two extensions. The logarithmic shape is recognizable in the corresponding regular increase of the coefficient values.

TABLE 2: COEFFICIENTS OF THE PWL TERM AFTER ADAPTATION

L	a_k
2	-0.936 0.259 1.066 0.156
4	-0.847 0.194 0.557 0.823 1.034 0.262
8	-1.261 -0.084 0.208 0.432 0.615 0.769 0.904 1.021 1.128 0.298

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A LINEARIZED SEARCH TO REFINE FREQUENCY ESTIMATES

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Abstract

A simple iterative method to improve frequency estimates in sinusoid detection, based in linearizing the error equations, is presented. The efficiency of the method to reach better estimates in low noise cases is illustrated by means of simulation examples, starting from both low and high resolution approaches. A similar approach formulated in the autocorrelation domain is also presented and its performance evaluated. We will finish the contribution considering the possibility of introducing a Wiener filtering formulation to handle high noise situations, and discussing other open lines and further work.

1. Introduction.

The problem of estimating the frequencies of a collection of sinusoids buried in additive noise is one of the classical problems in Signal Processing: it is of fundamental importance in many practical applications, such as communications, radar and sonar. A lot of estimation methods and algorithms have been proposed to solve this problem, as can be found in standard texts [1] [2]. Among these algorithms, those based on the periodogram offer very limited resolution in solving close sinusoids. Other methods, as those relying in signal subspace - noise subspace separation, give higher resolution and better performance but paying in computational burden.

To have the opportunity of applying a straightforward method and to improve its results by means of an also not very complicated procedure is, then, an interesting alternative; and even better if the proposed refinement algorithm can also improve the results of more complex initially applied techniques.

In this paper, we propose a very simple procedure of the above suggested kind, based in assuming that we have a first approach to the result that is good enough to allow the linearization of the error equations, and then applying gradient algorithms to reestimate the present frequencies (as well as the new amplitudes). The resulting algorithm is remarkably simple and very efficient for low noise situations, and particulary useful for short data registers. We will provide practical details of this approach, verifying, by means of simulation examples, that this procedure improves both periodogram and high resolution algorithms (we have chosen the Root MUSIC algorithm to obtain the first estimate). The combination of a low computational complexity algorithm, as the periodogram, with the proposed method achieves even better estimation accuracy than a direct high resolution algorithm, saving in computational cost.

This paper is organized as follows. In Section 2 the problem of sinusoid detection is formulated and the proposed method for improving the initial estimates is decribed. Simulation results that show the performance of this approach are presented in Section 3. Some extensions of this method are discussed in Section 4, and a Wiener filter formulation is suggested to reach better estimates in high noise cases. Finally, some conclusions and future avenues will be presented in Section 5.

2. Problem Formulation and Proposed Method.

Assume that the data consist of p real sinusoids corrupted by noise

$$x[m] = \sum_{i=1}^{p} A_i \cos[\omega_i m + \phi_i] + n[m] \qquad m = 0 \ldots N-1 \qquad (1)$$

$n[m]$ being an additive white Gaussian noise with power σ^2 and zero mean. The frequencies are assumed to be constant but unknown, and must be estimated (if they are available, amplitudes and phases can be estimated via a matrix pseudoinversion, for example).

In the present paper we suppose that p,the number of sinusoids, is known. Methods to estimate p such as Akaike Information Criterion (AIC) or Minimun Description Length (MDL) could be applied to get a value of p.

The proposed method acts as follows: let $\{A_i^o \cos(\omega_i^o k + \phi_i^o)\}$ be the estimated sinusoids after applying a preliminary estimation: we will have

$$x[k] = \sum_{i=1}^{p} A_i^o \cos[(\omega_i^o + \Delta\omega_i^o)k + \phi_i^o + \Delta\phi_i^o] + n[k]$$

$\{\Delta\omega_i^o, \Delta\phi_i^o\}$ being unknow errors in the estimates (we disregard the effect of possible $\{\Delta A_i^o\}$ errors); assuming that these errors are low enough,

$$\cos[(\omega_i^o + \Delta\omega_i^o)k + \phi_i^o + \Delta\phi_i^o] = \cos(\omega_i^o k + \phi_i^o)\cos(\Delta\omega_i^o k + \Delta\phi_i^o) -$$

$$-\mathrm{sen}(\omega_i^o k + \phi_i^o)\mathrm{sen}(\Delta\omega_i^o k + \Delta\phi_i^o) \cong \cos(\omega_i^o k + \phi_i^o) - \mathrm{sen}(\omega_i^o k + \phi_i^o)(\Delta\omega_i^o k + \Delta\phi_i^o)$$

(for a better approximation, k must go from -(N-1)/2 to (N-1)/2: this change is irrelevant for the purpose of our discussion).

