Diversity

Procesado de Señal en Comunicaciones Inalámbricas

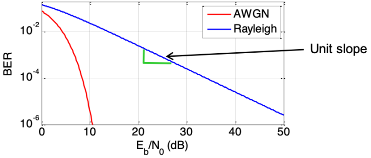
Curso 2023-2024

UNIVERSIDAD DE CANTABRIA

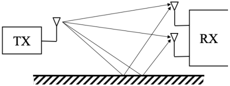
Contents

Fading and Diversity

- Even with the same average energy, the *variability* of the channel gain has dramatic effects
  - The worst channel realizations dominate the average BER



- If we take into account the channel variability, we can design TX/RX schemes mitigating the fading effect
  - Example: Two receive antennas



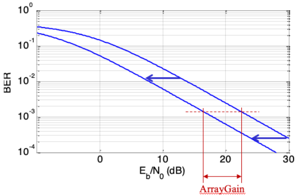
  - For not too close antennas, we have independent channels
  - Reduced probability of both channels being simultaneously bad
    - We can select the best antenna at RX: Antenna Selection

Exploiting Diversity

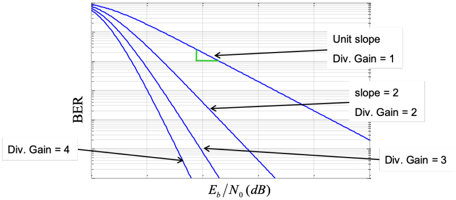
- **Key Idea**: TX/RX of information through several independent channel realizations
  - Application in wireless channel with multipath effects
  - Other systems: PLC (Power Line Communications), ADSL
- We can find independent channel realizations in different domains:
  - Spatial Diversity:
    - SIMO: Single Input Multiple Output. Several RX antennas
    - MISO: Multiple Input Single Output. Several TX antennas
    - MIMO: Multiple Input Multiple Output. Several TX and RX antennas
    - Practical coditions: Multipath channel with rich scattering. Isotropic antennas with $d \geq \lambda/2$ (5cm @ 3 GHz)
    - Smart processing needed !!
  - Fequency Diversity:
    - Use non-overlapping (separation $> B_c$) frequency bands
    - Information symbols *traveling* through all the frequency bands
  - Time Diversity:
    - Use different (separation $> T_c$) TX instants
    - Information symbols *traveling* through all the channel realizations

Key Metrics

- **Array Gain**: Increase in the average SNR with respect to the *original* case
  - The effect can be seen in channels with and without fading



- **Diversity Gain**: Increase in the slope of the BER (SER, outage probability) curve in fading channels



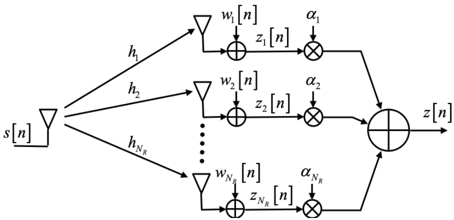
Key Metrics

- **Multiplexing Gain**: Increase in the number of information streams
  - In MIMO, the channel can be usually decomposed on several parallel channels. Therefore, we can transmit a different binary stream by each channel
  - There exists a tradeoff between diversity and multiplexing gain
- **Example**: Maximum Gain Values in MIMO Channels

	Array Gain	Diversity Gain	Multiplexing Gain
SISO (1x1)	1	1	1
SIMO (1xN_R)	N_R	N_R	1
MISO (N_Tx1)	N_T	N_T	1
MIMO (N_TxN_R)	$N_T N_R$	$N_T N_R$ (*)	min(N_T,N_R) (*)

(*) Not simultaneously achievable

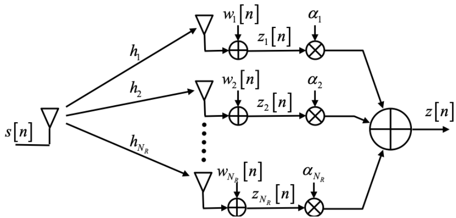
Discrete Equivalent Channel



$\mathbf{z}[n] = \mathbf{h}s[n] + \mathbf{r}[n]$

- $\mathbf{z}[n] = [z_1[n], z_2[n], \ldots, z_{N_R}[n]]^T$
- $\mathbf{h} = [h_1, h_2, \ldots, h_{N_R}]^T$ is the flat fading channel
- $s[n]$ is the information symbol
- $\mathbf{r}[n] = [r_1[n], r_2[n], \ldots, r_{N_R}[n]]^T$ is AWGN $\mathbf{r}[n] \sim \mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I})$
- In each receive branch: $\text{SNR}_i \propto \frac{|h_i|^2}{\sigma^2}$
- $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \ldots, \alpha_{N_R}]^T$ are the complex weights (coefficients) for the linear combination of the signals

