Universidad de Cantabria
ETSII Telecomunicación
2º Curso del Grado en Ingeniería de Tecnologías de Telecomunicación
Métodos Matemáticos para Telecomunicaciones

SOLUCIONES HOJA DE PROBLEMAS 5

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1. a) K=4/(3ab), E[**XY**]=0, E[**X**]=0, E[**Y**]=0 $\Rightarrow$ **X**,**Y** incorreladas

b) $f_X(x)=\begin{cases}4/(3a) & -a/2<x<-a/4\\ 2/(3a) & -a/4<x<a/4\\ 4/(3a) & a/4<x<a/2\\ 0 & \text{resto}\end{cases}$ $\qquad f_Y(y)=\begin{cases}4/(3b) & -b/2<y<-b/4\\ 2/(3b) & -b/4<y<b/4\\ 4/(3b) & b/4<y<b/2\\ 0 & \text{resto}\end{cases}$

$f_{XY}(x,y)\neq f_X(x)f_Y(y) \Rightarrow$ **X** e **Y** no son Independientes

c) $C_{ZW}=\frac{1}{2}(E[\mathbf{X}^2]-E[\mathbf{Y}^2])\text{sen}(2\theta)$

si $E[\mathbf{X}^2]>E[\mathbf{Y}^2] \Rightarrow \theta=\pi/4$

si $E[\mathbf{X}^2]<E[\mathbf{Y}^2] \Rightarrow \theta=-\pi/4$

d) $C_{ZW}=0$ si $E[\mathbf{X}^2]=E[\mathbf{Y}^2]$

$E[\mathbf{X}^2]=5a^2/48$, $E[\mathbf{Y}^2]=5b^2/48 \Rightarrow a=b$

2. a) $E[\mathbf{UV}]=\cos\theta\,\text{sen}\theta(\sigma_X^2-\sigma_Y^2+\eta_X^2-\eta_Y^2)+(\cos^2\theta-\text{sen}^2\theta)(r\sigma_X\sigma_Y+\eta_X\eta_Y)$

$E[\mathbf{U}]E[\mathbf{V}]=\cos\theta\,\text{sen}\theta(\eta_X^2-\eta_Y^2)+(\cos^2\theta-\text{sen}^2\theta)(\eta_X\eta_Y)$

U,**V** Incorreladas $\Rightarrow E[\mathbf{UV}]=E[\mathbf{U}]E[\mathbf{V}] \Rightarrow \theta=\frac{1}{2}\text{arcotg}(2r\sigma_X\sigma_Y/(\sigma_Y^2-\sigma_X^2))$

b) **U** es $N(\eta_U,\sigma_U)$ por ser combinación lineal de gaussianas con:

$\eta_U=\eta_X\cos\theta-\eta_Y\text{sen}\theta$

$\sigma_U^2=\cos^2\theta\sigma_X^2+\text{sen}^2\theta\sigma_Y^2$

3. a) $f_U(u)=ue^{-u^2/2}$ para $0<u<\infty$ (Rayleigh)

$f_{UY}(u,y)=ue^{-u^2/2}$ para $\begin{cases}0<u<\infty\\ 0<y<1\end{cases}$

b) $f_{ZW}(z,w)=\dfrac{1}{2\pi}e^{-\frac{(z^2+w^2)}{2}}$ para $\begin{cases}-\infty<z<\infty\\ -\infty<w<\infty\end{cases}$

$f_Z(z)$ y $f_W(w)$ son Gaussianas

4. a) $\Omega_{XY}=\{(0,0),(0,1),(1,1),(1,2)\}$

P(**X**=0,**Y**=0)=q^2 , P(**X**=0,**Y**=1)=pq , P(**X**=1,**Y**=1)=pq , P(**X**=1,**Y**=2)=p^2

P(**X**=0)=q , P(**X**=1)=p

P(**Y**=0)=q^2 , P(**Y**=1)=2pq , P(**Y**=2)=p^2

X e **Y** No son Independientes

b) $\eta_X=p$, $\eta_Y=2p$, $\sigma_X^2=pq$, $\sigma_Y^2=2pq$, $E[\mathbf{XY}]=p(1+p) \Rightarrow r_{XY}=1/\sqrt{2}$

X e **Y** No están Incorreladas

$\hat{\mathbf{Y}}=\mathbf{X}+p$

c) $\hat{\mathbf{Y}}=E[\mathbf{Y}\,|\,\mathbf{X}=x]=\mathbf{X}+p$

5. a) a=6/7 , b=2/7 $\Rightarrow$ $\hat{\mathbf{Y}}_1 = \frac{6}{7}\mathbf{X} + \frac{2}{7}$

b) $\hat{\mathbf{Y}}_2 = \frac{\mathbf{X}\text{-}1}{\ln\mathbf{X}}$

c) $r_{X\hat{Y}_1} = \frac{a}{|a|}$, $C_{X\hat{Y}_2} = \frac{1}{24}$

6. a) $f_Z(z)=pN(0,\sigma_1)+ qN(0,\sigma_2)$

b) $\eta_Z=0$, $\sigma_Z^2=p\sigma_1^2+q\sigma_2^2$

c) Estimador Lineal: $\hat{\mathbf{Z}} = (1-p)\mathbf{X}_2$, Estimador sin restricciones: $\hat{\mathbf{Z}} = (1-p)\mathbf{X}_2$

7. a) E[**Z**]=E[**X**]+cE[**Y**]=90ms

X, **Y** indep. $\Rightarrow$ $\sigma_Z^2=\sigma_X^2+c^2\sigma_Y^2$ =58.3ms^2

$r_{XZ} = \sqrt{\frac{3}{7}}$

b) $f_Z(z) = \frac{1}{20}\left[G\left(\frac{z}{5}-16\right) - G\left(\frac{z}{5}-20\right)\right]$

c) Estimador Lineal: $\hat{\mathbf{Z}}$ = c**Y** + η_X = 0.5**Y** +40

Estimador sin restricciones: : $\hat{\mathbf{Z}}$ = c**Y** + η_X = 0.5**Y** +40