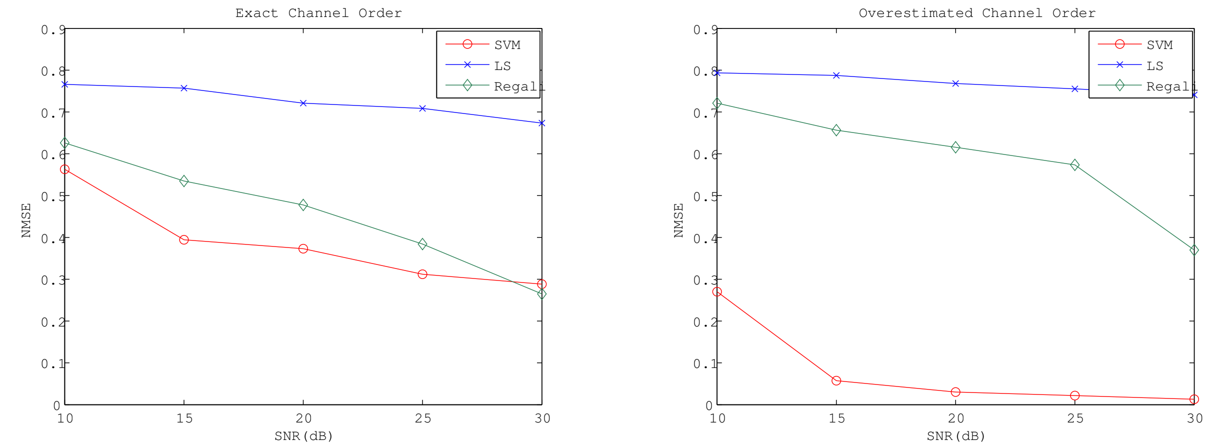
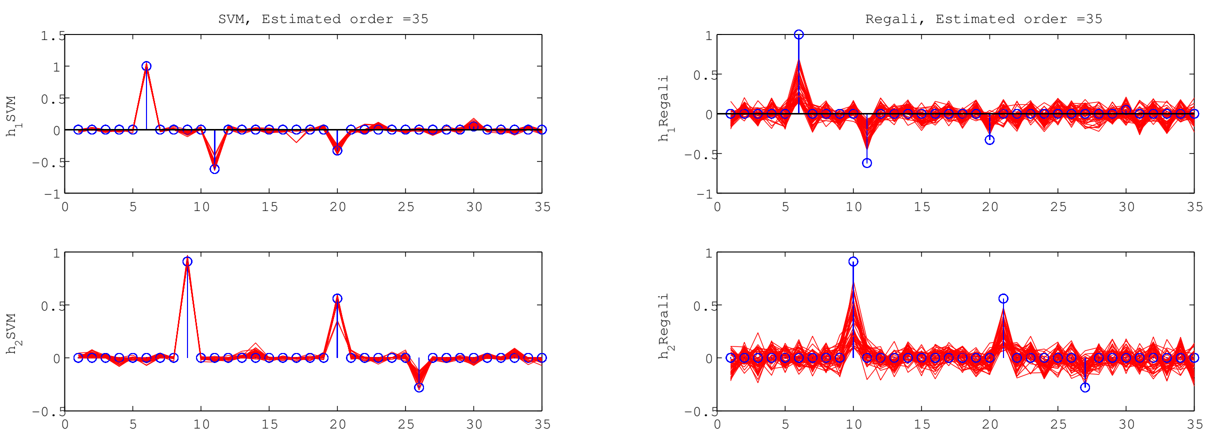


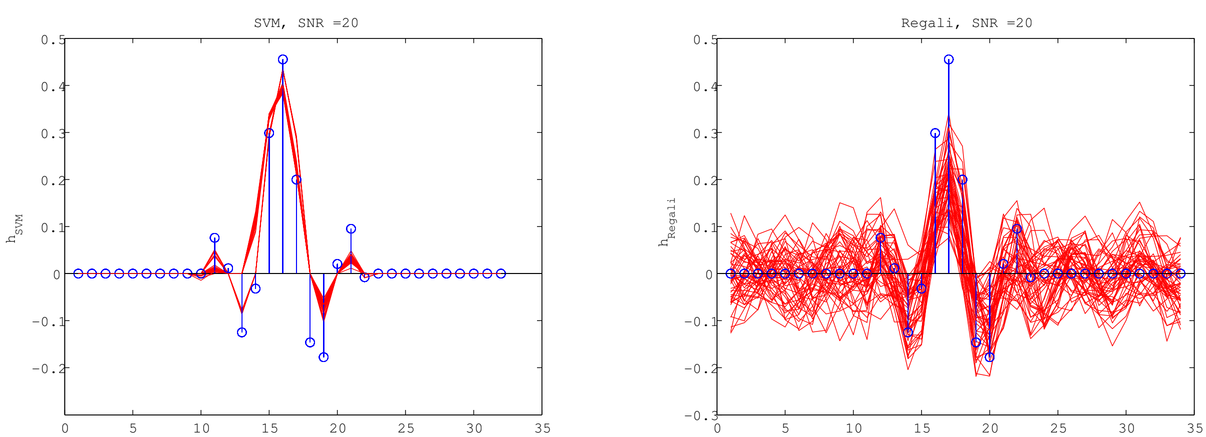
(a) Exact channel order (b) Overestimated channel order

Fig. 5. Comparison of robustness when the channel order is exact or overestimated by 10 taps



(a) SVM based method (b) Regalia method

Fig. 6. 50 trials of the SVM based method and Regalia method when overestimated by 10 taps and SNR=30dB



(a) SVM based method (b) Regalia method

Fig. 7. 50 trials of the SVM based method and Regalia method when overestimated by 20 taps and SNR=20dB