Let us call

$$y[k] = x[k] - \sum_{i=1}^{p} A_i^o \cos(\omega_i^o k + \phi_i^o)$$

using matrix notation, we arrive to

$$[kS \mid S]\begin{bmatrix}\Delta\omega \\ \text{---} \\ \Delta\phi\end{bmatrix} + n = y \qquad (2)$$

where all the elements are obvious; in particular, S is a Nxp matrix (N being the register length) having elements

$$S_{k,i} = -A_i^o \cos(\omega_i^o k + \phi_i^o)$$

Rewriting (2) as

$$M\delta + n = y \qquad (3)$$

for simplicity, it is clear that, if n is not too high, a pseudoinverse could be a reasonable solution for δ: to use one (or few) steps of a gradient algorithm could be even better in order to decrease the error in $\{\Delta\omega_i^o, \Delta\phi_i^o\}$; i.e., to apply

$$\delta(r+1) = \delta(r) + \mu M^T[y - M\delta(r)] \qquad (4)$$

starting from $\delta(0)=0$, until arriving to $\delta(R)$. After this, we can introduce

$$\omega_i^{(m+1)} = \omega_i^{(m)} + \Delta\omega_i^{(m)}(R)$$

$$\phi_i^{(m+1)} = \phi_i^{(m)} + \Delta\phi_i^{(m)}(R)$$

(m=0,1,...), iterating the process. In each step the amplitudes are reestimated (using a similar technique, for example).

The low computational complexity of this approach is obvious, even more if only one step in expression (4) is used (in this way, the method requires 2pN real multiplications per iteration); and the method also provides good performance as we show in Section 3 by means of some simulation examples (even with only one step).

3. Simulation Results.

In this section, we present some simulation results in order to evaluate the performance of this new approach, starting from both low resolution (periodogram) and high resolution (Root MUSIC) algorithms. Two signal scenarios consisting of two sinusoids with different frequency separation are considered; the SNR is defined as

$$\mathrm{SNR} = 10\log_{10}\left(\sum_{i=1}^{p}\frac{A_i^2}{2}\Big/\sigma^2\right) \qquad (\mathrm{dB})$$

In these simulations, a 25 point data record is generated 500 times at each SNR for the MonteCarlo experiments. The Root MUSIC method uses the eigendecomposition of a data covariance matrix of order M=12.

In the first experiment, we consider the example

$$x[k]=\cos(2\pi 0.3k)+\cos(2\pi 0.32k+\pi/4)+n[k] \qquad k=0...24$$

With a record length of 25, the periodogram resolution is 1/25=0.04, which is twice the frequency spacing of the sinusoids; so, a high resolution method is needed as a first estimate. The number of iterations empirically selected for the proposed method has been 15 and in each iteration one step LMS type algorithms are used to reestimate both the frequencies and the amplitudes. We have found that a parameter $\mu_1=10^{-4}$ to reestimate frequencies and $\mu_2=10^{-2}$ to reestimate amplitudes give a good tradeoff between robustness and speed of convergence.These parameters have demonstrated to be robust enough to ensure convergence for a wide number of examples. Nevertheless, some problems of convergence have been found when initial estimates are very close each other.

Figure 1 shows the performance of the method when applied to the above mentioned example (for comparison we have included also the Cramer-Rao bound [7] for unbiased estimators). In calculating the mean square error (MSE), the lower frequency estimate is assigned to f_1=0.30, and we assume that the estimates are unbiased, that is, the mean of the estimates are the true frequencies.

