

SOLUCIONES HOJA DE PROBLEMAS 3

1. a)

$$F_Y(y) = \begin{cases} 0 & y < -b_2 \\ 1/8 & -b_2 \leq y < -b_1 \\ 1/2 & -b_1 \leq y < b_1 \\ 7/8 & b_1 \leq y < b_2 \\ 1 & y \geq b_2 \end{cases}$$

b)  $\varepsilon = \text{Var}[(Y - X)] = \frac{3}{4}b_1^2 + \frac{1}{4}b_2^2 - \frac{1}{3}b_1 - \frac{1}{3}b_2 + \frac{1}{6}$

c)

$$\left. \begin{aligned} \frac{\partial \varepsilon}{\partial b_1} &= 0 \\ \frac{\partial \varepsilon}{\partial b_2} &= 0 \end{aligned} \right\} \Rightarrow b_1 = \frac{2}{9}, b_2 = \frac{2}{3}$$

2. a)  $E[Z] = 3 - 4G(a/\sigma_0) = 0 \Rightarrow a \approx 0.67\sigma_0$

b)  $\text{Var}[Z] = E[Z^2] - E[Z]^2 \approx 1 - 0.47^2 = 0.78$

3. a)

$$f_Y(y) = \begin{cases} \frac{a-1}{2a}\delta(y+1) + \frac{1}{2a} + \frac{a-1}{2a}\delta(y-1) & y \in [-1, 1] \\ 0 & \text{resto} \end{cases}$$

b)  $P(|Y|=1) = P(\{Y=-1\} \cup \{Y=1\}) = P(Y=+1) + P(Y=-1) = (a-1)/a$

c)  $\sigma_X^2 = \frac{a^2}{3}, \sigma_Y^2 = \frac{3a-2}{3a} \Rightarrow \frac{\sigma_X^2}{\sigma_Y^2} = \frac{a^3}{3a-2}$

4. a)  $f_Y(y) = \frac{1}{\pi\sqrt{a^2 - y^2}}$  para  $|y| < a$     b)  $F_Y(y) = \begin{cases} 0 & y \leq -a \\ \frac{1}{2} + \frac{1}{\pi} \arcsen\left(\frac{y}{a}\right) & -a \leq y \leq a \\ 1 & y \geq a \end{cases}$

5. a)  $P(|I - \eta_I| < \varepsilon) \geq 1 - \frac{\sigma_I^2}{\varepsilon^2} = 0.9 \Rightarrow \varepsilon = \frac{\sigma_I}{\sqrt{0.1}} = 1.581i_0$  pues  $\eta_I = \frac{i_0}{2}(e-2)$  y  $\sigma_I^2 = i_0^2/4$   
por tanto el intervalo es  $[\eta_I - \varepsilon, \eta_I + \varepsilon] = [-1.22i_0, 1.94i_0]$

b)  $[0, 1.23i_0]$  pues  $F_I(i) = \begin{cases} 0 & i < 0 \\ \frac{1}{2} \left[ 1 + \ln\left(1 + \frac{i}{i_0}\right) \right] & 0 \leq i \leq i_0(e-1) \\ 1 & i > i_0(e-1) \end{cases}$

6. a)  $n=6000$  veces,  $p=1/6$ ,  $q=5/6$

$$\text{aprox. DeMoivre-Laplace } P(k_1 \leq \mathbf{X} \leq k_2) \approx G\left(\frac{k_2 - np + 0.5}{\sqrt{npq}}\right) - G\left(\frac{k_1 - np - 0.5}{\sqrt{npq}}\right)$$

$$\eta=np=1000, \sigma=(npq)^{1/2}=28.86 \Rightarrow P(980 \leq \mathbf{X} \leq 1005) \approx G(0.19) - G(-0.71) = 0.3364$$

$$\text{b) } P(\mathbf{X}=1005) = \binom{n}{1005} p^{1005} q^{n-1005} \approx \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(1005-\eta)^2}{2\sigma^2}} = 0.0136 \text{ con } \eta=np \text{ y } \sigma^2=npq$$