Discrete Equivalent Channel



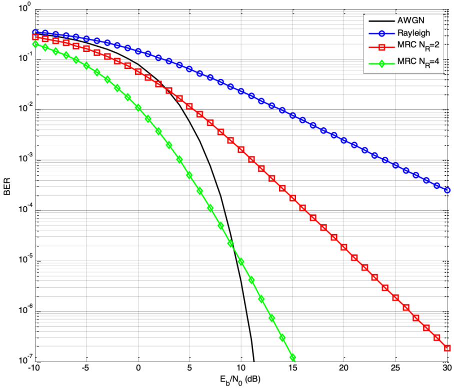
Optimal Receiver:

- $\mathrm{z[n]} = \boldsymbol{\alpha}^H \mathbf{z}[n]$
- The weights maximizing the SNR after combining are:
  - MRC (Maximum Ratio Combining): $\boldsymbol{\alpha} = \mathbf{h}/\|\mathbf{h}\|$
  - Requires CSIR (Channel State Information at the Receiver)
  - Equivalent SISO Channel: $z[n] = \|\mathbf{h}\| s[n] + r[n]$
  - The resulting SNR is $\mathrm{SNR} = \sum_{i=1}^{N_R} \mathrm{SNR}_i$
    - For i.i.d channels: $\mathrm{SNR} = N_R \mathrm{SNR}_i \quad \Rightarrow \quad$ Array Gain $= N_R$
  - Same concept and derivation as the matched filter

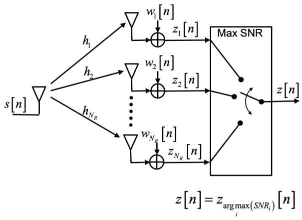
MRC: Diversity Gain in Rayleigh Channels

$$\text{Diversity Gain} = N_R$$

Rayleigh Channel and MRC. BPSK Modulation and N_R receive antennas



Other SIMO Techniques

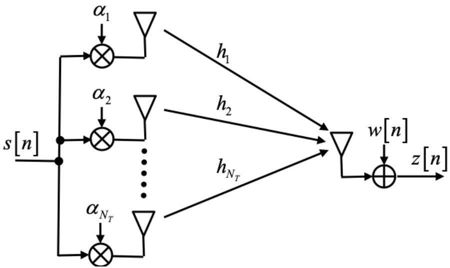


- Antenna Selection:
  - Simpler than MRC (only one branch)
  - Slightly worse performance than MRC
  - For i.i.d. Rayleigh channels:
    - Array Gain $= \sum_{i=1}^{N_R} \frac{1}{i}$
    - Diversity Gain $= N_R$
- Equal Gain Combining
  - Similar to MRC, but constant modulus weights (phase changes): $\alpha_i = e^{-j\phi_i}$
- Threshold Combining, Mixed Approaches, etc.

MISO Systems

- Multiple Transmit Antennas
  - Total power distributed among transmit antennas
  - Design and performance depend on CSIT (Channel State Information at the Transmitter)
  - How to obtain CSIT?
    - Channel Feedback: Channel estimated (pilots, preambles, blind techniques) at the RX and sent back to TX
    - Reciprocity: Semiduplex link with same carrier $\mathbf{H}_{TR} = \mathbf{H}^*_{RT}$
  - MISO with CSIT: MRC at TX
  - MISO without CSIT: Orthogonal Space-Time Block Codes (OSTBC, Alamouti)

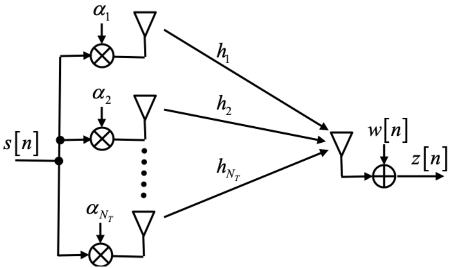
MISO Systems with CSIT



$z[n] = \boldsymbol{\alpha}^H \mathbf{h} s[n] + r[n]$

- $z[n]$ is the received signal
- $r[n] \sim \mathcal{CN}(0, \sigma^2)$
- $\mathbf{h} = [h_1, h_2, \ldots, h_{N_R}]^T$ is the flat fading channel
- $s[n]$ is the information symbol
- $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \ldots, \alpha_{N_R}]^T$ are the complex weights (coefficients) to be applied to the information signal

