Orthogonal Frequency Division Multiplexing (OFDM)

Procesado de Señal en Comunicaciones Inalámbricas

Curso 2023-2024

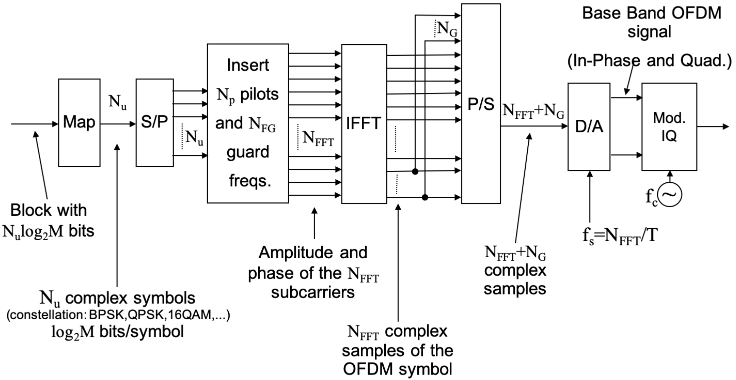
UNIVERSIDAD DE CANTABRIA

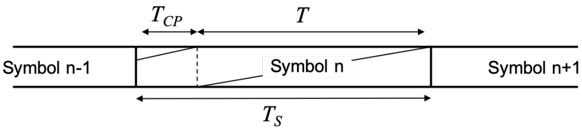
Contents

Inter-Symbol Interference

- We have already seen how to exploit diversity in order to combat fading (time-variant/selective channels)
- We still don't know how to combat ISI in frequency selective channels
- Under frequency selectivity, the discrete equivalent baseband signal model is $x[n] = h[n] * s[n] + r[n]$
  - A convolution in time domain is equivalent to a product in frequency domain (Isn't it?)
- Consider a block of N_c samples $[x[0], \ldots, x[N_c - 1]]$
  - $N_c/f_s \ll T_c$: The channel is time-invariant and frequency selective
  - $N_c/f_s \gg 1/B_c$: The block is much longer than the duration of the channel impulse response
  - What happens in the frequency domain?
  - In other words: How do we get a product in frequency domain?

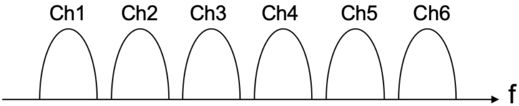
OFDM Block Diagram with Cyclic Prefix



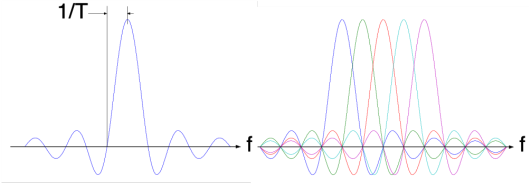


What are we doing?

- Traditional (analog) Frequency Multiplexing (filtering + guard frequencies)

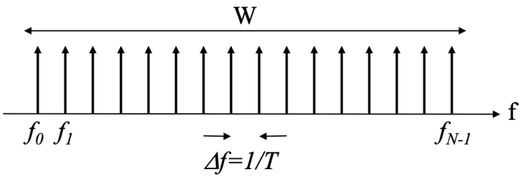


- OFDM: Overlapped Orthogonal Subcarriers
  - No ICI: Inter-Carrier Interference

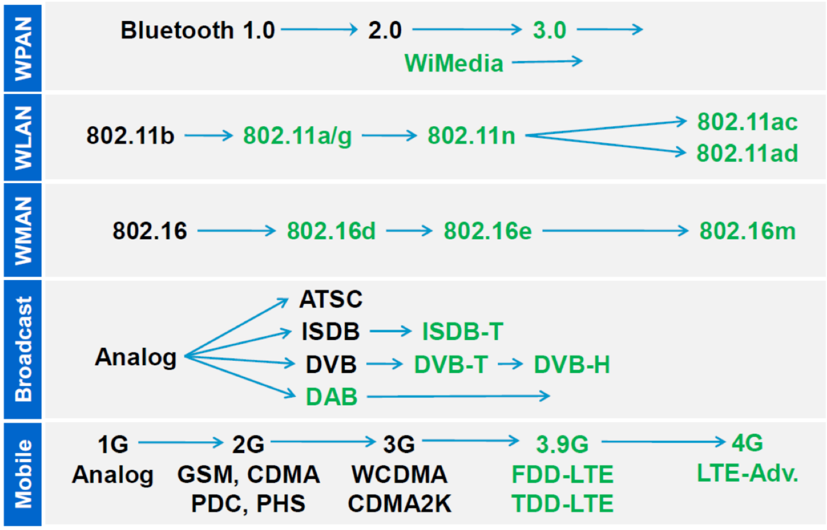


System Parameters

- Total bandwidth W divided into N_c subchannels
- Symbol and binary rates: R_s bauds, R_b bps
- Symbol and binary rate per subchannel: R_s bauds, R_b/N_c bps
- Duration of the OFDM symbol: $T = 1/R_s = \frac{N \log_2 M}{R_b}$
- N_c subcarriers with frequencies $f_k = f_0 + k/T$ for $k = 0, \ldots, N_c - 1$

