Time-Variant and Selective Channels

Procesado de Señal y Comunicaciones

Curso 2023-2024

UNIVERSIDAD DE CANTABRIA

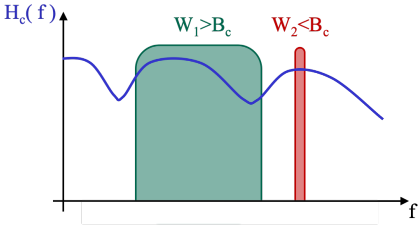
Contents

Time Domain

- **Does the channel change with time?**:
  - **No**: Invariant channel, stationary
  - **Yes**: Time-Variant Channel. Non stationary, fading effect
  - **When can we consider the channel as Time-Invariant?**
    - When the Coherence Time (T_c) is greater than the frame period
    - Therefore we need $T_c > KT_s$ (K symbols of period T_s in a frame)

Frequency Domain

- **What about its frequency response?**:
  - **Flat**: Not Frequency Selective, no dispersive, without memory
  - **Non Flat**: Frequency Selective Channel, dispersive, with memory
  - **When can we consider the channel as Flat?**
    - When the Coherence Bandwidth (B_c) is greater than the signal bandwidth (W)
    - Therefore we need $B_c > W$



Fundamental Trade-off:

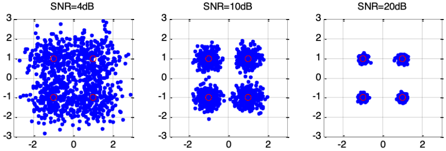
$$W \propto R_s = \frac{1}{T}$$

Ideal Channel

- Flat and Time-Invariant channel
  - Its effect reduces to a constant attenuation and delay/phase-change
  - It does not introduce ISI (Inter-symbol Interference)
- Ideal Discrete Equivalent Channel

$$z[n] = s[n] * h[n] = h \cdot s[n-d]$$

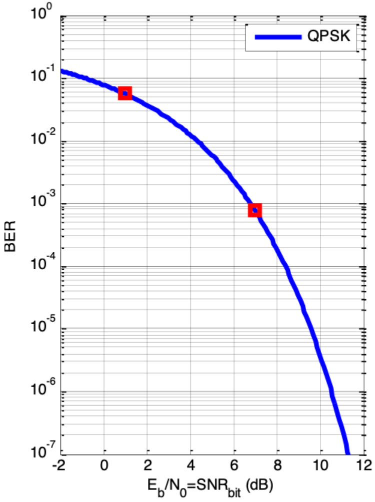
- After perfect time synchronization, phase recovery, and automatic gain control (AGC): $z[n] = s[n] + r[n]$



Bit Error Rate

$$\frac{E_b}{N_0} = \frac{\text{SNR}}{\log_2 M} \overset{\text{QPSK}}{=} \frac{\text{SNR}}{2}$$

$$P_e(\text{bit}) \overset{\text{QPSK}}{=} \mathcal{Q}\left(\sqrt{\frac{2E_b}{N_0}}\right) = \mathcal{Q}(\sqrt{\text{SNR}})$$



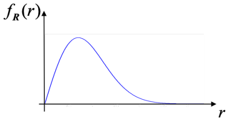
Flat Channel with Fading

- The channel changes with time
- We consider:
  - Flat Fading Channel
  - AWGN
  - Perfect processing at the receiver (AGC, synchronization, etc)
- Channel Effects:
  - Attenuation and delay/phase-change varying with time
  - It does not introduce ISI

Channel Model: Rayleigh

- Received signal: $z[n] = Hs[n] + r[n]$
  - H is a complex random variable representing the channel response
- Rayleigh Channel
  - $H \sim \mathcal{CN}(0, \sigma_r^2)$
  - Amplitude gain: $|H|$ follows a Rayleigh distribution:

$$f_R(r) = \frac{2r}{\sigma_r^2} e^{-r^2/\sigma_r^2} \qquad r \geq 0$$



  - Power gain: $|H|^2$ follows an exponential distribution
  - Typical for Non Line of Sight (NLOS) wireless environments

Channel Model: Rice

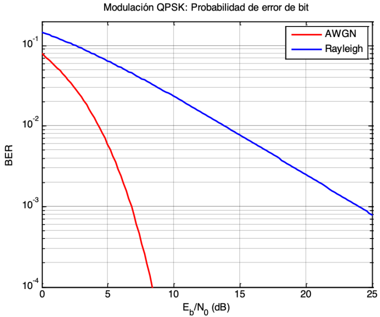
- Received signal: $z[n] = Hs[n] + r[n]$
  - H is a complex random variable representing the channel response
- Rice Channel
  - $H \sim \mathcal{CN}(\alpha, \sigma_r^2)$
  - Rice factor: $K_{\text{rice}} = \frac{\alpha^2}{\sigma_r^2}$
    - Equivalent to Rayleigh channel for $K_{\text{rice}} = 0$
    - Equivalent to AWGN (no fading) channel for $K_{\text{rice}} = \infty$
  - Typical for Line of Sight (LOS) wireless environments

BER in Rayleigh Channels

- Received signal: $z[n] = Hs[n] + r[n]$
- For each channel realization (fixed H) we have a different BER
- We need to average all the different BERs
- For Rayleigh channels, typical modulations, and not too low SNRs:

$$P_e(\text{bit}) \propto \text{SNR}^{-1}$$

- Much worse than the AWGN case
- Slope of the BER curve
- Effect of the coherence time (T_c) ?



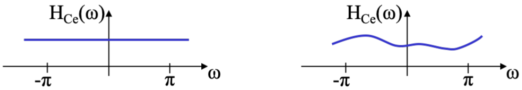
Frequency Selective Time Invariant Channels

- Causes for frequency selective channels
  - Multipath channel in radio communiactions
  - Multiple Reflections in cable transmissions
  - Very High Speed Communications
- We consider:
  - In band Frequency Selective Channel
  - Time-Invariant (stationary) channel
  - Noise AWGN
  - Perfect processing at the receiver (AGC, synchronization, etc)

Channel Effects

- Time-domain
  - Equivalent discrete channel with memory
  - Time-dispersion: The energy of each symbol is dispersed to others

- Frequency-domain
  - Frequency selectivity: The discrete equivalent channel is no flat



Intersymbol Interference (ISI) $\Rightarrow$ BER $\uparrow\uparrow$

Detection with ISI

- ML (Maximum Likelihood) receiver
  - Optimal Solution
  - Decoding of a sequence of L symbols: Viterbi algorithm
- Equalization
  - Suboptimal solution
  - Wiener Filter (MMSE equalizer)
    - LMS application if the channel varies slowly
    - Non-blind approach based on pilots
    - Blind (decision feedback equalizer or DFE) approach
- Other Approaches:
  - ISI-resistant modulations: OFDM