Formulario

Métodos Matemáticos para Telecomunicaciones

Grado en Ingeniería de Tecnologías de Telecomunicación
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Universidad
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Grupo de
Tratamiento Avanzado de Señal

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Combinatoria

$V_{n,r} = \frac{n!}{(n-r)!}$ $\quad VR_{n,r} = n^r$

$C_{n,r} = \binom{n}{r} = \frac{n!}{(n-r)!r!}$ $\quad CR_{n,r} = \binom{n+r-1}{r} = \frac{(n+r-1)!}{(n-1)!r!}$

$P_n = V_{n,n} = n!$ $\quad PR_r^{r_1,r_2,\ldots,r_n} = \frac{r!}{r_1!r_2!\ldots r_n!}$

Axiomas de Kolmogorov

Dado $< \Omega, \mathcal{F}, P >$ ligado a un experimento aleatorio:

i) $P(A) \geq 0, \qquad \forall A \in \mathcal{F}.$

ii) $P(\Omega) = 1.$

iii) $A, B \in \mathcal{F}, \quad A \cap B = \emptyset \quad \Rightarrow \quad P(A \cup B) = P(A) + P(B).$

Teoremas de la Multiplicación y de la Probabilidad Total

$$P(A \cap B) = P(A|B)P(B) = P(B|A)P(A) \text{ con } P(A) \neq 0, P(B) \neq 0$$

$$P(B) = \sum_{i=1}^{N} P(B|A_i)P(A_i) \text{ con } \{A_1, A_2, \ldots, A_N\} \text{ partición}$$

Distribución Uniforme: $x_1 < x_2$

$$f_X(x) = \begin{cases} \frac{1}{x_2 - x_1} & x_1 \leq x \leq x_2 \\ 0 & \text{resto} \end{cases} \qquad F_X(x) = \begin{cases} 0 & x < x_1 \\ \frac{x - x_1}{x_2 - x_1} & x_1 \leq x \leq x_2 \\ 1 & x > x_2 \end{cases}$$

$$E[X] = \frac{x_1 + x_2}{2} \qquad \sigma_X^2 = \frac{(x_2 - x_1)^2}{12}$$

Distribución Exponencial: $c > 0$

$$f_X(x) = \begin{cases} ce^{-cx} & x \geq 0 \\ 0 & x < 0 \end{cases} \qquad F_X(x) = \begin{cases} 1 - e^{-cx} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$E[X] = \frac{1}{c} \qquad \sigma_X^2 = \frac{1}{c^2}$$

Distribución Gaussiana o Normal: $\mathcal{N}(\eta, \sigma^2)$

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\eta)^2}{2\sigma^2}}, \quad -\infty < x < \infty$$

Distribución Bernoulli: p, $q = 1 - p$

$$f_X(x) = q\delta(x) + p\delta(x-1) \qquad F_X(x) = qu(x) + pu(x-1)$$

$$E[X] = p \qquad \sigma_X^2 = pq$$

Distribución Binomial: $B(n,p)$, $q = 1 - p$

$$f_X(x) = \sum_{k=0}^{n} \binom{n}{k} p^k q^{n-k} \delta(x-k) \qquad F_X(x) = \sum_{k=0}^{n} \binom{n}{k} p^k q^{n-k} u(x-k)$$

$$E[X] = np \qquad \sigma_X^2 = npq$$

Distribución Poisson: $a > 0$

$$f_X(x) = \sum_{k=0}^{\infty} e^{-a} \frac{a^k}{k!} \delta(x-k) \qquad F_X(x) = \sum_{k=0}^{\infty} e^{-a} \frac{a^k}{k!} u(x-k)$$

$$E[X] = a \qquad \sigma_X^2 = a$$

Teorema del Límite Central

Dado $Y = \sum_{i=1}^{N} X_i$ con $X_1, X_2, \ldots, X_N$ v.a. independientes, entonces

$$\lim_{N\to\infty} P(y_1 < Y \leq y_2) = \mathcal{G}\left(\frac{y_2 - \eta_Y}{\sigma_Y}\right) - \mathcal{G}\left(\frac{y_1 - \eta_Y}{\sigma_Y}\right)$$

con $\quad \eta_Y = \sum_{i=1}^{N} \eta_i \quad$ y $\quad \sigma_Y^2 = \sum_{i=1}^{N} \sigma_i^2$

Aproximaciones de v.a. Binomial

$$B(n,p) \approx \begin{cases} \mathcal{P}oisson(np) & \text{si} \quad n \gg 1, np < 5 \\ \mathcal{N}(np, npq) & \text{si} \quad n \gg 1, np \gg 1 \end{cases}$$

Ley de Los Grandes Números

n realizaciones indep. de exp. aleatorio. Suceso A con $P(A) = p$.

$$\lim_{n\to\infty} P(|f_A - p| \leq \epsilon) = \lim_{n\to\infty} 2\mathcal{G}\left(\frac{n\epsilon}{\sqrt{npq}}\right) - 1 = 1 \quad \forall \epsilon > 0$$

Teorema de la Multiplicación y Media de una Función de dos v.a.'s

$$f_{XY}(x,y) = f_X(x|y) f_Y(y) = f_Y(y|x) f_X(x)$$

$$E\left[g(X,Y)\right] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y) f_{XY}(x,y)dxdy$$

