Signal Space Representation

Procesado de Señal en Comunicaciones Inalámbricas

Curso 2023-2024

UNIVERSIDAD DE CANTABRIA

Contents

Signal Space

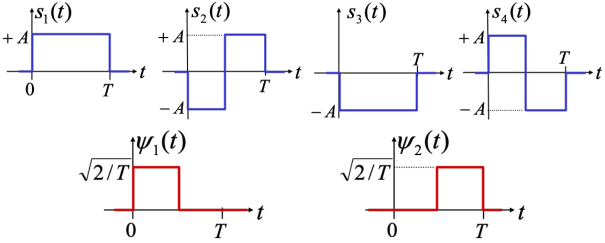
- **Internal product**: $\langle s_1(t), s_2(t)\rangle = \int_a^b s1(t)s_2^*(t)dt$
- **Orthogonality**: $s_1(t) \perp s_2(t) \Leftrightarrow \langle s_1(t), s_2(t)\rangle = 0$
- **Norm** ($\sqrt{\text{Energy}}$): $\|s(t)\| = \sqrt{\langle s(t), s(t)\rangle} = \left(\int_a^b |s(t)|^2 dt\right)^{1/2}$
- **Orthonormal Basis**: $\{\Psi_1(t), \Psi_2(t), \ldots, \Psi_N(t)\}$ orthonormal iff
  - Ortho: $\langle \Psi_i(t), \Psi_j(t)\rangle = 0, \qquad i \neq j$
  - Normal: $\|\Psi_i(t)\| = 1, \qquad \forall i$
- **Representation**: Signal as a vector in N-dimensional space

$$s(t) = \sum_{i=1}^{N} a_i \Psi_i(t) \qquad s(t) \equiv [a_1, a_2, \ldots, a_N]^T$$

Orthonormal Basis

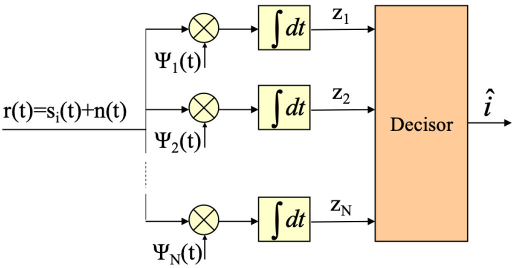
- Given a set of M signals: $\{s_1(t), s_2(t), \ldots, \mathbf{s}_M(t)\}$, they can be represented by means of an orthonormal basis $\{\Psi_1(t), \Psi_2(t), \ldots, \Psi_N(t)\}$ with dimension $N \leq M$
- We can now analyze, study and manipulate the signals in a N-dimensional vector space

$$s_k(t) = \sum_{i=1}^{N} a_{k,i}\Psi_i(t) \quad \equiv \quad [a_{k,1}, a_{k,2}, \ldots a_{k,N}]^T, \qquad k = 1, \ldots, M$$



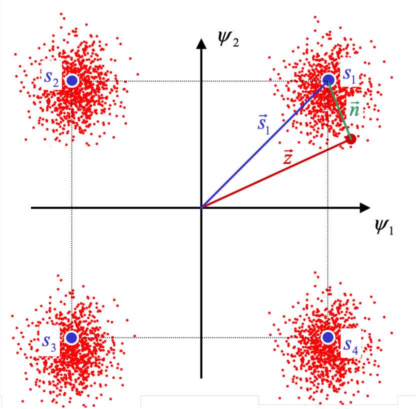
Optimal receiver for M symbols

- M equiprobable signals (symbols) in AWGN: $\{s_1(t), s_2(t), \ldots, \mathbf{s}_M(t)\}$
- Orthonormal basis $\{\Psi_1(t), \Psi_2(t), \ldots, \Psi_N(t)\}$ with $N \leq M$
- Receiver based on N correlators (matched filters for $\Psi_i(t)$)
- Decisor: Minimum Distance Criterion: $\hat{i} = \arg\min_k (\|\mathbf{s}_k - \mathbf{z}\|)$
  - Symbol representation: $\mathbf{s}_i = [a_{i,1}, a_{i,2}, \ldots, a_{i,N}]^T$
  - Noise representation $\mathbf{n} = [n_1, n_2, \ldots, n_N]^T$
  - Received Signal representation: $\mathbf{z} = [z_1, z_2, \ldots, z_N]^T = \mathbf{s}_i + \mathbf{n}$



Receiver Analysis

- Received signal: $\mathbf{z} = \mathbf{s}_i + \mathbf{n}$
  - $\mathbf{n}$ is a Gaussian random vector $\mathcal{N}(0, \sigma^2 \mathbf{I})$
  - Independent components with zero mean and variance $\sigma^2 = N_0/2$
  - If the symbol s_i is transmitted: $\mathbf{z}|\mathbf{s}_i \sim \mathcal{N}(\mathbf{s}_i, \sigma^2 \mathbf{I})$



Optimal Decoding

- Given the received signal ($\mathbf{z} = \mathbf{s}_i + \mathbf{n}$), we need to determine which symbol was transmitted
- This is a classification problem:
  - Minimum error criterion (MAP): $P(\mathbf{s}_i|\mathbf{z}) = \frac{f(\mathbf{z}|\mathbf{s}_i)P(\mathbf{s}_i)}{f(\mathbf{z})}$
  - For equiprobable symbols reduces to maximizing $f(\mathbf{z}|\mathbf{s}_i)$ (ML)
  - In the AWGN case, it further reduces to the minimum distance criterion: $\hat{i} = \arg\min_k (\|\mathbf{s}_k - \mathbf{z}\|)$
- The probability of error will depend on:
  - Distance among symbols: $d_{i,j} = \|\mathbf{s}_i - \mathbf{s}_j\|$
  - Noise variance: $\sigma^2 = N_0/2$
  - Mean Bit Energy: $E_b = E_s/\log_2(M)$
  - Symbol Energy: $E_{s_i} = \|\mathbf{s}_i\|^2$
  - Finally: $P_e = g(d, \sigma) = h(E_b, N_0)$

Analysis and Simulation of Communication Signals

- Complete System:
  - Includes Modulation, Amplifiers, Filters, Channel, Demodulation . . .
  - Sampling Frequency: $f_s \geq 2 f_{\max}$: Impractical in most of the cases
- Low-Pass Equivalent:
  - Analysis of the baseband signals
  - The effect of channel and TX/RX parts is modeled by their low-pass equivalent
  - Sampling Frequency moderate: In practice $f_s = R_s L$ with $L > 2$
- Equivalent Discrete System:
  - Analysis in Signal Space
  - Sampling Frequency: $f_s = R_s$ One sample per symbol

Consider WiFi: $f_c = 2.4GHz, W = 20MHz$