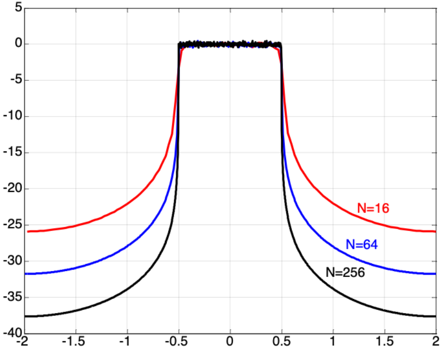


Standards: Evolution towards OFDM



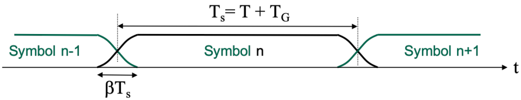
Out of Band Spectrum

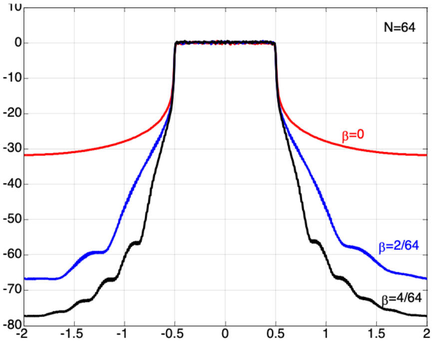
- Abrupt phase transitions at the extremes of the OFDM symbol
- Out of band PSD decays very slowly



Time Windowing

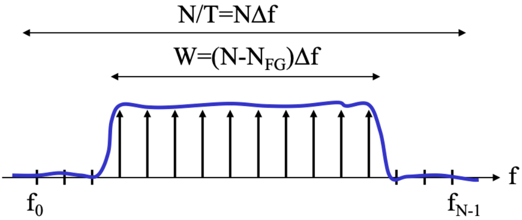
- Out of band PSD decays very slowly
- Solution: Time Windowing
  - At the cost of a small decrease of multipath inmunity: $\tau \leq T_G - \beta T_s$





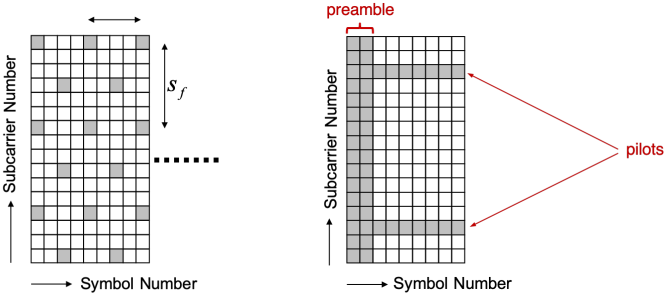
Guard Frequencies

- Out of band PSD decays very slowly
- Solution: N_{FG} guard subcarriers in each OFDM symbol
  - At the cost of a decrease in the effective binary rate



Channel Estimation and Synchronization

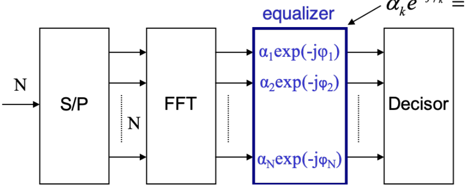
- The receiver needs to know the channel. We need N_p pilot subcarriers to estimate the channel
- Moreover, we need to track the channel in time and frequency
- Different strategies for continuous or frame transmissions
  - The number and spacing (in time and frequency) is directly related to T_c and B_c



Receiver

- The use of long symbol periods enables the use of a cyclic prefix
- The ISI problem is avoided. Each subcarrier has a flat channel
- Very simple channel estimation and equalization
- Simple combination with MIMO techniques in each subcarrier

$$\alpha_k e^{-j\varphi_k} = \frac{1}{\hat{H}_c(f_k)} = \frac{1}{\hat{r}_k e^{j\hat{\varphi}_k}}$$



Diversity

- The use of OFDM solves the ISI problem, each information symbol is sent over a subcarrier, which *sees* a flat channel
- The channel response will be better for some subcarriers and worse for others (frequency selectivity)
- We need to exploit the frequency diversity by distributing each information symbol among several subcarriers
- This problem is partially solved by Channel Coding / Error Correcting Codes
- Alternative: Linear Precoding of the information symbols
  - Instead of directly transmitting one information symbol over one subcarrier, we transmit a linear combination of information symbols
  - More details in the Live Script