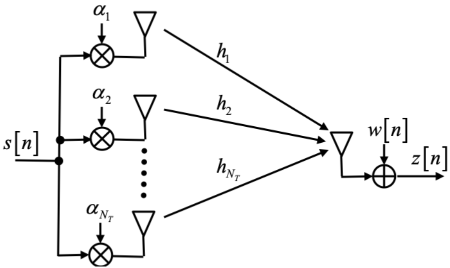
MISO Systems with CSIT



Optimal Receiver:

- Maximum SNR (minimum BER) for $\boldsymbol{\alpha} = \frac{\mathbf{h}}{\|\mathbf{h}\|}$
- MRT: Maximum Ratio Transmission (equivalent to MRC at TX)
- More power in the *best* antennas
- Phase changes to ensure coherent combination at RX

MISO Systems with CSIT

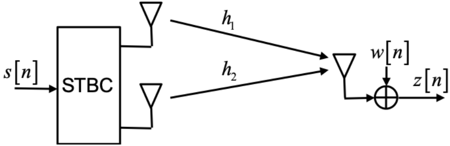


MRT Performance:

- Same as MRC: $\text{SNR}_{\text{MRT}} = \sum_{i=1}^{N_T} \text{SNR}_i$
- Array Gain (i.i.d. channel): N_T
- Diversity Gain (Rayleigh i.i.d. channel): N_T

MISO Systems without CSIT

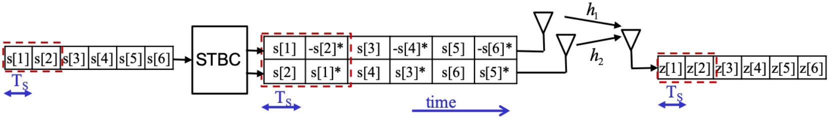
- If the channel is not available at the TX, we need some smart transmission scheme to ensure that each symbol travels through all the spatial channels
- Space-Time Block Coding (STBC)



  - Particular Case: Orthogonal STBC (OSTBC)
    - Simplest Detection: Each symbol is decoded independently of others
    - Optimal receiver based on MRC
    - Particular case: Alamouti Coding: $N_T = 2$. Only OSTBC with TX rate $R = 1$

MISO Systems without CSIT

- Alamouti Transmitter
  - Coding Matrix: $\mathbf{C}_{\text{Alamouti}} = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix}$



- Alamouti Receiver
  - Observations:

$$\mathbf{z} = \begin{bmatrix} z[1] \\ z^*[2] \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} s[1] \\ s[2] \end{bmatrix} + \begin{bmatrix} r[1] \\ r[2] \end{bmatrix} = \mathbf{Hs} + \mathbf{r}$$

  - Equivalent channel with orthogonal columns: $\mathbf{H}^H\mathbf{H} = \|\mathbf{h}\|^2 \mathbf{I}_2$
  - Multiplying the received vector by $\mathbf{H}^H$ yields (with $\mathbf{r}' = \mathbf{H}^H\mathbf{r}$):

$$\mathbf{y} = \begin{bmatrix} y[1] \\ y[2] \end{bmatrix} = \mathbf{H}^H\mathbf{z} = \|\mathbf{h}\|^2 \begin{bmatrix} s[1] \\ s[2] \end{bmatrix} + \begin{bmatrix} r'[1] \\ r'[2] \end{bmatrix}$$

MISO Systems without CSIT

- Alamouti Performance:
  - The channel knowledge is only required at the receiver (CSIR)
  - After scaling the observations: $\frac{\mathbf{y}}{||\mathbf{h}||^2} = \mathbf{s} + \mathbf{r}''$
    - Equivalent noise $\mathbf{r}'' \sim \mathcal{CN}(\mathbf{0}, \mathbf{C})$
    - Covariance matrix: $\mathbf{C} = \sigma^2 \mathbf{H}^H \mathbf{H} / ||\mathbf{h}||^2 = \sigma^2 \mathbf{I}_2$
    - The SNR does not change $\Rightarrow$ Array Gain $= 1$
  - Full diversity with i.i.d. Rayleigh channels:

$$\boxed{\text{Diversity Gain} = N_T N_R = 2N_R}$$

BER in MISO Systems

Rayleigh Channel and BPSK Modulation with $N_T = 2$ antennas