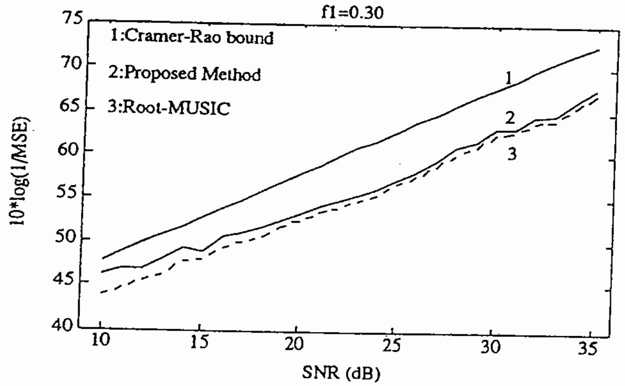

Figure1: Detection performance for Example 1

Two comments can be made. First at all, the iterative method appears to perform well for a low noise situation. Second, for SNR levels between 10 and 15 dB, some trials diverge; since we reject these cases to represent the corresponding curve in Figure 1, a bias appears in the frequency estimates, which becomes clear in Figure1. However, for higher SNR´s, this phenomenon disappears: Figure 2 shows the probability of divergence defined as

T=number of cases of divergence/total number of simulations

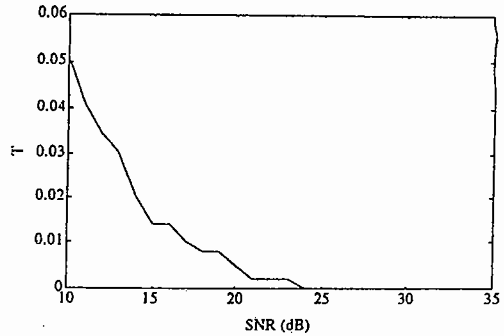

Figure 2: Estimated probability of divergence

Results obtained for f_2 are very similar. Table1 shows the statistics (mean, standard deviate) for SNR= 10, 20, 30 dB.

Table 1.a

SNR	f1 (σ_1)	f2 (σ_2)
10dB	0.2979 (0.0072)	0.3213 (0.0069)
20dB	0.2997 (0.0023)	0.3204 (0.0024)
30dB	0.3000 (8.7*e-4)	0.3200 (8.8*e-4)

Table 1.b

SNR	f1 (σ_1)	f2 (σ_2)
10dB	0.2983 (0.0056)	0.3213 (0.0056)
20dB	0.2999 (0.0021)	0.3203 (0.0022)
30dB	0.3000 (8.1*e-4)	0.3200 (8.5*e-4)

Statistics from 500 simulations for the first example. Mean and the rms (in brackets): 1.a) Root-MUSIC 1.b) Proposed Method

For the second experiment we have selected

$$x[k]=\cos(2\pi 0.3k)+\cos(2\pi 0.34k+\pi/4)+n[k] \qquad k=0...24$$

In this example the frequency spacing is higher: a Fourier based method can be used to reach a first estimate. An 1024 point FFT is used to compute the periodogram, the frequency-bin space being smaller than 0.001, which usually is accurate enough for most applications. The frequency estimates are chosen as the two largest samples of the FFT. For the selected case, the sinusoids are not orthogonal in the observation interval: due to this fact, the first estimation (that obtained with the periodogram) is strongly biased: we will see that the proposed algorithm works in this case reducing this large bias, although increasing the variance (the grid established by the FFT produces a very small variance in the estimates).

The results obtained applying (60 iterations) our method are illustrated in Figure 3, that shows how the bias is the dominant error in the MSE for the first estimate obtained with the periodogram: that is the reason because the MSE for the initial estimate remains near constant. The iterative method in this case tends towards alleviating the bias, producing a nearly efficient estimator for some SNR values.

Statistics (mean, standard deviate) for 5,10 and 20 dB are presented in Table 2. Results obtained for f_1=0.30 are very similar to these.

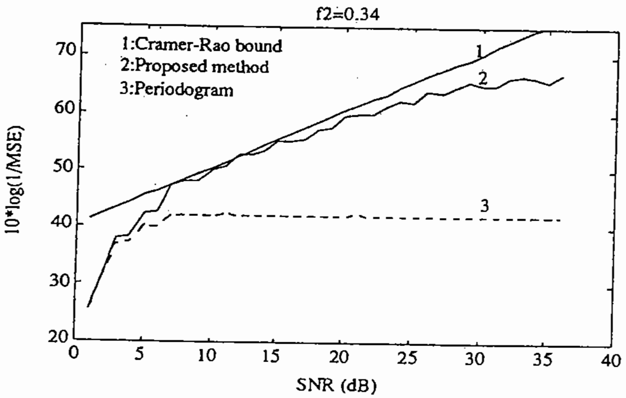

Figure3: Detection performance for Example 2

Table 2.a

SNR	f_1 (σ_1)	f_2 (σ_2)
5dB	0.2935 (0.0023)	0.3476 (0.0022)
10dB	0.2936 (0.0013)	0.3476 (0.0012)
20dB	0.2937 (3.5*e-4)	0.3477 (0)