PAPR

- With the standard formulation, the OFDM signal has a high PAPR
  - Peak to Average Power Ratio: $\text{PAPR} = \frac{\max(|x(t)|^2)}{E[|x(t)|^2]}$
  - In time domain, the OFDM signal is the combination of N_c independent symbols $\Rightarrow$ Gaussian Distribution
  - Amplitude of OFDM signal $\sim$ Rayleigh
  - Instantaneous power: $\sim$ Exponential
  - High peaks with high probability
    - Problems with AD's and DA's ranges
    - Problems with Power Amplifiers
- Solutions to reduce the PAPR:
  - Clipping: Spectral Regrowth and BER degradation
  - Back-off: Amplifiers in linear operation zone
  - Design of symbol sequences with low PAPR: Computationally complex, reduced R_b
  - Linear Precoding: Make the time samples look like symbols again (see Live Script)

Multiple Access

- The use of OFDM allows Multiple Access without the drawbacks of conventional FDMA systems
- OFDMA (Orthogonal Frequency Domain Multiple Access)
  - A predetermined subset of carriers is assigned to each user
- FH-OFDMA (Frequency Hopping Orthogonal Frequency Domain Multiple Access)
  - A subset of carriers is assigned to each user in each time slot
  - The assignment follows a predetermined pattern/code
  - In combination with channel coding or linear precoding of the symbols, provides frequency diversity

DAB

- DAB (Digital Audio Broadcast)

	Mode I	Mode II	Mode III	Mode IV
Number of Subcarriers (N)	1536	384	192	768
Subcarrier Spacing (Δf)	1 KHz	4 KHz	8 KHz	2 KHz
Symbol Period (T_s)	1.246 ms	311.5 μs	155.8 μs	623 μs
Guard Time (T_G)	246 μs	61.5 μs	30.8 μs	123 μs
Carrier (f_0)	$<$375 MHz	$<$1.5 GHz	$<$3 GHz	$<$1.5 GHz
TX Separation (f_0)	$<$96 Km	$<$24 Km	$<$12 Km	$<$48 Km

- DQPSK Modulation
- Binary Rate $R_b = 2N/T_s \simeq 2.47$ Mbps
- Bandwidth $W = N\Delta f \simeq 1.536$ MHz
- Convolutional Code (between 1/4 and 8/9)

DVB-T

- DVB-T (Digital Video Broadcast - Terrestrial)

	2k Mode I	8k Mode
Number of useful Subcarriers (N_u)	1705	6817
Subcarrier Spacing (Δf)	4.464 KHz	1.116 KHz
FFT Time (T)	224 μs	896 μs
Guard Time (T_G)	7-56 μs	28-224 μs
Guard Frequencies	171	687

- QPSK, 16QAM or 64QAM Modulation
- Binary Rate: From $R_b = 12.2$ Mbps (2k, QPSK) to $R_b = 44.3$ Mbps (8k, 64QAM)
- Reed-Solomon (204,188) and Convolutional (1/2, 2/3, 3/4, 5/6 or 7/8)

802.11 WiFi

- 802.11 a/g: WLAN standard with OFDM @ 2.4 Ghz (802.11 g) and 5 GHz (802.11 a)

Number of useful Subcarriers (N_u)	52
Subcarrier Spacing (Δf)	312.5 KHz
Symbol Period (T_s)	4 μs
Guard Time (T_G)	800 ns
Bandwidth	16.56 MHz

  - BPSK, QPSK, 16QAM or 64QAM Modulation
  - Binary Rate: From $R_b = 6$ Mbps to $R_b = 54$ Mbps (8k, 64QAM)
  - Convolutional Code (1/2, 2/3, 3/4)
- 802.11 n: MIMO 4x4 + channel aggregation (20/40 MHz) $\Rightarrow$ 600 Mbps
- 802.11 ac: MU-MIMO 8x8 + 256QAM + (40/80/160 MHz) $\Rightarrow$ 6.24 Gbps
- 802.11 ad @ 60GHz: SC/OFDM + 2.16 GHz + beamforming $\Rightarrow$ 6.76 Gbps

802.16 (WiMAX)

- WMAN standard with OFDMA @ 2.5 GHz, 3.5 GHz and 5.8 GHz

Number of Subcarriers (N)	128	512	1024	2048
Subcarrier Spacing (Δf)	10.94 KHz			
Symbol Period (T_s)	102.9 μs			
Guard Time (T_G)	11.4 μs			
Bandwidth (MHz)	1.25	5	10	20

  - MIMO with QPSK, 16QAM or 64QAM Modulation
  - Binary Rate up to 100 Mbps
  - Convolutional Code (1/2, 2/3, 3/4, 5/6)
  - Convolutional Turbo-Code (1/2, 2/3, 3/4, 5/6)
  - Frames with 1 preamble symbol for synchronization
  - Mobile WiMAX up to 120 Km/h (802.16e)