Teorema Fundamental: $Z = g(X,Y)$, $W = h(X,Y)$

i) $Z = g(X,Y)$ y $W = h(X,Y)$ no ctes. en Ω_{XY}.

ii) X e Y continuas.

$$f_{ZW}(z,w) = \sum_i \frac{f_{XY}(x_i,y_i)}{|J(x_i,y_i)|} \quad \text{con} \quad \begin{cases} J(x,y) = \begin{vmatrix} \frac{\delta z}{\delta x} & \frac{\delta z}{\delta y} \\ \frac{\delta w}{\delta x} & \frac{\delta w}{\delta y} \end{vmatrix} = \frac{1}{J(z,w)} \\ (x_i,y_i) \quad \text{raíces de} \begin{cases} z = g(x,y) \\ w = h(x,y) \end{cases} \end{cases}$$

Estimador Mediante Una Recta con Mínimo MSE

$$\hat{Y} = aX + b \quad \text{con} \quad \begin{cases} a_{\text{opt}} = r_{XY}\frac{\sigma_Y}{\sigma_X} = \frac{C_{XY}}{\sigma_X^2} = \frac{E[XY]-E[X]E[Y]}{E[X^2]-E[X]^2} \\ b_{\text{opt}} = E[Y] - a_{opt}E[X] \end{cases}$$

Relaciones Trigonométricas

$$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$$
$$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$$
$$\cos\left(x \pm \pi/2\right) = \mp \sin(x)$$
$$\sin\left(x \pm \pi/2\right) = \pm \cos(x)$$
$$\cos(2x) = \cos^2(x) - \sin^2(x)$$
$$\sin(2x) = 2\sin(x)\cos(x)$$
$$2\cos(x) = e^{jx} + e^{-jx}$$
$$2j\sin(x) = e^{jx} - e^{-jx}$$
$$2\cos(x)\cos(y) = \cos(x-y) + \cos(x+y)$$
$$2\sin(x)\sin(y) = \cos(x-y) - \cos(x+y)$$
$$2\sin(x)\cos(y) = \sin(x-y) + \sin(x+y)$$
$$2\cos^2(x) = 1 + \cos(2x)$$
$$2\sin^2(x) = 1 - \cos(2x)$$

Integrales Indefinidas: Funciones Algebráicas Racionales

$$\int (a+bx)^n dx = \frac{(a+bx)^{n+1}}{b(n+1)} \quad n>0$$

$$\int \frac{dx}{a+bx} = \frac{1}{b}\ln(|a+bx|)$$

$$\int \frac{xdx}{a+bx} = \frac{-1}{b^2}(a\ln(|a+bx|)-bx)$$

$$\int \frac{dx}{(a+bx)^n} = \frac{-1}{(n-1)b(a+bx)^{n-1}} \quad n>1$$

$$\int \frac{dx}{a^2+b^2x^2} = \frac{1}{ab}\tan^{-1}\left(\frac{bx}{a}\right)$$

$$\int \frac{xdx}{a^2+x^2} = \frac{1}{2}\ln(a^2+x^2)$$

$$\int \frac{x^2dx}{a^2+x^2} = x - a\tan^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right)$$

Integrales Indefinidas: Funciones Trigonométricas

$$\int \cos(x)dx = \sin(x)$$

$$\int x\cos(x)dx = \cos(x) + x\sin(x)$$

$$\int x^2\cos(x)dx = 2x\cos(x) + (x^2 - 2)\sin(x)$$

$$\int \sin(x)dx = -\cos(x)$$

$$\int x\sin(x)dx = \sin(x) - x\cos(x)$$

$$\int x^2\sin(x)dx = 2x\sin(x) - (x^2 - 2)\cos(x)$$

Integrales Indefinidas: Funciones Exponenciales y Logarítmicas

$$\int e^{ax}dx = \frac{e^{ax}}{a}$$

$$\int xe^{ax}dx = e^{ax}\left[\frac{x}{a} - \frac{1}{a^2}\right]$$

$$\int x^2e^{ax}dx = e^{ax}\left[\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right]$$

$$\int x^3e^{ax}dx = e^{ax}\left[\frac{x^3}{a} - \frac{3x^2}{a^2} + \frac{6x}{a^3} - \frac{6}{a^4}\right]$$

$$\int e^{ax}\sin(x)dx = \frac{e^{ax}}{a^2+1a}\left[a\sin(x) - \cos(x)\right]$$

$$\int e^{ax}\cos(x)dx = \frac{e^{ax}}{a^2+1a}\left[a\cos(x) + \sin(x)\right]$$

$$\int \ln(x)dx = x\ln(x) - x$$

$$\int x\ln(x)dx = \frac{x^2\ln(x)}{2} - \frac{x^2}{4}$$

Series

$$\sum_{n=1}^{N} n = N(N+1)/2$$

$$\sum_{n=1}^{N} n^2 = N(N+1)(2N+1)/6$$

$$\sum_{n=0}^{N} \binom{N}{n} = \sum_{n=0}^{N} \frac{N!}{n!(N-n)!} = 2^N$$

$$\sum_{n=0}^{N} \binom{N}{n} a^n b^{N-n} = (a+b)^N$$

$$\sum_{n=k}^{\infty} r^n = \frac{r^k}{1-r} \qquad 0 < r < 1$$

$$\sum_{n=0}^{\infty} \frac{a^n}{n!} = 1 + a + \frac{a^2}{2!} + \frac{a^3}{3!} + \cdots = e^a$$