Table 2.b

SNR	f_1 (σ_1)	f_2 (σ_2)
5dB	0.2990 (0.0047)	0.3411 (0.0050)
10dB	0.2995 (0.0028)	0.3405 (0.0030)
20dB	0.2996 (9.7*e-4)	0.3405 (9.5*e-4)

Statistics from 500 simulations for the second example. Mean and the rms (in brackets): 2.a) Periodogram 2.b) Proposed Method

Figure 4 shows the improvement obtained with the proposed method (starting from the periodogram), and compares its results with those of a Root-MUSIC, for the second example (from 10 to 30 dB). In Table 3 the results obtained with the Root-MUSIC are presented in order to compare them with those presented in Table 2.b. It is clear that the proposed algorithm provides a better performance: in spite of the 60 iterations needed to reach a good estimate, the computational burden of the proposed method is much lower than that of Root-MUSIC.

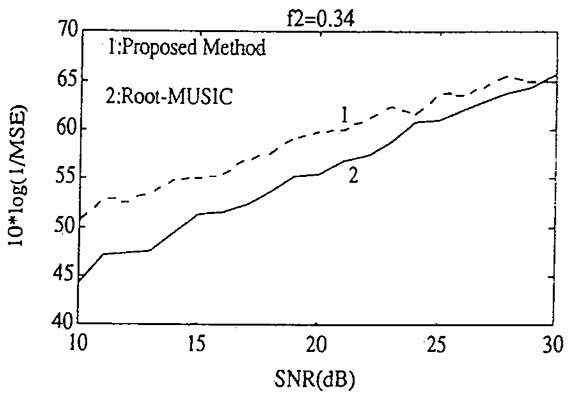

Figure 4: Comparison detection performance for Example 2

Table 3

SNR	f1 (σ_1)	f2 (σ_2)
5dB	0.2987 (0.0074)	0.3410 (0.0081)
10dB	0.3001 (0.0041)	0.3397 (0.0043)
20dB	0.3000 (0.0016)	0.3400 (0.0016)

Statistics from 500 simulations for the second example. Mean and the rms (in brackets) of the estimates obtained with the Root-MUSIC

4. Some Extensions and Improvements of the Algorithm.

The same linearization procedure can be used in the autocorrelation domain. Following the same steps as those detailed in Section 2, we arrive to a very similar expression

$$[kS]\Delta\omega = y \qquad (5)$$

y being a column vector of dimension T with elements

$$y[k] = \sum_{i=1}^{p} \frac{(A_i^0)^2}{2} \cos\left[\omega_i^0 k\right] \; - \; r_{xx}[k] \qquad k = 1\ldots T$$

and S, a Txp matrix with elements

$$S_{k,i} = \frac{(A_i^0)^2}{2} \mathrm{sen}\left[\omega_i^0 k\right]$$

For simplicity we can rewrite (5) as

$$M\delta = y$$

and now, we can apply the above described procedure.

The effect of the noise is alleviated by rejecting the term $r_{xx}[0]$. On the other side, the number of estimates T of the autocorrelation function has to be carefully selected to get accurate estimates of r_{xx}, since a bad estimate in r_{xx} degrades the method.

This approach has been tested against that working in the time domain, showing that it requires longer data lengths (to get good estimates of r_{xx}). This approach has proved to be less robust, and critical with respect to the selection of T (the number of samples of r_{xx}).

One of the main drawbacks of the gradient type algorithm proposed in Section 2 is its degraded performance in high noise situations. To apply an iterative version of the Wiener filter seems a simple and straightforward possibility to handle these cases. These techniques have

been extensively applied to the problem of bandlimited signal extrapolation (all of them being a generalization of the algorithm proposed in [3]).

