

SOLUCIONES HOJA DE PROBLEMAS 4

1. a) $c=1/210$

$$b) F_X(x) = \begin{cases} 0 & x < 2 \\ c(5x^2 + 12.5x - 45) & 2 < x < 6 \\ 1 & x > 6 \end{cases} \quad F_Y(y) = \begin{cases} 0 & y < 0 \\ c(2y^2 + 32y) & 0 < y < 5 \\ 1 & y > 5 \end{cases}$$

$$c) f_X(x) = \begin{cases} c(10x + 12.5) & 2 < x < 6 \\ 0 & \text{resto} \end{cases} \quad f_Y(y) = \begin{cases} c(4y + 32) & 0 < y < 5 \\ 0 & \text{resto} \end{cases}$$

d) $P(3 < X < 4, Y > 2) = 3/20$

e) $P(X > 3) = 23/28$

f) $P(X + Y > 4) = 33/35$

$$g) F_{XY}(x, y) = \begin{cases} 0 & x < 2 \text{ ó } y < 0 \\ c(x^2y + 0.5y^2x - 4y - y^2) & 2 < x < 6, 0 < y < 5 \\ c(5x^2 + 12.5x - 45) & 2 < x < 6, y > 5 \\ c(2y^2 + 32y) & x > 6, 0 < y < 5 \\ 1 & x > 6, y > 5 \end{cases}$$

h) No son independientes

2. b) $C_{XY} = pq(q-p)$

$$c) C_{XY} = 0 \Rightarrow \begin{cases} p = 0, q = 1 \\ q = 0, p = 1 \Rightarrow X, Z \text{ son independientes} \\ p = q = 1/2 \end{cases}$$

3.

$$F_{XY}(x, y) = \begin{cases} 0 & x < 0 \text{ ó } y < 0 \\ 2xy & (x, y) \in R \\ 2x - x^2 & 0 < x < 1, y > 1 \\ 2x - x^2 + 2y - y^2 - 1 & 0 < x < 1, 1 - x < y < 1 \\ 2y - y^2 & x > 1, 0 < y < 1 \\ 1 & x > 1, y > 1 \end{cases}$$

$$F_X(x) = F_{XY}(x, \infty) = \begin{cases} 0 & x < 0 \\ 2x - x^2 & 0 < x < 1 \\ 1 & x > 1 \end{cases}$$

$$F_Y(y) = F_{XY}(\infty, y) = \begin{cases} 0 & y < 0 \\ 2y - y^2 & 0 < y < 1 \\ 1 & y > 1 \end{cases}$$

4. a) \mathbf{X} v.a. uniforme en $[0, T] \Rightarrow f_X(x) = 1/T$ para $0 < x < T$
 $f_Y(y|x)$ uniforme en $[0, T-x] \Rightarrow f_Y(y|x) = 1/(T-x)$ para $0 < y < T-x$
 $f_{XY}(x, y) = f_X(x)f_Y(y|x)$
b) $E[\mathbf{XY}] = T^2/12$
 $E[\mathbf{X}] = T/2$
 $F_Y(y) = \ln(T/y)/T$ para $0 < y < T \Rightarrow E[\mathbf{Y}] = T/4$
 $E[\mathbf{XY}] \neq E[\mathbf{X}]E[\mathbf{Y}] \Rightarrow \mathbf{X}$ e \mathbf{Y} están correladas
c) $P(\mathbf{Y} < x) = \ln(2)$

5. a) $f_Y(y|x) = \begin{cases} \frac{1}{2x} & \text{para } -x < y < x \\ 0 & \text{resto} \end{cases}$ $f_{XY}(x, y) = f_X(x)f_Y(y|x) = \begin{cases} \frac{1}{2x} & \text{en } R \\ 0 & \text{resto} \end{cases} = \begin{cases} 0 < x < 1 \\ -x < y < x \end{cases}$
b) $E[\mathbf{XY}] = 0$
c) $f_Y(y) = \begin{cases} -\frac{1}{2} \ln(|y|) & \text{para } -1 < y < 1 \\ 0 & \text{resto} \end{cases}$
 $E[\mathbf{X}] = 1/2, E[\mathbf{Y}] = 0, E[\mathbf{XY}] = 0 \Rightarrow E[\mathbf{XY}] = E[\mathbf{X}]E[\mathbf{Y}] \Rightarrow \mathbf{X}, \mathbf{Y}$ incorreladas

6. a) $k = 2/\pi$
b) $f_X(x) = \begin{cases} \frac{2}{\pi} \sqrt{1-x^2} & |x| < 1 \\ 0 & \text{resto} \end{cases}$ $f_Y(y) = \begin{cases} \frac{4}{\pi} \sqrt{1-y^2} & 0 < y < 1 \\ 0 & \text{resto} \end{cases}$
 $f_{XY}(xy) \neq f_X(x)f_Y(y) \Rightarrow \mathbf{X}, \mathbf{Y}$ no son independientes
c) $E[\mathbf{XY}] = 0$
 $P(0.5 < x^2 + y^2 < 1) = 1/2$