The equivalence between the problem of harmonic extrapolation and sinusoid detection allows us to apply an identical approach. From this point of view, we can rewrite the problem of sinusoid detection as

$$Bz + n = x$$

where z is the data register without noise and B is the band defining operator

$$B = F^{-1}PF$$

here, F is a DFT matrix, P is a selection matrix (having only some nonzero values, just unity, at its diagonal: those corresponding to the present frequencies). We can try to estimate z by means of a Wiener filtering of x

$$\hat{z} = Wx$$

where W is selected to minimize the mean square value of

$$e = Bz - \hat{z} = Bz - Wx$$

the Orthogonality Principle leads to the Wiener solution given by

$$W = E\left[Bzx^H\right]E^{-1}\left[xx^H\right]$$

E indicates mean value and ^{H} Hermitian transpose. Writing B as before, and accepting that spectral lines are uncorrelated, we arrive to

$$\hat{z} = F^{-1}PF\left[F^{-1}PF + E\left[nn^H\right]\right]^{-1} x \qquad (6)$$

In practical situations, P is unknow: therefore we need to determine it in solving the equation. That is accomplished using the Wiener filter in combination with the iterative bandwith thresholded technique [4] [5] [6]. In this way, we can use (introducing the spectral vector Z=Fz)

$$P_n = \mathrm{diag}\left\{T_n\left[|Z_n|^2\right]\right\} \qquad (7)$$

where the subindex n refers to iteration and T_n is the iterative threshold introduced in [3]. If we use a frequency version, we can rewrite (6) in the form

$$Z_{n+1} = \mathrm{diag}\left\{T_n\left[|Z_n|^2\right]\right\}F\left[F^{-1}\mathrm{diag}\left\{T_n\left[|Z_n|^2\right]\right\}F + E\left[nn^H\right]\right]^{-1} x \qquad (8)$$

This approach avoids the need for assuming a priori knowledge of the number of sinusoids. Once the filtered version of x is obtained, we can apply directly the proposed method of this paper refining the estimates. This method achieves better results in high noise situations and, therefore, can be used as a first estimate in these cases. However, more extensive simulations are needed before presenting conclusions.

5. Conclusions.

Based on linearizing the error equations starting from a first estimate, a simple method to improve frequency estimates in sinusoid detection is presented. The resulting algorithm is simple and useful for short data registers and low noise situations. It is shown by simulations that the method improves slightly the estimates obtained with a high resolution method as Root-MUSIC, for low noise cases. However, this approach is more useful for methods that impose a pre-specification of possible frequencies, such as periodogram and band-limited extrapolation. The combination of a low computational complexity algorithm, as the periodogram, with the proposed method achieves lower variance than a high resolution algorithm.

Finally, to avoid the effect of noise, a Wiener filtered version of the data is proposed as a first estimate. This approach achieves better results in high noise situations.

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DATA DEPENDENT FILTERING APPLIED TO DUAL FILTER DETECTORS FOR FREQUENCY ESTIMATION IN DIGITAL COMMUNICATIONS

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Abstract: *This paper approaches the problem of frequency estimation in environments with large and non-stationary Doppler. Data-dependent techniques based on the adaptive GSLC concept are applied to the Dual Filter Detector scheme to improve its performance in terms of sensibility to the noise, image spectrum and interferences.*

1.- INTRODUCTION.

The estimation of the frequency error in digital communication is a process prior to any other synchronism and parameter recovery step. This problem is specially important in polar or sunsynchronous orbit satellites, where both the Doppler shift and the Doppler rate can have very stringent specifications.

Several techniques have been described for frequency estimation, but most of them cannot cope with frequency errors greater than the symbol rate. Among the techniques available one seems well suited for error frequency detection and correction: the one based in the Dual Filters Detectors first introduced by Alberty and Hespelt [1].

This technique applies a closed loop architecture for frequency correction: a frequency error detector (the DFD) generates an error signal (e) which is weighted to input an NCO in a feedback loop to bring the signal to the desired frequency:

$$f_{NCO} = f_{NCO} + \mu e \qquad (1)$$

In this paper, a new scheme based in the DFD's is suggested. The error function and the filter design are modified to improve the performance of the system.

2.- THE DUAL FILTER DETECTOR.

The Dual Filter Detector or DFD is a non-linear algorithm that follows the block diagram of fig.1. It consists of two filters $\underline{w}_1$ and $\underline{w}_2$; one operating at frequencies higher than the desired central frequency and the other one operating at frequencies below it. Both filters have mirror frequency responses with respect to the central frequency f_0.

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